A simple example for the results of this section is the transition $J=1\rightarrow 2$, $K=1$, $M_J=0$ in CH_3I. The frequency of this Stark component is

$$\nu=30,\,132+\mathcal{E}^2 1.79\cdot 10^{-4}, \tag{76}$$

where $\mathcal{E}$ is measured in volt/cm. The Zeeman pattern is

$$\delta\nu=\begin{cases} 3\eta-0.313\eta^2 \\ 0\;-0.313\eta^2 \\ -3\eta-0.313\eta^2. \end{cases} \tag{77}$$

(γ) $H_m\sim H_q$

To solve the secular equation of order $2I+1$ in this case it is convenient to start with the $IJKM_JM_J$ representation for $K\neq 0$. H_q is then diagonal for fixed M_J the diagonal elements being (34) the off diagonal elements come from $(M_I|I_1|M_I\pm 1)$ only. $E_m^{(\mathrm{mol})}$ is given by (64). For $K=0$ we must take the $\bar{M}_JM_I$ representations. The diagonal elements are then (34) for $M_J\neq 1$, (69) for $M_J=1$; the off-diagonal elements for $M_J=1$ can be obtained from (75). $E_m^{(\mathrm{mol})}$ is given by (71) and (72).

(C) Intermediate Field Strength. $H_e\sim H_q\sim H_m$

Here we have three operators of equal order no two of which commute. In this case the secular quotation of order $(2J+1)(2I+1)$ must be solved numerically for each value of the parameters. The matrix elements in the $IJKM_JM_I$ representation have all been given in (30), (31), (37) and (33).

I wish to thank Dr. A. Roberts for drawing my attention to this problem.

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Range-Energy Relation for Protons in Nuclear Emulsions

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An experimental range-energy relation in Ilford C-2 emulsion has been obtained for protons up to 39.5 Mev. In the region from 17 to 33 Mev the relation for dry emulsion is fitted by the empirical equation $E_{(\mathrm{Mev})}=0.251R_{(\mu)}{}^{0.581}$. Variations in water content due to changes in atmospheric humidity make several percent difference in range. The range in Ilford glass is found to be 18±4 percent longer than in dry C-2 emulsion.

INTRODUCTION

THE ranges of protons of energies up to 13.1 Mev in nuclear track emulsions were measured by Lattes, Fowler, and Cuer,[1] with 50μ thick Ilford B1 emulsion. Extrapolations[2] of these data were used by Bishop[3] to calculate masses of mesons whose range in emulsion and initial momentum are known. The region of interest corresponded to proton ranges of approximately 4000μ, or 31-Mev energy. For tracks of this length the extrapolation gave energy values accurate to only ±8 percent.

In the present study the ranges of protons up to 39.5 Mev were measured in Ilford nuclear emulsions. C2 plates used in this study are stated by Ilford Ltd. to have identical chemical composition to B1 plates.

METHOD

Monoenergetic protons were obtained from the circulating beam of the 184-in. Berkeley cyclotron in an arrangement shown in Fig. 1. Protons which struck a $\frac{1}{8}$ in.$\times\frac{1}{16}$ in.$\times$3 in. copper ribbon target at, say, 30 inches from the cyclotron center scattered in all directions with a variety of energies. A C2 plate placed behind a short channel at 80 inch radius recorded those protons which left the target in a backward direction with such an energy that their paths had approximately 25 inches radius of curvature in the cyclotron magnetic field. With an accurate knowledge of the field it was possible to calculate the energy of a proton entering normal to the edge of the plate. Plates were put at 80 inch radius for all exposures, and the target radius was varied to obtain different proton energies.

Plates were tilted approximately 3° from horizontal so that protons would enter the surface of the emulsion. Distortion of the emulsion upon processing prevents accurate determination of entrance angle if one uses particles coming through the edge of the emulsion. This tilt, plus scattering of the protons in the emulsion made it difficult to find tracks in 200μ plates which stayed in the emulsion for 7000μ. Hence, the study was not extended beyond ~39 Mev.

Plates were processed in 3:1 D-19 at 68°F for 20 to 25 minutes, followed by shortstop and commercial hardening fixer. Plates suffered 50±5 percent thickness shrinkage in processing, but negligible transverse

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[1] Lattes, Fowler, and Cuer, Proc. Phys. Soc. London **59**, 883 (1947).
[2] V. Camerini and C. M. G. Lattes, Ilford technical data (revised March 27, 1948).
[3] A. S. Bishop, Phys. Rev. **75**, 1468 (1949).

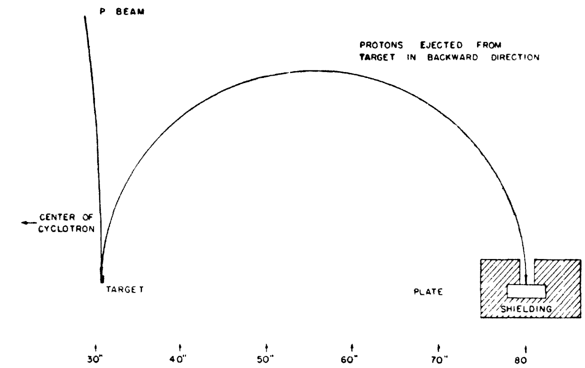

FIG. 1. Schematic diagram of arrangement for range-energy experiment.

shrinkage. Track lengths were measured both by eyepiece reticle and by dial indicator attached to the microscope stage. Corrections were made for angle of dip of protons in the emulsion.

RESULTS

Results of the measurements are presented in Figs. 2 and 3. It will be noted that the data fit a straight line on log-log plot between 17 and 35 Mev and fit the data of Lattes at 8 Mev. A least squares analysis gives for this portion the relation:

$$E_{(\mathrm{Mev})} = 0.251 R_{(\mu)}{}^{0.581}, \tag{1}$$

where the energy E is expressed in Mev and the range R is measured in microns.

The original values predicted by Camerini and Lattes,[4] and a recent relativistically correct extrapolation made by Aron,[5] are indistinguishable from the present experimental results up to 35 Mev.

The effect of widely different humidity conditions on plates before exposure is indicated by the 3 points at 33.5 Mev, and the 2 points at 17.6 Mev. It was necessary to evacuate plates for at least 6 minutes before exposure so that moisture conditions were not accurately known. No difference was found between 6 minutes and 6 hours evacuation of 100μ emulsions which had previously come to equilibrium in a normal Berkeley atmosphere (75°F, 60 percent R.H.) and these plates are therefore called "dry" in the present study. However, the 17.6-Mev point shows difference between 6 minutes evacuation of such a plate and 6 minutes evacuation of a plate which had been exposed to several hours of rainy weather (95 percent R.H.). Large changes occurred as shown at the 33.5-Mev point, when 200μ plates were given 6 minutes evacuation after 24 hours exposure to 93 percent R.H. at 75°F in a $NH_4H_2PO_4$ constant humidity chamber. The 80 percent humidity point was obtained with a plate which had been maintained for 24 hours over wet NH_4CL.

An approximate value for the range of protons in the glass backing of C2 plates was found by measuring the length of tracks which had traveled most of their range in the glass. Protons from the monoenergetic (±100 kev) beam of the Berkeley linear accelerator were used in this experiment. The range of the protons of approximately 30-Mev energy was found to be 18±4 percent larger in glass than in dry C2 emulsion.

[4] V. Camerini and C. M. G. Lattes, Ilford technical data (1947).

[5] W. Aron (private communication).

DISCUSSION

The numbers used above to illustrate the calculation of momentum from target and plate radius are only approximate because the protons did not follow a circular orbit in the radially decreasing cyclotron field. Protons leaving the target at a cyclotron radius r_1 at a small angle θ_1 from the backward direction, enter the plate (r_2) at a different small angle θ_2 to perpendicular, and have a momentum given by the expression

$$\langle Hr\rangle_{\mathrm{Av}} = \frac{1}{r_1\cos\theta_1 + r_2\cos\theta_2}\int_{r_1}^{r_2} Hr dr.$$

Stepwise integration is required to evaluate the integral. Figure 4 shows a group of values of θ_1 determined graphically by stepwise trajectory plotting starting

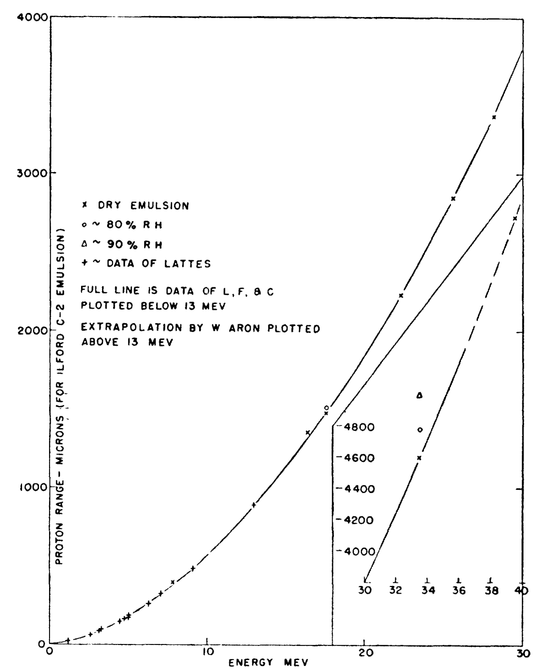

FIG. 2. Range of protons *vs.* energy.

E (Mev)	R (μ)	
7.8	389	±2.7
16.4	1358	8.1
17.6	1465	4.5
17.6	1497	6.0
22.3	2244	8.9
25.6	2849	7.1
28.2	3369	10.0
33.5	4597	13.8
33.5	4762	24.0
33.5	4996	10.0
39.5	6123	18.0

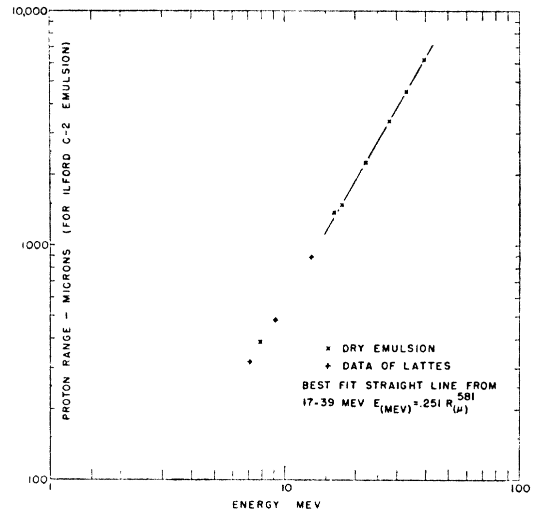

FIG. 3. Power law approximation for range-energy relation.

with various values of θ_2. In the range-energy study only tracks entering the emulsion at $0°\pm 5°$ were measured, so that the difference between $\cos\theta_1$ and $\cos\theta_2$ was negligible, and the expression for $\langle Hr\rangle_{\mathrm{Av}}$ could be written

$$\langle Hr\rangle_{\mathrm{Av}}=\frac{\sec\theta_2}{r_1+r_2}\int_{r_1}^{r_2} Hrdr.$$

The value of the integral was obtained by numerical integration using absolute field calibration at 4 points by nuclear induction to 4-gauss accuracy in the 14,000-gauss field, and flip coil measurements[6] of relative fields at 1-inch intervals along a radius. The relative values at the different points in the flip coil measurements were estimated to be accutate to 0.1 percent. The field is shown as a function of cyclotron radius in Fig. 5. It was assumed, in the integration, that the cyclotron field had azimuthal symmetry.

The chief limit to the accuracy of the results was due to scatter in range of supposedly monoenergetic protons. The mean square scatter of 1 percent to 3 percent in ranges was partly caused by inaccuracies in measurement of entrance angle, partly by "skipping" of protons along the emulsion so that the actual beginning in the emulsion was missed, and sometimes caused by surface fog which prevented exact determination of the beginning of a track. Approximately half, however, could be

[6] D. Sewell (private communication).

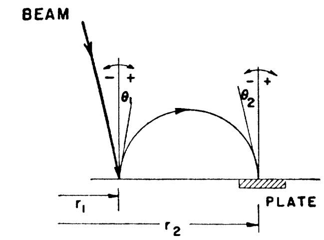

FIG. 4. Emission angle *vs.* entrance angle.

R_2	θ_2	R_1	θ_1
80″	0	27.3	−1° 45′
80″	−3°	27.4	+1° 15′
80″	+3°	27.6	−5° 0′
80″	0	33.0	−2° 35′
80″	−3°	33.1	+1° 25′
80″	+3°	33.2	−4° 45′
80″	0	37.7	−1° 45′
80″	−3°	37.7	+1° 25′
80″	+3°	37.8	−4° 40′

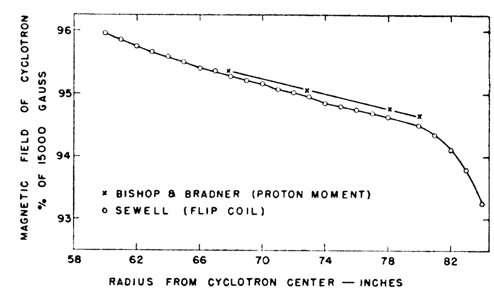

FIG. 5. Cyclotron magnetic field *vs.* radius.

accounted for theoretically by straggling. Probable error of the mean due to scatter in range was less than 0.7 percent for each energy value. However, it seems unwise to claim better than ± 2 percent accuracy for the range-energy relation because of incomplete control of emulsion moisture, and ignorance of possible variations in emulsion composition.

The point at 16.4 Mev which lies 2 percent above the curve may be caused by "skipping" or perhaps an error in the radius to which the target was set. All other points lie within $\frac{1}{2}$ percent of the curve defined by the relation (1). The relation is probably accurate to better than 2 percent for any batch of dry B1, B2, C2, C3 or G5 plates.

The authors wish to express their appreciation of the stimulating interest of Professor E. O. Lawrence, and of the cooperation of the cyclotron crew.

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