thus in effect reducing the efficiency of the circuit. Therefore, a procedure was devised for measuring N_0, the disintegration rate of the source, without the determination of the counting rates N_1 and N_2 in the individual channels.

The total number of coincidences N is given, under conditions discussed elsewhere,[3] by the well-known equation,[4]

$$N=N_g+N_c=(N_1N_2/N_0)+2N_1N_2\tau, \tag{1}$$

where τ is the resolving time. It is apparent from this equation that it is not necessary to measure N_1 and N_2 if a measurement of N_c and τ is made. For strong sources the measurement of N_c is necessary in any case and therefore involves no additional effort. The difference in the procedure is simply the substitution of a determination of τ for the determination of N_1 and N_2. The disintegration rate is then given by:

$$N_0=N_c/2\tau N_g. \tag{2}$$

It was in the determination of τ that it became evident that the resolving time had to be redefined. The meaning of τ and N_g becomes ambiguous when there are (1) pulses of unequal size with varying probability of being recorded as coincidences and (2) randomly distributed delays in the counters and the coincidence circuit.

Fortunately, the whole difficulty of the definition of τ and N_g can be avoided as shown in reference 1 by the equation:

$$N_c=N_0\int_{-\infty}^{+\infty}N_g(T)dT, \tag{3}$$

where T is the time delay deliberately put in either channel, $N_g(T)=N(T)-N_c$ as a function of T. Equation (3) gives N_0 to any desired accuracy[3] in terms of directly measurable quantities.

The integral in Eq. (3) is proportional to N_0 and independent of any time delays in the source or in the equipment. Therefore, it gives (1) a definition of the resolving time to be used in determining the presence of genuine coincidences and in the measurement of decay constants, and (2) the value of N_0 regardless of any kind of time delays, whether they are in the source or in the equipment. That is, N_0 can be determined for a source when there is a delay between the related particles emitted, such as by the parent and the daughter in a radioactive decay series.

* This author's work supported by the ONR.
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On the Motion of Particles in a Traveling Wave Linear Accelerator*

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IN the linear motion of a point charge, unless the energy of the charge changes by mc^2 or greater in a distance of the order of e^2/mc^2,[1] the particle's own field, which gives rise to radiative effects, may be neglected in the Lorentz equations of motion

$$dp^\mu/ds=(e/mc^2)F^{\mu\nu}p_\nu$$
$$(p^\mu=dx^\mu/ds=\text{energy}-\text{momentum vector}/mc^2). \tag{1}$$

Thus, in a linear accelerator we may safely regard $F^{\mu\nu}$ as the external field alone.

Now, a traveling wave field moving with phase velocity β_w along the $x'(\equiv x)$ direction is written as a function of the Lorentz invariant phase $\phi=k_\mu x^\mu=\omega t-kx$. Only the component $F^{14}=-F^{41}=\epsilon f(\phi)$ of the field is effective in changing the energy of particles in motion along the x direction. Here ϵ is the magnitude of the electric field in the x direction. Integration of Eq. (1) with β_w constant will give the energy of a particular particle from the equation

$$E-\beta_w(E^2-1)^{\frac{1}{2}}+U(\phi)=H, \tag{2}$$

where

$$E\equiv(1-\beta^2)^{-\frac{1}{2}}=(\text{kinetic plus rest energy})/mc^2$$

and

$$U(\phi)=\frac{e\epsilon/k}{mc^2}\int^\phi f(\phi)d\phi=\epsilon\int^\phi f(\phi)d\phi. \tag{3}$$

$p=mc^2(E^2-1)^{\frac{1}{2}}$ is the algebraic momentum of the particle and changes sign if the direction of motion reverses. The term $\beta_w(E^2-1)^{\frac{1}{2}}$ appears in Eq. (2) when we use a fixed system of coordinates with respect to which the wave field is moving. It can be transformed away by using a Lorentz system moving with respect to the fixed system with velocity β_w in the x direction. The term mc^2H is a constant of the motion (with respect to ϕ), the Hamiltonian of the particle, and from Eq. (2) we find that p and $-\phi/k$ are conjugate momentum and coordinate.

Equation (2), when solved for the energy E, gives the following results:

$$E(\phi)=\begin{cases}(1-\beta_w^2)^{-1}\{\chi(\phi)\pm\beta_w[\chi^2(\phi)-(1-\beta_w^2)]^{\frac{1}{2}}\}\cdots\beta_w\neq1,\\ \frac{1}{2}\{\chi(\phi)+[1/\chi(\phi)]\}\cdots\beta_w=1,\end{cases} \tag{4}$$

where

$$\chi(\phi)=H-U(\phi). \tag{5}$$

Plots of $(E(\phi)-1)^{\frac{1}{2}}=p/mc$ are given by Slater[2] for two cases of a sinusoidal field $\epsilon\sin\phi$: $\beta_w=0.5$ and $\beta_w=1$. Equations (4) represent the kinematical trajectory in energy-wave phase space of a particle under the influence of the field. An important consequence of these equations is that a particle has no chance of attaining unlimited kinetic energy unless the accelerating wave is traveling with the speed of light ($\beta_w=1$), for $|\chi(\phi)|$ must be bounded in the physical problem (if $k\neq0$). Even for $\beta_w=1$, the initial conditions (which determine H) must be such that a certain real relative phase ϕ exists for which $U(\phi)\rightarrow H$, in order that unlimited energy be kinematically possible.

In Eq. (4), if $\beta_w\neq1$, we see that only when $\chi^2(\phi)-(1-\beta_w^2)$ is positive will there be real (and hence physical) solutions for the energy $E(\phi)$. If $\beta_w>1$, any particle can kinematically assume all values of relative phase. This is nothing more than the physical fact that a particle can never travel as fast as the field in this case —it will continually slip back in relation to a point moving with the wave. If $\beta_w<1$, there can be (depending on H) certain relative phases for a given particle for which the radical is imaginary. Particles in this category can never advance, or fall behind the wave, beyond a phase for which the radical vanishes. Physically this means that certain particles may be, so to speak, "locked" to the wave in closed orbits. For $\beta_w=1$, those particles will be locked for which ϕ is real in the equation $U(\phi)=H$.

The dynamical solution of Eq. (1), that is, energy as a function of distance traveled in the field, is obtained by integrating

$$dE/dx=(e/mc^2)F^{14}=kdU/d\phi,$$

which gives

$$\int\frac{dE}{(dU/d\phi)(E)}=kx.$$

For a sinusoidal field, the shape function $f(\phi)$ is a circular function so that

$$\left(\frac{dU/\epsilon}{d\phi}\right)^2+(U/\epsilon)^2=1$$

and we have

$$(\pm)\int_{E_0}^{E(x)}\frac{dE}{[\epsilon^2-\{E-\beta_w(E^2-1)^{\frac{1}{2}}-H\}^2]^{\frac{1}{2}}}=k(x-x_0). \tag{6}$$

We must bear in mind that if the momentum $(E^2-1)^{\frac{1}{2}}$ changes sign, the integral will lead to a negative contribution to x. The integral in Eq. (6) is, in general, elliptic.

For a $\beta_w=1$ accelerator we have observed that very high energies are theoretically attainable by some of the particles. The energy of those particles for which ϕ is real in $U(\phi)=H$ will, for a sinusoidal field at least, approach asymptotically a linear function of distance, as may be seen in the following:

$$E^2 \gg 1: \quad E-(E^2-1)^{\frac{1}{2}} \rightarrow 1/2E.$$

Thus, for high energy,

$$\begin{pmatrix} \beta_w=1 \\ E^2\gg 1 \end{pmatrix}: \quad \frac{dE}{dx} \rightarrow k[\epsilon^2-\{(1/2E)-H\}^2]^{\frac{1}{2}},$$

$$\left[\begin{array}{l} \beta_w=1 \\ E^2\gg 1 \\ E\gg 1/2H \end{array}\right]: \quad \frac{dE}{dx} \rightarrow k[\epsilon^2-H^2]^{\frac{1}{2}}=\text{constant}.$$

A more general solution of the dynamical motion, which is equivalent to Eq. (6) for a sinusoidal field, is that obtained by finding $\phi(x)$. This solution is:

$$\left.\begin{array}{ll} \beta_w\neq 1: & (1-\beta_w^2)^{-1}\displaystyle\int_{\phi_0}^{\phi(x)} d\phi \left\{ \frac{\pm\beta_w \chi(\phi)}{[\chi^2(\phi)-(1-\beta_w^2)]^{\frac{1}{2}}}-1 \right\}=kx, \\ \beta_w=1: & \displaystyle\int_{\phi_0}^{\phi(x)} \frac{d\phi}{\chi^2(\phi)}-[\phi(x)-\phi_0]=2kx, \end{array}\right\} \quad (7)$$

where the $\pm$ sign depends on which branch of the $E(\phi)$ curve in Eq. (4) the particle is orbiting. Here there are no restrictions on the shape function $f(\phi)$ of the field, from which $\chi(\phi)$ is obtained. When Eq. (7) is solved for $\phi(x)$, we can then use Eq. (3) to find $U[\phi(x)]$ and finally Eq. (2) to get $E(x)$. The expressions in Eq. (7) might be called the equations of bunching.

It is a general result of the foregoing solutions that the energy spectrum of a beam of particles from a traveling wave linear accelerator is a strong function of the initial phase spread of the beam so that pre-bunching is a necessity for any appreciable final energy homogeneity of the beam.

Detailed considerations of all the above questions will be taken up in a forthcoming paper.

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Note on Nuclear Models

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NUCLEAR models in which the protons are concentrated on or near, or uniformly distributed over, the surface of a sphere have been recently proposed by several investigators.[1-3] For the same value of Coulomb repulsion energy, a given nucleus represented by such a neutron-core model will be smaller than that by the usually assumed model in which the protons are uniformly distributed over the volume of a sphere. In other words, the new model will yield a smaller value of r_0 in the formula $r=r_0A^{\frac{1}{3}}$ for nuclear radii than the customary model, when it is computed either from the semi-empirical binding-energy equation[4] or from a comparison of the binding energies of contiguous isobars.[5]

Assume the protonic charge be uniformly distributed over the surface. Then, the constant coefficient of the Coulomb energy term in the semi-empirical equation[4] is $a_3=\frac{1}{2}(e^2/r_0)$ instead of $a_3\approx(3/5)(e^2/r_0)$ so that with the value of a_3 given in Lapp and Andrews' book, we find, for this neutron-core model,

$$r_0=1.23_+\times 10^{-13}\text{ cm},$$

and, for the customary model, $r_0=1.48\times 10^{-13}$ cm. Also, the difference in Coulomb energy of two contiguous isobars due to the additional repulsion of the extra proton in the Coulomb field of the isobar with the lower Z is now $C=(Z+\frac{1}{2})e^2/r$ instead of $C=(6/5)Ze^2/r$ so that, with Bethe's value of C for ${}_7N^{13}-{}_6C^{13}$, we have, for this neutron-core model,

$$r_0=1.34_-\times 10^{-13}\text{ cm},$$

instead of 1.47×10^{-13} cm obtained by Bethe for the customary model. The two results from the new model do not appear to agree as well as those from the customary model. This, however, is not entirely unexpected, since for a nucleus as light as C^{13} our new model with charge spread over a surface layer cannot be anything but a very crude model, and we should expect that, for the isobar N^{13} having the extra proton, a model with a proton on the surface of the C^{13} model should be closer to the true situation. Thus, by considering the charge of the extra proton to remain unspread, we have $C=Ze^2/r$, giving $r_0=1.23_-\times 10^{-13}$ cm, which brings the two results in agreement again.

Comparing the above results with the value $r_0\approx 1.5\times 10^{-13}$ cm for most α-emitters as given by Gamow's theory of α-decay, one may, at first sight, stick to the customary model. However, on our proton-shell model, one should expect that the radius, R, of a nucleus of mass number A as defined in Gamow's theory should be equal to our model radius, r, for a nucleus of mass number $(A-4)$ plus an effective radius, r_α, of α-particle. Since r_α cannot be well defined, and since our formula

$$r=1.23\times 10^{-13}A^{\frac{1}{3}}\text{ cm} \quad (1)$$

is not expected to hold for very light nuclei, we may calculate r_α from the values of R and r. A more accurate calculation of the nuclear radii of α-emitters have been recently made by Preston.[6] His values of R give r_0 that range from 1.28 (ThC–ThC″) to 1.62×10^{-13} cm (Th$-$MsTh$_1$) with a great majority of cases having r_0 around 1.5×10^{-13} cm. Using these values of R, we find r_α to range from 0.28 to 2.4×10^{-13} cm with a great majority of cases within 1.7 to 1.8×10^{-13} cm. These latter values are somewhat less than 1.96×10^{-13}, the value given by (1). This is reasonable since ${}_2He^4$ is very tightly bound as is well known (it is the only element hitherto unaffected by neutron bombardment).

Recently, Fernbach *et al.*,[7] using data of scattering of fast neutrons by nuclei from Li to U, have shown that $r_0=1.37\times 10^{-13}$ cm. Weisskopf and Ewing[8] have also obtained the best fit of data on the anomalous scattering of protons by medium weight nuclei by assuming $r_0=1.3\times 10^{-13}$ cm. These values are in better agreement with ours than what the customary model gives.

Therefore, it appears likely that the proton-shell neutron-core model is closer to the true nuclear structure than the customary model.

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Measurement of Some Weak γ-Ray Intensities

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WE have measured the intensities of some weak high energy γ-rays by counting the photo-protons produced by the disintegration of deuterium in an ionization chamber counter.[1] The 2.62 Mev γ-ray from Rd Th was used for energy calibration. The intensities have been measured relative to other γ-ray lines