# Conservation of Flux by a Superconducting Torus

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A superconducing torus was subjected to a magnetic field normal to the plane of the torus and the field distribution across the plane of the superconducting torus was determined experimentally. The magnetic field near the axis of the torus was found to have the same direction as the applied field, but near the inner surface of the torus the field reversed direction in such a manner that the total flux enclosed by the torus was zero. It is concluded that the magnetic field distribution about a superconducting torus in weak magnetic fields can be adequately described by classical electrodynamics and the assumption of perfect diamagnetism.

## 1. INTRODUCTION

USING the principle of superposition of magnetic fields and postulating the conservation of flux, one of the authors[1] has derived theoretical expressions for the magnetic field about a superconducting torus subjected to magnetic cycling. In order to subject to test the validity of the expressions obtained, an experimental determination of the field distribution about a torus was made.

The theoretical distribution of the magnetic field ($H$) for the case of an external uniform magnetic field ($H_0$) applied perpendicularly to the plane of a superconducting torus is shown in Fig. 1. This plot is for a torus whose mean radius $R$ is 22.4 times the radius of the toroidal wire. Since the field is symmetrical about the axis of the torus, a two-dimensional plot of field intensity distribution along a diameter of the torus is given.

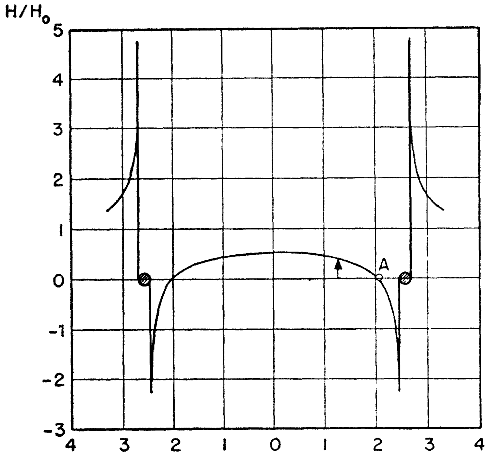


FIG. 1. Field distribution along a diameter of a superconducting torus for the case of an applied field $H_0$. The field direction and intensity are given by vectors drawn normal to the diameter and terminating on the curve. The position of the toroidal wire is indicated by the shaded circle. Example: at a distance $R/2$ from the center of the torus, the field is in the same direction as the applied field and equal to $0.405H_0$.

[1] Jules de Launay, "Electrodynamics of a superconducting torus," Naval Research Laboratory Report No. P-3441 (April, 1949).

The distribution in Fig. 1 is the one chosen for test. However, since precision measurement of field intensity and direction is beset with difficulties, it was decided to measure flux changes upon the application of $H_0$. Figure 2 gives the flux distribution to be expected.

The most significant characteristics of this flux distribution are:

(1) The location ($r$) of the maximum flux (point $A$), corresponding to the point $A$ of Fig. 1.
(2) The slope of the curve between the point $A$ and the inner surface of the torus.
(3) The slope of the curve external to the torus.

These will be most influenced by the transition from the state of normal conduction to superconductivity.

## 2. EXPERIMENTAL DESIGN

The theoretical treatment given by de Launay[1] is valid only so long as the torus is composed entirely of superconducting material and penetration depth is negligible. The experimental investigation, therefore, was restricted to the application of small fields (few gauss) to a torus cooled to the superconducting state under small (earth, uncompensated) applied magnetic field. By measuring the inductance of a very small coil wrapped directly upon the torus, transition of the

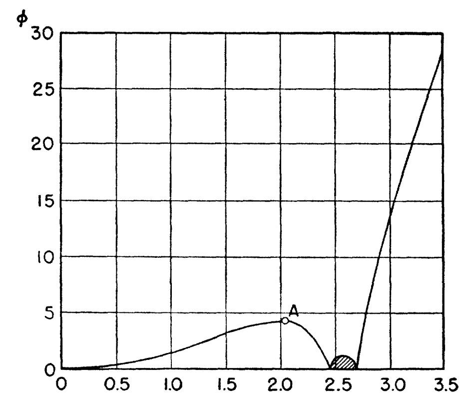


FIG. 2. Flux distribution obtained by integrating the distribution of Fig. 1 from the center of the torus outward ($r$). The position of the toroidal wire is indicated by the shaded semicircle. The flux $\phi$ (arbitrary units) is plotted as ordinate. The polar coordinate ($r$) is plotted as abscissa.

material into the superconducting state could be determined.

For the measurement of the flux distribution the torus was mounted in a solenoid and the e.m.f. generated in small concentric co-planar pick-up coils was measured when a 2000-cycle alternating current was passed through the solenoid. The mean diameters of the measuring coils were selected to yield information in the regions shown in Fig. 1. Construction data is given in Table I.

The field intensity in the solenoid was determined by comparison with a secondary standard. With the torus removed, the uniformity of the field was verified experimentally by measurement with the pick-up coils. For constant excitation current through the solenoid, the ratio of generated e.m.f. of coil area was determined. For this ratio the maximum deviation from the mean was 0.5 percent.

## 3. RESULTS

As the temperature of the lead torus was gradually lowered, the magnetic field about the torus passed gradually from an almost uniform distribution, corresponding to high resistivity of the lead, to a distribution indicating almost complete conservation of flux within the torus, corresponding to very high conductivity of the lead. At the transition temperature a very definite increase in slope of the flux distribution near the coil was observed. This was interpreted as the change to be expected by the transition to perfect conductivity and the change from a skin effect field penetration to the condition of superconductivity. A sudden decrease in the self-inductance of the small test coil wound on the torus verified that the abrupt change in slope and the onset of superconductivity occurred simultaneously. Below the transition temperature no change in flux distribution with further lowering of the temperature was noted. An absolute determination of the transition temperature was not attempted. However, by the use of a He gas thermometer the transition temperature was estimated to be about 7°K and the transition interval to be less than 0.1°K wide.

TABLE I. Geometry of lead torus and copper measuring coils used in experiment.

| | Mean radius cm | Wire diameter cm | Number of turns |
|---|---|---|---|
| Torus | 2.575 | 0.230 | 1 |
| Coil 1 | 3.227 | 0.010 | 10 |
| Coil 2 | 2.832 | 0.010 | 10 |
| Coil 3 | 2.437 | 0.010 | 10 |
| Coil 4 | 2.237 | 0.010 | 10 |
| Coil 5 | 2.037 | 0.010 | 10 |
| Coil 6 | 1.839 | 0.010 | 10 |
| Coil 7 | 1.447 | 0.010 | 10 |
| Coil 8 | 0.850 | 0.010 | 10 |

A comparison of the experimental results with the theoretical predictions is given in Fig. 3. The solid curve is a plot of the theoretical prediction and the circles indicate the experimental values determined with the measuring coils.

Instrumentation limited the absolute accuracy of the experimental results to approximately 2 percent. This accuracy was sufficient to verify the correctness of procedure and magnitude of effect but was inadequate for a precise test of the theoretical distribution. However, since the contour of the distribution is the important test, by conducting a complete test under constant conditions, the relative flux distribution can be obtained to a higher order of accuracy. This type of data is presented in Figs. 3 and 4, where the experimental point at $A$ is made to coincide in magnitude with the theoretical curve.

In Fig. 3 the experimental data agree well with the theoretical prediction. Particularly noticeable is the region external to the torus where, because of the large slope, the theory is subjected to a good test. The scale of the plot of Fig. 3, however, is inadequate to show the goodness of fit in the internal region of the torus.

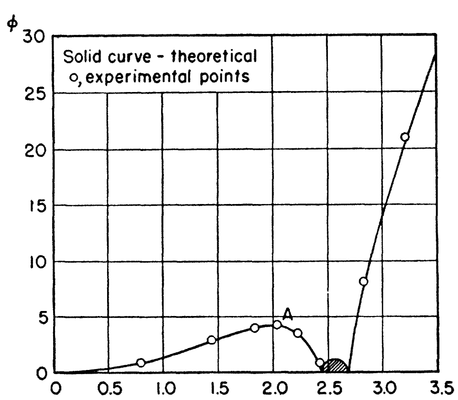


FIG. 3. Comparison of experimental results with theory. For inspection of contour, the experimental and theoretical magnitudes are forced to equality at the one point $A$.

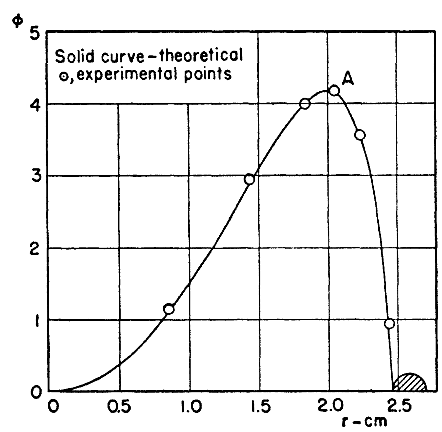


FIG. 4. Comparison of experimental results with theory for the inner region of the torus. This plot reproduces the inner portion of the curve of Fig. 3 on an expanded scale in order to show goodness of fit.

Figure 4 presents a plot of the internal region of the torus on an expanded scale. The high correlation of the experimental results to the theory is evident.

The experimental results of this investigation establish the validity of the assumptions of the principle of superposition of magnetic fields and the conservation of flux as used by de Launay[1] in describing the electromagnetic behavior of a superconducting torus under magnetic cycling. It is, therefore, felt that the results of his calculations can be applied with confidence to experimental investigations of the magnetic field distributions about superconducting tori.

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# A New Method for Particle Injection into Accelerators

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The purpose of this note is to describe briefly a new method by which electrons have been injected into a betatron. This new method has made possible the production of a reasonable electron beam in the betatron phase of operation of the non-ferromagnetic synchrotron now in construction* at the General Electric Research Laboratory. This betatron is characterized chiefly by a small aperture (vertical aperture = 0.05 orbit diameter, radial aperture = 0.025 orbit diameter) and low rate of acceleration (about 10 to 15 volts per orbital turn). It is believed that the method can be applied with success to other accelerators, especially those employing small relative apertures and small energy increase per orbital turn.

Most betatrons seem to work and there have been many attempts to explain satisfactorily how the electrons were injected so that they did not subsequently hit the structure of the injector itself after a few revolutions. These explanations have included many possible effects, e.g., resonance conditions between orbital, vertical, and radial oscillation frequencies, effects caused by injected current flux linkage, space-charge considerations either in the "donut" or near the injector gun, etc.[1] In our first attempt to obtain a betatron beam in the non-ferromagnetic synchrotron we found that, while these phenomena may provide a successful means of injection in conventional betatrons, they did not appear to provide enough of an effect to give us an appreciable beam. We were, however, successful in developing a scheme that did provide efficient injection and which appears to have the very desirable properties of being readily understood and easily applied. This scheme involves two steps; the first is to provide a means of rapidly damping the radial or vertical oscillations for as large a number of revolutions as possible, and the second is to remove this damping agent before it causes subsequent undamping. We have found that the first of these conditions can be very easily met, as shown below, by proper control of the azimuthal variation of the index, $n$.[2] The second of these conditions is easily obtained by making the azimuthal pattern of $n$ time-dependent in the proper way; in our case the azimuthal variation (producing beam damping) is reduced to zero some time late in the gun injection pulse.

The best method by which rapid beam damping can be produced depends upon the particular design of the betatron and, for definiteness, the following description is limited to a discussion of the conditions which pertain to our non-ferromagnetic betatron. The injector gun is displaced from the orbit in the $Z$ (axial) direction (perpendicular to the radius and to the tangent of the orbit). The value of $n$ is (at injection) nominally 0.25; this, however, has been made adjustable over wide limits. In this case one expects vertical ($Z$) oscillations of about one-half the orbital frequency so that after two orbital turns the injected electrons will find themselves back at the gun, and, if the gun has any finite physical size, will be intercepted. By experimentally tracing the beam we have found this to be true only when great care was exercised in providing an index, $n$, which was quite uniform in azimuth. When a particular azimuthal "bump" in $n$ was provided, the $Z$ oscillation amplitude was found to change markedly even after two orbital turns. A qualitative explanation for this phenomenon is shown in Fig. 1. This scheme is one which is similar to those considered by Langmuir and Davis.[3] One full $Z$ oscillation cycle is shown for two different conditions of $n(\theta)$; first, when $n$ is uniform and equal to 0.25 one gets the solid curve with no damping or undamping. The injector gun is shown in its proper position, and it will be noticed that, apart from small damping effects not considered here,[4] after two orbital

* The construction and development of this accelerator has been supported in part by ONR under Contract N7onr-332 Task I.

[1] See for example D. W. Kerst, Phys. Rev. 74, 503 (1948).

[2] In this definition the axial magnetic field is proportional to the minus $n$th power of the radius.

[3] R. V. Langmuir and L. Davis, Phys. Rev. 75, 1457 (1949). Also private communication.

[4] D. W. Kerst and R. Serber, Phys. Rev. 60, 47 (1941), showed the damping of oscillations caused by the increase of magnetic field (and hence restoring force) during a few orbital turns. How-