predicts a large number of π^--absorptions in silver which produce no observable prongs (except possibly recoil nuclei) while model I predicts very few. Both models lead to a substantial number of fast protons too energetic to be accounted for by an evaporation process.

Meson absorptions giving rise to fast protons have been observed in photographic plates by Perkins[8] and by Cheston and Goldfarb[9] at Rochester. The data appear to fit reasonably well with our model II but the results are still inconclusive.

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* Preliminary results were reported by Professor R. E. Marshak at the Idaho Springs Cosmic Ray Conference in June 1949.
** AEC Predoctoral Fellow.

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Upper and Lower Bounds of Eigenvalues

TOSIO KATO
Physics Department, Faculty of Science, Tokyo University, Tokyo, Japan
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THE Ritz variational method gives only upper bounds of eigenvalues of Hermitian operators. Many authors[1] attempted to generalize the method and derived various formulas to estimate lower bounds of eigenvalues. But it seems that as yet the mutual relations and relative merits of these formulas have not been fully discussed from a systematic point of view.

We carefully examined these formulas and reached the conclusion that the formula of Temple[1] is the most precise one among them notwithstanding that it is the oldest as well as the simplest one.

Moreover, we could generalize the Temple formula to the case of higher eigenvalues of operators which are not necessarily bounded below. Let λ be a non-degenerate eigenvalue of a Hermitian operator H and let $\alpha<\beta$ be two numbers such that the interval (α,β) contains λ but no other points of the spectrum of H. Let w be an approximate eigenfunction and calculate

$$\eta=(w,Hw)/\|w\|^2,\quad \epsilon=\|(H-\eta)\|/\|w\|.$$

Then we can show that

$$\eta-\frac{\epsilon^2}{\beta-\eta}\leqq\lambda\leqq\eta+\frac{\epsilon^2}{\eta-\alpha},\tag{1}$$

provided $\epsilon^2<(\eta-\alpha)(\beta-\eta)$.

This formula is symmetric with respect to upper and lower bounds, as it should be. If in particular λ is the lowest eigenvalue, we can put $\alpha=-\infty$ and (1) reduces to the Ritz-Temple formula. It should be noted that (1) gives λ within the error of the order ϵ^2, which is very small if ϵ is small, i.e. w is a good approximate eigenfunction. Also we can show that (1) is in precision not behind any of the formulas cited above,[1] so long as the latter is not incorrect. In fact, it can even be shown that (1) is the best possible estimate if η, ϵ, α and β are the only available data.

Another advantage of (1) is that it can be applied to higher eigenvalues without the preliminary procedure of orthogonalization which is necessary in the Ritz method.

We can also estimate the error of the approximate eigenfunction w. Further (1) can be generalized to the case of degenerate eigenvalues.

These formulas proved to be very useful in approximate solution of eigenvalue problems of various kinds. For instance, they give much more narrow range of errors than hitherto supposed in the calculation of eigenvalues by the relaxation method.[2] Also we hope that they can successfully be applied to problems of quantum mechanics. Their application to the eigenvalue problem of the deuteron with the meson potential is now in progress.

Detailed account will appear shortly in Journal of the Physical Society of Japan.

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Neutrons from $Li^7(p,n)Be^7$ *

V. R. JOHNSON, M. J. WILSON LAUBENSTEIN, AND H. T. RICHARDS
University of Wisconsin, Madison, Wisconsin
December 23, 1949

IF nuclear forces are charge independent, one expects Be^7 to have a low lying level equivalent to the well-known 478-kev level in its mirror nucleus, Li^7. Earlier work by one of us[1] on the neutrons from $Li^7(p,n)Be^7$ failed to detect such a level. However, with the old Ilford halftone emulsions then available, the observed neutron group was very asymmetric and was approximately 700 kev in half-width; hence neutrons of energy corresponding to Be^7 excited could easily have been missed if of low intensity.

Recently, Grosskreutz and Mather[2] reported from this same reaction neutron groups corresponding to levels in Be^7 at 205 kev, 470 kev, and 745 kev, and comparable in intensity to the main group. Because of the importance of this result on the charge independence hypothesis of nuclear forces, we have re-examined this spectrum with the improved post-war nuclear emulsions. Thin (30–60 kev thick) targets of metallic lithium evaporated upon a tantalum backing were bombarded by 3.34-Mev and 3.96-Mev homogeneous protons from the Wisconsin electrostatic generator. 100 micron thick Eastman NTA emulsions (glass-backed) were mounted 15 cm from the target and at 0° and 60° to the proton beam.

After processing, the tracks were measured in a microscope with an oil immersion objective to achieve minimum depth of focus. The observed recoil proton energy, E_p, is equal to $E_n\cos^2\theta$, where E_n is the neutron energy and θ is the angle of recoil with respect to the incident neutron. If the incident neutron is along the x axis and y is in the plane of the emulsion, then

$$\tan\theta=(R_z^2+R_{xy}^2\sin^2\chi)^{\frac{1}{2}}/R_{xy}\cos\chi,$$

where R_z is the z projection of the track, R_{xy} is the projection on the xy plane, and χ is the angle R_{xy} makes with the x axis. R_z is the only one of these quantities difficult to measure accurately. Experimentally we observed the uncertainty in R_z to be about 0.5 micron, which uncertainty must be multiplied by a factor 2.5 because of the shrinkage of these concentrated emulsions upon processing. To minimize this resultant uncertainty in θ, we therefore accepted only tracks with a dip angle in the processed emulsion of $\leqq 3°$. Recoils out to $\chi=15°$ were, however, accepted since

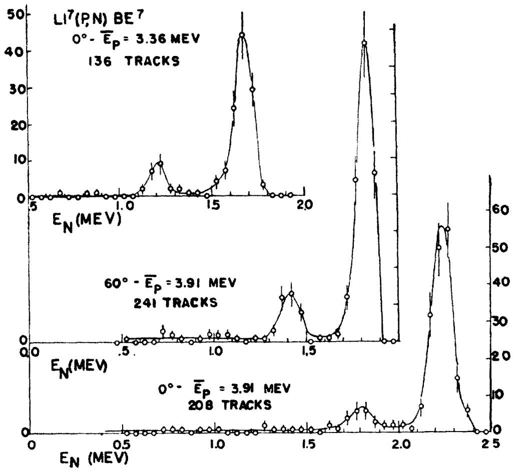

FIG. 1. Energy of neutrons from $Li^7(p,n)Be^7$.

this angle could be measured by our goniometer eyepiece to better than a degree.

Our observed mean ranges (appropriately corrected for recoil angle) were used in conjunction with the accurately known neutron energies to obtain three points on a range energy curve for the NTA emulsions. These data agreed within a few percent with the data for Ilford "Nuclear Research" emulsions,[3] so the shape of the latter curve was used to extrapolate our data to lower energies.

Our data plotted in energy intervals is shown in Fig. 1. Two groups of neutrons are clearly resolved on all the data and correspond to a level in Be^7 at 435±15 kev. This value is in agreement with that recently reported[4] for this same level as produced in the $B^{10}(p,\alpha)Be^7$.

The observed relative intensity of the low energy group (when corrected for $n-p$ scattering cross section) is 0.09±0.02 for the 0°, $E_p=3.91$-Mev data; 0.16±0.03 for the 60° data; and 0.17±0.05 for the 0°, $E_p=3.31$-Mev data. The indicated statistical uncertainty in the above ratio is, we feel, less by probably a factor of two than the true uncertainty. The cause of this additional uncertainty is the difficulty of applying uniformly for short and long tracks the criteria that the dip angle $\leq 3°$.

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1 See Fig. 1 of LADC-32 or MDDC-39(Perlman, Richards, and Speck).
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3 Lattes, Fowler, and Cuer, Proc. Phys. Soc. London 59, 883 (1947).
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The Electromagnetic Concentration of Be^{10}

C. P. KEIM, H. W. SAVAGE, AND BOYD WEAVER
Y-12 Research Laboratory, Carbide and Carbon Chemicals Corporation, Oak Ridge, Tennessee
December 23, 1949

BE^{10} present as a small fraction in Be^9 irradiated in the pile has been successfully enriched by the electromagnetic process. The weak specific activities in the beryllium charge material used in the electromagnetic separation indicated that the Be^{10} was present to the extent of about one part in 500,000. After the Be^{10} was enriched in the calutron, the specific activity of the enriched Be^{10} showed that it had been given an enrichment factor* of 870 and 1200 in the two samples processed. This enriched material has made it possible to determine the interesting beta-spectrum of Be^{10}. (These spectral results are being reported by D. J. Hughes, Argonne National Laboratory, and P. R. Bell, Oak Ridge National Laboratory, in separate letters.)

A few milligrams of beryllium enriched in Be^{10} were obtained from the calutron separations. Although calutron data indicated enrichment factors in excess of 3000, some cross contamination took place to dilute the product. Among the factors which contributed to the success of this enrichment were: a careful preparation of anhydrous beryllium chloride; the use of specially prepared spectroscopically pure isotope collectors; accurate process control to obtain high ion beam resolution; quantitative refinement of the enriched isotopic beryllium; and strict adherence to safety precautions to protect personnel from beryllium compounds.

Although this achievement was the result of the combined efforts of a large part of the personnel in the Isotope Research and Production Division, Y-12 Research Laboratory, particular credit is due W. C. Davis, who prepared some of the beryllium chloride and who did the chemistry on some of the enriched beryllium, L. O. Love, who with his associates, conducted the calutron operations, and S. F. Fairbourne, who with Mr. Love, directed certain chemical and calutron services. Especial appreciation is due D. J. Hughes and C. Eggler, Argonne National Laboratory, and P. R. Bell, Oak Ridge National Laboratory, who supplied the irradiated beryllium and some of the beryllium chloride to be used as charge material in the calutron and who provided the enrichment factors based on the specific activities they obtained.

This report is based on research carried out under contract for the AEC by the Isotope Research and Production Division at the Carbide and Carbon Chemicals Corporation, Y-12 Research Laboratory, Oak Ridge, Tennessee.

* Enrichment factor is the ratio Be^{10}/Be^9 in the product beryllium to the ratio Be^{10}/Be^9 in the starting beryllium.

General Relation between Genuine and Chance Coincidences and Its Application to Measurement of High Activity Sources

Z. BAY*
George Washington University, Washington, D. C.
AND
M. M. SLAWSKY
National Bureau of Standards, Washington, D. C.
December 19, 1949

IN an attempt to set up a useful standard radioactive source it was decided to use the coincidence method for the measurement of about one millicurie of Co^{60}. The problem of measuring a large source by this method presents several experimental difficulties. In the process of resolving these difficulties a very general relationship between the genuine coincidences N_g, and chance coincidences N_c, was derived by one of us.[1] At the same time the necessity of measuring the number of events registered in the individual channels of the coincidence device was eliminated.

In general, the difficulties of this type of measurement can be attributed to the fact that the equipment is required to have the highest possible resolution and efficiency. For this reason we used two electron multipliers feeding directly into a high resolution coincidence circuit.[2] It was felt that any addition to the coincidence circuit, in order to measure the number of counts in each channel, would have the undesirable effect of (1) distorting the pulse shape, thus increasing the response time and (2) discriminating against some coincidences which otherwise would have been recorded,