predicts a large number of π^--absorptions in silver which produce no observable prongs (except possibly recoil nuclei) while model I predicts very few. Both models lead to a substantial number of fast protons too energetic to be accounted for by an evaporation process.

Meson absorptions giving rise to fast protons have been observed in photographic plates by Perkins[8] and by Cheston and Goldfarb[9] at Rochester. The data appear to fit reasonably well with our model II but the results are still inconclusive.

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* Preliminary results were reported by Professor R. E. Marshak at the Idaho Springs Cosmic Ray Conference in June 1949.

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Upper and Lower Bounds of Eigenvalues

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THE Ritz variational method gives only upper bounds of eigenvalues of Hermitian operators. Many authors[1] attempted to generalize the method and derived various formulas to estimate lower bounds of eigenvalues. But it seems that as yet the mutual relations and relative merits of these formulas have not been fully discussed from a systematic point of view.

We carefully examined these formulas and reached the conclusion that the formula of Temple[1] is the most precise one among them notwithstanding that it is the oldest as well as the simplest one.

Moreover, we could generalize the Temple formula to the case of higher eigenvalues of operators which are not necessarily bounded below. Let λ be a non-degenerate eigenvalue of a Hermitian operator H and let $\alpha<\beta$ be two numbers such that the interval (α, β) contains λ but no other points of the spectrum of H. Let w be an approximate eigenfunction and calculate

$$\eta=(w, Hw)/\|w\|^2, \quad \epsilon=\|(H-\eta)\|/\|w\|.$$

Then we can show that

$$\eta-\frac{\epsilon^2}{\beta-\eta}\leqq\lambda\leqq\eta+\frac{\epsilon^2}{\eta-\alpha}, \tag{1}$$

provided $\epsilon^2<(\eta-\alpha)(\beta-\eta)$.

This formula is symmetric with respect to upper and lower bounds, as it should be. If in particular λ is the lowest eigenvalue, we can put $\alpha=-\infty$ and (1) reduces to the Ritz-Temple formula. It should be noted that (1) gives λ within the error of the order ϵ^2, which is very small if ϵ is small, i.e. w is a good approximate eigenfunction. Also we can show that (1) is in precision not behind any of the formulas cited above,[1] so long as the latter is not incorrect. In fact, it can even be shown that (1) is the best possible estimate if η, ϵ, α and β are the only available data.

Another advantage of (1) is that it can be applied to higher eigenvalues without the preliminary procedure of orthogonalization which is necessary in the Ritz method.

We can also estimate the error of the approximate eigenfunction w. Further (1) can be generalized to the case of degenerate eigenvalues.

These formulas proved to be very useful in approximate solution of eigenvalue problems of various kinds. For instance, they give much more narrow range of errors than hitherto supposed in the calculation of eigenvalues by the relaxation method.[2] Also we hope that they can successfully be applied to problems of quantum mechanics. Their application to the eigenvalue problem of the deuteron with the meson potential is now in progress.

Detailed account will appear shortly in Journal of the Physical Society of Japan.

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Neutrons from $Li^7(p,n)Be^7$ *

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IF nuclear forces are charge independent, one expects Be^7 to have a low lying level equivalent to the well-known 478-kev level in its mirror nucleus, Li^7. Earlier work by one of us[1] on the neutrons from $Li^7(p,n)Be^7$ failed to detect such a level. However, with the old Ilford halftone emulsions then available, the observed neutron group was very asymmetric and was approximately 700 kev in half-width; hence neutrons of energy corresponding to Be^7 excited could easily have been missed if of low intensity.

Recently, Grosskreutz and Mather[2] reported from this same reaction neutron groups corresponding to levels in Be^7 at 205 kev, 470 kev, and 745 kev, and comparable in intensity to the main group. Because of the importance of this result on the charge independence hypothesis of nuclear forces, we have re-examined this spectrum with the improved post-war nuclear emulsions. Thin (30–60 kev thick) targets of metallic lithium evaporated upon a tantalum backing were bombarded by 3.34-Mev and 3.96-Mev homogeneous protons from the Wisconsin electrostatic generator. 100 micron thick Eastman NTA emulsions (glass-backed) were mounted 15 cm from the target and at 0° and 60° to the proton beam.

After processing, the tracks were measured in a microscope with an oil immersion objective to achieve minimum depth of focus. The observed recoil proton energy, E_p, is equal to $E_n\cos^2\theta$, where E_n is the neutron energy and θ is the angle of recoil with respect to the incident neutron. If the incident neutron is along the x axis and y is in the plane of the emulsion, then

$$\tan\theta=(R_z^2+R_{xy}^2\sin^2\chi)^{\frac{1}{2}}/R_{xy}\cos\chi,$$

where R_z is the z projection of the track, R_{xy} is the projection on the xy plane, and χ is the angle R_{xy} makes with the x axis. R_z is the only one of these quantities difficult to measure accurately. Experimentally we observed the uncertainty in R_z to be about 0.5 micron, which uncertainty must be multiplied by a factor 2.5 because of the shrinkage of these concentrated emulsions upon processing. To minimize this resultant uncertainty in θ, we therefore accepted only tracks with a dip angle in the processed emulsion of $\leqq 3°$. Recoils out to $\chi=15°$ were, however, accepted since