in series with a rectifier, for which we assume a relation

$$V_A - V_B = f = ri + bi^2 \tag{3}$$

between the difference of potential $f$ and the current $i$.

At a temperature $T$, electromotive forces $\varphi$ will be generated in the circuit, for the frequency band $d\nu$, and may contribute to the d.c. electromotive force. We assume

$$\begin{gathered}\varphi = \varphi_0 + \varphi_\nu \\ \langle\varphi\rangle_{\mathrm{Av}} = \varphi_0 \quad \langle\varphi_\nu\rangle_{\mathrm{Av}} = 0 \quad \langle\varphi_\nu{}^2\rangle_{\mathrm{Av}} = 0,\end{gathered} \tag{4}$$

thus splitting the d.c. component of $\varphi_0$ from the oscillating term $\varphi_\nu$.

These electromotive forces produce a current $i$ with a d.c. component $i_0$ and an oscillating component $i_\nu$

$$\varphi_\nu = (R + r + jX) i_\nu \tag{5}$$

and

$$\langle\varphi\rangle_{\mathrm{Av}} = \varphi_0 = (R+r) i_0 + b\langle i_\nu{}^2\rangle_{\mathrm{Av}} \tag{6}$$

assuming $X=0$ for direct currents.

The second principle of thermodynamics requires that $i_0$ be zero on the average, in order to give no average potential difference, $V_A - V_B$, so that

$$\langle i_0\rangle_{\mathrm{Av}} = 0$$

and Eq. (5) yields, after averaging,

$$\langle\varphi\rangle_{\mathrm{Av}} = \varphi_0 = b\langle i^2\rangle_{\mathrm{Av}}. \tag{7}$$

This means that the d.c. component of the fluctuation exactly cancels the rectified voltage due to the thermal oscillating current. Furthermore, the Nyquist formula (1) must apply to the $r$ term of the rectifier [Eq. (3)] as well as to the resistor $R$. Suppose for a moment that the thermal noise in the rectifier is stronger than in a resistance of similar $r$-value. This would mean that the noise from the rectifier should heat up the resistor $R$ and produce, after a while, a difference in temperature between rectifier and resistor; this would again be a contradiction of the second principle. This proves that

$$\langle\varphi_\nu{}^2\rangle_{\mathrm{Av}} = 4(R+r)kTd\nu, \tag{8}$$

a formula similar to (1), applied to the whole circuit. Hence

$$\langle i_\nu{}^2\rangle_{\mathrm{Av}} = \frac{\langle\varphi_\nu{}^2\rangle_{\mathrm{Av}}}{(R+r)^2+X^2} = 4\frac{R+r}{(R+r)^2+X^2}kTd\nu \tag{9}$$

and finally with the help of Eqs. (7) and (9)

$$\varphi_0 = b\langle i_\nu{}^2\rangle_{\mathrm{Av}} = 4b\frac{R+r}{(R+r)^2+X^2}kTd\nu. \tag{10}$$

The direct component of the noise, corresponding to the frequency band $d\nu$, depends on the properties $R$, $X$ of the whole circuit. There is no rectified current in the circuit, and no direct voltage across the rectifier. For an actual electric circuit, $X$ depends on $\nu$, and an integration performed on Eq. (10), from $\nu=0$ to $\nu=\infty$ yields the total direct $\varphi_0$-term. Application of the formula to special examples always shows a complete compensation of direct average voltage.

The whole procedure is an example of the general method of detailed balancing discussed by Bridgman.[1]

[1] P. W. Bridgman, Phys. Rev. 31, 101 (1928).

## Nuclear Recoil Momentum in Pair Production

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RECENTLY[1] Modesitt and Koch investigated the production of electron pairs by $\gamma$-ray quanta in an air filled cloud chamber. They made measurements of the momentum transferred to nuclei in the creation of pairs by quanta of energies ranging from 1.02 to 19.5 Mev. The momentum imparted to the nucleus is related to the distance from the nucleus at which the electron pairs are formed. The data on recoil momenta of nuclei then constitute a test of a feature of the theory of pair production.

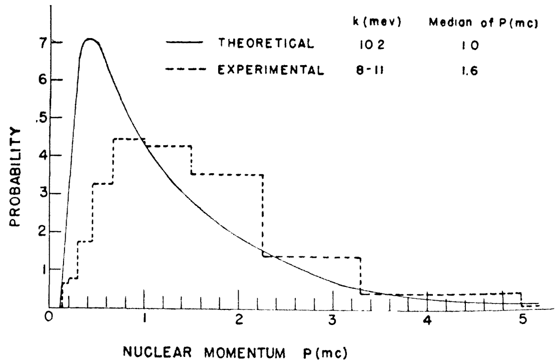


FIG. 1. The theoretical and experimental probability distribution curve of nuclear recoil momentum for pair production in air.

Bethe and Heitler[2] have given the theory of pair production by a $\gamma$-ray quantum in the Coulomb field of a nucleus. For large enough quantum energy the theory predicts a simple probability distribution for the nuclear recoil momentum[3]

$$\phi(p)dp = \mathrm{const.}dp/p,$$

provided

$$1 \gg p/mc \gg \delta/mc = mc^2k/2E_+E_-.$$

Here $\delta$ is the minimum possible momentum transfer. However, the quantum energies used in the experiments[1] are too small to justify comparison with the simple formula above, since the minimum value of $\delta/mc$ (at say $k=20\ mc^2$) is 0.1, which hardly allows $p$ to satisfy the conditions.

In order to obtain a formula which can justly be compared with experiment an exact numerical calculation of $\phi(p)$ was made for a quantum energy typical for the experiments, $k=20$ cm$^2$. Bethe[3] has given, by simple alterations of his Eq. (32), a form of the differential cross section for pair production which is convenient for integration. Part of the necessary integrations in calculating $\phi(p)$ were performed analytically by Bethe. Further work then consisted essentially of numerical integration.

The theoretical probability curve at $k=10.2$ Mev and the experimental results for the range 8 to 11 Mev are shown in Fig. 1. The calculated median momentum 1.0 $mc$ and the experimental 1.6 $mc$ are in disagreement. The validity of the Born approximation is hardly in doubt since the pair production took place in air; furthermore, screening is negligible in the range of recoil momenta involved. It is clear from the figure that the simple $dp/p$ law at this energy is a poor approximation to the actual theoretical curve.

At higher quantum energies the theoretical distribution function approaches the simple asymptotic formula given by Bethe. The concentration of momentum transfers in the neighborhood of the minimum momentum increases with increasing quantum energy and the median, consequently, decreases. In the data of Modesitt and Koch there is no such variation of the median momentum with quantum energy. The form of their histograms is essentially the same from 3 to 19.5 Mev. Their set of observed median momenta are larger than predicted by the theory, the difference being greatest for highest energy range, 17.0 to 19.5 Mev. This disagreement between theoretical predictions and experiment has not been explained.

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[1] G. E. Modesitt and H. W. Koch, Phys. Rev. 77, 175 (1950).
[2] H. Bethe and W. Heitler, Proc. Roy. Soc. 146A, 83 (1934).
[3] H. Bethe, Proc. Camb. Phil. Soc. 30, 524 (1934).