Electromagnetic Interactions of a Vector Meson with a Scalar Excited State*

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A relativistically invariant formalism is presented for a theory of a meson possessing a vector and a scalar state of different mass which are tightly coupled by the electromagnetic field. It is found that the theory reduces formally to the theory of a five-dimensional vector meson, with the constant which couples the states playing the role of a fifth component of the energy-momentum vector. With a large coupling constant there is a high probability for producing excited-state mesons, and hence decay photons, in electromagnetic processes; e.g., by Rutherford scattering on passage of a beam of mesons through heavy material. The production of many photons in nucleon-nucleon scattering, such as has been reported at Berkeley, might possibly be interpreted as resulting from the decay of excited-state mesons which are produced directly. It is also found that the existence of a scalar excited state aggravates the high energy divergences of vector meson cross sections for such electromagnetic processes as Compton effect, so that the validity of the Christy-Kusaka argument against the μ-meson having spin 1 is not affected by the possible existence of a scalar excited state.

I. INTRODUCTION

RECENT experimental work has forced the abandonment of the idea that by the assumption of one simple-type meson field (such as vector, scalar, etc.) all the phenomena of cosmic rays and nuclear forces could be explained. Instead all evidence points to a picture of great complexity. It was therefore considered worthwhile to investigate the consequences of a theory which treated particles possessing several states of different spin and mass. In particular, it is the purpose of this paper to develop a convenient formalism for a relativistically invariant field theory of a meson possessing a vector state and a scalar state of differing mass. The states are assumed connected by the electromagnetic field and our main interest will be centered on the electromagnetic cross sections of this meson.

In 1941 Christy and Kusaka[1] showed that cosmic-ray (μ) mesons must have spin 0 or $\frac{1}{2}$ since the electromagnetic interaction of high energy vector mesons is much too strong to be compatible with experimental evidence.

We shall inquire whether the assumption of a scalar excited state with an appropriately chosen coupling to the vector ground state can alter this conclusion. We might expect that the assumption of excited states would lower the theoretical cross section for a vector meson if we consider the latter as being simply the ground state of a system of two tightly bound spin $\frac{1}{2}$ particles. This system may be supposed to have a number of discrete states and a continuum of unbound states, so that for the limit of infinitely high energy the system behaves like two free spin $\frac{1}{2}$ particles, hence has a relatively weak interaction with the electromagnetic field. In this paper, however, we restrict ourselves to the consideration of a single scalar excited state and find that in this case the additional terms lead to an increase in the high energy cross sections. Moreover, the additional mechanism for photon production which is provided by the possibility of Rutherford scattering to the excited state followed by decay to the ground state also leads to an amount of burst production incompatible with cosmic-ray evidence.

It should also be mentioned here that some work on the problem of a "vector-scalar" meson has been done by the Russian physicist V. L. Ginsburg[2] who used the usual wave theory formalism. He, however, never developed the theory much beyond the qualitative stage.

II. DEVELOPMENT OF MATRIX FORMALISM

A. Introduction of Coupling

In this paper we shall use the matrix formalism originated by Kemmer.[3] While this formalism at first appears unnecessarily complex, the spur techniques which it makes possible are invaluable in a calculation of this kind. We first write down the relativistically invariant Lagrangian density.

$$\begin{aligned} L &= L_{\text{vector}} + L_{\text{scalar}} + L_{\text{coupling}}; \\ \left.\begin{aligned} L_{\text{vector}} &= i\hbar c[\psi_v{}^+ B_\mu{}^v(\partial\psi_v/\partial X_\mu) + (M_v c/\hbar)\psi_v{}^+\psi_v]; \\ L_{\text{scalar}} &= i\hbar c[\psi_s{}^+ B_\mu{}^s(\partial\psi_s/\partial X_\mu) + (M_s c/\hbar)\psi_s{}^+\psi_s]; \end{aligned}\right\} \\ L_{\text{coupling}} &= -g\hbar c(\psi_v{}^+\gamma\psi_s + \psi_s{}^+\gamma^+\psi_v). \end{aligned}$$

Here ψ_v is the ten component vector meson wave function (four-vector potential and associated antisymmetric tensor); ψ_s is the five component scalar wave function (scalar potential and its four-gradient); M_v and M_s are respectively the vector and scalar masses; the $B_\mu{}^v$'s are Kemmer's ten by ten matrices; the $B_\mu{}^s$'s are his five by five matrices. γ is a ten by five matrix which we introduce to couple the vector and scalar states. We shall deduce its properties later. Also

$$\psi^+ = i\psi^* N_4. \quad (N_\mu = 2B_\mu{}^2 - 1).$$
$$\gamma_{ij}{}^+ = \gamma_{ji}{}^*.$$

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[1] R. Christy and S. Kusaka, Phys. Rev. 59, 414 (1941).

[2] V. L. Ginsburg, J. Phys. U.S.S.R. 8, 33 (1944).

[3] N. Kemmer, Proc. Roy. Soc. London 173, 91 (1939).

The B's follow the commutation rule:

$$B_\mu B_\nu B_\lambda + B_\lambda B_\nu B_\mu = B_\mu \delta_{\nu\lambda} + B_\lambda \delta_{\nu\mu}. \quad (1)$$

The omission of the superscript v or s implies that the law is true both for vector and scalar matrices.

We can now express the Lagrangian in a neater form by adopting a 15 by 15 formalism. We set:

$$L = \hbar c\{i[\psi^+ B_\mu(\partial\psi/\partial X_\mu) + \psi^+ K\psi] - g\psi^+ D\psi\}, \quad (2)$$

where

$$B_\mu = \left(\begin{array}{c|c} B_\mu{}^v & \\ \hline & B_\mu{}^s \end{array}\right),$$

$$K = \left(\begin{array}{c|c} \begin{matrix} \frac{M_v c}{\hbar} & \\ & \ddots \end{matrix} & \\ \hline & \begin{matrix} \frac{M_s c}{\hbar} & \\ & \ddots \end{matrix} \end{array}\right), \quad \psi = \left(\begin{array}{c} \psi_{v1} \\ \vdots \\ \psi_{v10} \\ \hline \psi_{s1} \\ \vdots \\ \psi_{s5} \end{array}\right), \quad D = \left(\begin{array}{c|c} 0 & \gamma \\ \hline \gamma^+ & 0 \end{array}\right). \quad (3)$$

Our equations of motion become:

$$B_\mu(\partial\psi/\partial X_\mu) + (K + igD)\psi = 0, \quad (4)$$

$$(\partial\psi^+/\partial X_\mu)B_\mu - \psi^+(K + igD) = 0. \quad (5)$$

It is obvious that the new B_μ's still obey the commutation rules (1). It should also be noted that the rest masses of the vector and scalar states are no longer given simply by M, since the coupling introduces a "mesomatic" mass which we shall derive later.

B. Properties of the Coupling Matrix

We next consider the properties of D as determined by the conditions of relativistic invariance. First we consider invariance of the Lagrangian (2) under an infinitesimal Lorentz rotation. As Pauli[4] has shown, it is sufficient that L be invariant under the transformation $\psi' = S\psi$; $\psi^{+\prime} = \psi^+ S^{-1}$ where

$$S = 1 + \tfrac{1}{2}\mathcal{E}_{\mu\nu}(B_\mu B_\nu - B_\nu B_\mu),$$

($\mathcal{E}_{\mu\nu} = -\mathcal{E}_{\nu\mu}$ = magnitude of rotation) provided that the B's obey the commutation rules (1). The first two terms in (2) are invariant just as in the pure scalar or vector case. In order that the term $\psi^+ D\psi$ remain invariant we must require:

$$(B_\mu B_\nu - B_\nu B_\mu)D - D(B_\mu B_\nu - B_\nu B_\mu) = 0. \quad (6)$$

At this point it will prove useful to summarize some of the formulas easily derivable from the commutation rules (1). These are:

$$\begin{array}{ll} B_\mu B_\nu B_\mu = 0;\ (\mu \neq \nu) & N_\mu = 2B_\mu{}^2 - 1; \\ B_\mu{}^2 B_\nu + B_\nu B_\mu{}^2 = B_\nu;\ (\mu \neq \nu) & N_\mu B_\mu = B_\mu N_\mu = B_\mu; \\ B_\mu{}^3 = B_\mu; & N_\mu B_\nu = -B_\nu N_\mu;\ (\mu \neq \nu). \end{array} \quad (7)$$

[4] W. Pauli, Rev. Mod. Phys. 13, 227 (1941).

We now consider the problem of invariance under a reflection about the X_ν axis, i.e., $X_\mu' = -X_\mu$ ($\mu \neq \nu$); $X_\nu' = X_\nu$. Pauli has shown that the proper transformation law is $\psi' = \pm N_\nu \psi$, the sign being dependent on the desired reflection properties of ψ. It can be shown that $\psi' = N_\nu \psi$ is the desired law for scalar or pseudo-vector; $\psi' = -N_\nu \psi$ for pseudo-scalar or vector. We are interested in vector coupled to scalar so our transformation matrix is:

$$S = \left(\begin{array}{c|c} -N_\nu{}^v & \\ \hline & N_\nu{}^s \end{array}\right).$$

Using this and (3) we find that invariance of $\psi^+ D\psi$ requires:

$$N_\nu D + DN_\nu = 0. \quad (8)$$

From (6) and (8) we can derive the properties of D. From (6):

$$B_\mu B_\nu D + DB_\nu B_\mu - B_\nu B_\mu D - DB_\mu B_\nu = 0.$$

Multiply by $B_\mu{}^2$ from front and rear and remember $B_\mu B_\nu B_\mu = 0$; $B_\mu{}^3 = B_\mu$. Then

$$B_\mu B_\nu D B_\mu{}^2 + B_\mu{}^2 D B_\nu B_\mu = 0.$$

But since N_μ anti-commutes with D and B_ν,

$$\begin{aligned} B_\mu B_\nu D B_\mu{}^2 = B_\mu B_\nu D(N_\mu + 1/2) &= B_\mu B_\nu(-N_\mu + 1/2)D \\ &= B_\mu(N_\mu + 1/2)B_\nu D = B_\mu B_\nu D. \end{aligned}$$

Therefore

$$B_\mu B_\nu D + DB_\nu B_\mu = 0. \quad (9a)$$

Similarly, multiplying (6) front and rear with B_μ

$$B_\mu D B_\nu + B_\nu D B_\mu = 0. \quad (9b)$$

Also we have from (8):

$$N_\mu D + DN_\mu = (2B_\mu{}^2 - 1)D + D(2B_\mu{}^2 - 1) = 0.$$

Therefore

$$B_\mu{}^2 D + DB_\mu{}^2 = D. \quad (9c)$$

Multiplying (9c) front and rear with B_μ

$$2B_\mu D B_\mu = B_\mu D B_\mu = 0. \quad (9d)$$

It can be shown that rules (9a–d) and the condition that D contain only terms coupling vector to scalar, i.e., no diagonal terms which only represent a change of mass, uniquely determine D. This may be most easily seen by considering the usual wave theory formalism where it is apparent that the requirement of relativistic invariance allows one to choose only an expression of the form $g\boldsymbol{\psi}\cdot\nabla_4\varphi$ as a bilinear, first-derivative coupling term in the Lagrangian. ($\boldsymbol{\psi}$ here is the vector four-potential, φ is the scalar potential.)

From this uniquely determined D one can, by using a special representation, also show:

$$DB_\mu D = 0; \quad D^2 B_\mu + B_\mu D^2 = B_\mu; \quad D^3 = D. \quad (9e)$$

It will now be observed that rules (9a–e) are precisely of the form of the commutation rule (1); i.e., that (1) is still valid if we consider $D=B_5$. Thus we may regard Eqs. (4) and (5) as a five-dimensional analog of the usual four-dimensional vector meson theory. The fifteen component wave function may be interpreted as a five-vector with associated ten antisymmetric tensor. Such a five-dimensional vector would have four possible orientations of spin—the three vector states and the one scalar state. It should be noted that in Eq. (4) the constant gh appears in a way exactly analogous to another momentum component, i.e., $g\hbar=-i\hbar(\partial/\partial X_5)$. This symmetry often leads to simplifications in the carrying out of computations.

C. Development of Canonical Formalism

We are now in a position to set up the energy-momentum tensor, Hamiltonian, etc. For this purpose we use the usual four-dimensional representation.

From Eqs. (4) and (5) we see that the current may be defined as:

$$S_\mu=e(\psi^+B_\mu\psi),\ \text{since}\ (\partial S_\mu/\partial X_\mu)=0. \tag{10}$$

This means that charge density is proportional to $\psi^+B_4\psi$, and that in a way exactly analogous to that used in the Dirac theory we may define the expectation value of an operator ω by $\bar{\omega}=\int\psi^+\overline{\overline{\omega B_4}}\psi d\tau$. Here the double bar represents some type of appropriate symmetrization. Remembering that the momentum p_ν is given by $(\hbar/i)(\partial/\partial X_\nu)$ we are led to assume for the energy-momentum tensor:

$$T_{\mu\nu}=(\hbar c/i)\psi^+B_\nu(\partial\psi/\partial X_\mu),$$

satisfying the requirement $(\partial T_{\mu\nu}/\partial X_\nu)=0$.

This, however, is not a symmetric form of the energy-momentum tensor, so we now apply the standard symmetrization procedure[4] and obtain after some work:

$$\theta_{\mu\nu}=i\psi^+(B_\nu B_\mu+B_\mu B_\nu-\delta_{\mu\nu})Mc^2\psi. \tag{11}$$

From the energy-momentum tensor we can write down the spin matrix (from the angular momentum):

$$S_{ik}=\frac{B_iB_k-B_kB_i}{i}, \tag{12}$$

and the Hamiltonian:

$$H=\theta_{44}=-i\psi^+N_4Mc^2\psi=\psi^*Mc^2\psi. \tag{13}$$

D. Supplementary Condition and Second Quantization

Our next step should be to quantize the field. We must, however, be careful only to quantize those components which represent independent states of the system. For while we have a fifteen component wave function it is clear that there are only eight independent states of the system—four positive ones (three vector and one scalar), and four negative ones. Since the charge density is given by $e\psi^+B_4\psi$ and we wish to quantize the theory in terms of charged particles, it is convenient to eliminate the seven components corresponding to a zero eigenvalue of B_4^2, and quantize $B_4^2\psi$ directly. We see, in fact, that the seven components corresponding to a zero eigenvalue of B_4^2 may be expressed in terms of the eight components corresponding to eigenvalue 1. First, we multiply the wave Eq. (4) from the left by $1-B_4^2$:

$$(1-B_4^2)[\partial_\mu B_\mu\psi+(K+igD)\psi]=0.$$

Here we have written $\partial_\mu=\partial/\partial X_\mu$. But

$$B_4^2B_\mu+B_\mu B_4^2=B_\mu+B_4\delta\mu_4. \tag{7}$$

Therefore

$$\{\partial_kB_kB_4^2+(1-B_4^2)K+igDB_4^2\}\psi=0,$$

k to be summed from 1–3, and

$$\psi=K^{-1}(K-\partial_kB_k-igD)B_4^2\psi.$$

Similarly,

$$\psi^+=\psi^+B_4^2(K+\partial_kB_k-igD). \tag{14}$$

We now quantize the field in the usual way and set:[5]

$$\begin{aligned}B_4^2\psi=1/h^{\frac{3}{2}}\int\sum_{k=1}^{4}\{&a_k(\mathbf{p})B_4^2U_k^+)\exp(-iE_k(\mathbf{p})t/\hbar)\\&+b_k^*(\mathbf{p})B_4^2U_k^-(\mathbf{p})\exp(iE_k(\mathbf{p})t/\hbar)\}\\&\times\exp(i\mathbf{p}\cdot\mathbf{r}/\hbar)d\mathbf{p},\\ \psi^*B_4^2=1/h^{\frac{3}{2}}\int\sum_{k=1}^{4}\{&a_k^*(\mathbf{p})U_k^+(\mathbf{p})^*B_4^2\exp(iE_k(\mathbf{p})t/\hbar)\\&+b_k(\mathbf{p})U_k^-(\mathbf{p})^*B_4^2\exp(-iE_k(\mathbf{p})t/\hbar)\}\\&\times\exp(-i\mathbf{p}\cdot\mathbf{r}/\hbar)d\mathbf{p}.\end{aligned} \tag{15}$$

Here $a_k(\mathbf{p})$ and $a_k^*(\mathbf{p})$ are annihilation and creation operators for positive mesons of momentum $\mathbf{p}$ and polarization k and $b_k(\mathbf{p})$ and $b_k^*(\mathbf{p})$ are the corresponding quantities for negative mesons of momentum $-\mathbf{p}$ and polarization k. They satisfy the usual commutation rules, e.g.,

$$[a_k(\mathbf{p})a_l^*(\mathbf{p}')-a_l^*(\mathbf{p}')a_k(\mathbf{p})]=\delta_{kl}\delta_{\mathbf{p},\mathbf{p}'}.$$

The $U_k(\mathbf{p})$'s are particular 15 component c-number solutions of the wave Eq. (4), corresponding to a momentum $\mathbf{p}$ and polarization k.

Substituting (14) and (15) in our fundamental Eq. (4), and remembering that $B_4(\mathbf{p}\cdot\mathbf{B})B_4=0$:

$$\begin{aligned}&[-E_k(\mathbf{p})B_4+Mc^2+(\mathbf{p}\cdot\mathbf{B}+g\hbar D)M^{-1}\\&\qquad\times(\mathbf{p}\cdot\mathbf{B}+g\hbar D)]B_4^2U_k^+(\mathbf{p})=0,\\&[E_k(\mathbf{p})B_4+Mc^2+(\mathbf{p}\cdot\mathbf{B}+g\hbar D)M^{-1}\\&\qquad\times(\mathbf{p}\cdot\mathbf{B}+g\hbar D)]B_4^2U_k^-(\mathbf{p})=0.\end{aligned} \tag{16}$$

It should be noted that $E_k(\mathbf{p})$ is different for vector and scalar states, as will be shown later. We now choose the U_k's to be orthogonal and normalize in such a way

[5] F. Booth and A. H. Wilson, Proc. Roy. Soc. London **175**, 490 (1940). Much of the subsequent calculation parallels this paper.

that the Hamiltonian (13) represents the energy of the system.

$$U_k^*Mc^2U_l=E_k\delta_{kl}=U_k^*B_4^2(Mc^2+(\mathbf{p}\cdot\mathbf{B}+g\hbar D)M^{-1}\times(\mathbf{p}\cdot\mathbf{B}+g\hbar D))B_4^2U_l. \quad (17a)$$

Multiplying (16) from the left by $U_l^*B_4^2$ and using (17a) we find:

$$U_l^*B_4U_k=\rho_{kl}. \quad (17b)$$

We have used the fact that $B_4^3=B_4$. Here

$$\rho_{kl}=\delta_{kl} \text{ for } k=1\cdots 4,$$
$$=-\delta_{kl} \text{ for } k=5\cdots 8,$$

where $k=1\cdots 4$ applies to positive, $k=5\cdots 8$ to negative states. We now consider the eight linearly independent vectors $B_4^2U_k$ as forming an eight by eight non-singular matrix B_4^2U. We note from (17b) that the inverse matrix for B_4^2U is given by $\rho(U^*B_4^2)B_4$. The equation $(B_4^2U)(B_4^2U)^{-1}B_4=B_4$ then gives the inverse orthogonality relation:

$$\sum_{k,\,l=1\cdots 8} B_4^2U_k\rho_{kl}U_l^*B_4^2=B_4. \quad (18)$$

E. Introduction of the Electromagnetic Field and Writing Down of the Hamiltonian

The next step is to introduce the electromagnetic field which we do in the usual gauge-invariant way, replacing $\partial\psi/\partial X_k$ with $(\partial\psi/\partial X_k)-(ie/\hbar c)A_k\psi$ and $\partial\psi^+/\partial X_k$ with $(\partial\psi^+/\partial X_k)+(ie/\hbar c)\,A_k\psi^+$ where A_k is the 4-vector potential of the electromagnetic field:

$$\mathbf{A}=1/h^{\frac{3}{2}}\sum_{\mathbf{k},\lambda}(2\pi\hbar^2c/k)^{\frac{1}{2}}\mathbf{e}_{\mathbf{k},\lambda}[q_{\mathbf{k},\lambda}\exp(i\mathbf{k}\cdot\mathbf{r}/\hbar)+q_{\mathbf{k},\lambda}^*\exp(-i\mathbf{k}\cdot\mathbf{r}/\hbar)]. \quad (19)$$

Here q and q^* are the photon annihilation and creation operators.

Substituting (19), (15), and (14) into the Hamiltonian (13), integrating over the volume, and expanding in powers of the electric charge e, we obtain:

$$\bar{H}_0=\int H_0d\mathbf{r}=c^2\sum_{k,\,l=1\cdots 4}\int[a_l^*(\mathbf{p})U_l^+(\mathbf{p})^*+b_l(\mathbf{p})U_l^-(\mathbf{p})^*]B_4^2[M-(i/c)\mathbf{p}\cdot\mathbf{B}-(ig\hbar/c)D]\times N_4M^{-1}[M-(i/c)\mathbf{p}\cdot\mathbf{B}-(ig\hbar/c)D]B_4^2\times[a_k(\mathbf{p})U_k^+(\mathbf{p})+b_k^*(\mathbf{p})U_k^-(\mathbf{p})]d\mathbf{p}. \quad (20a)$$

$$\bar{H}_1=-\frac{1}{2\pi}\frac{e}{(\hbar c)^{\frac{1}{2}}}\frac{1}{(|\mathbf{p}'-\mathbf{p}|)^{\frac{1}{2}}}\sum_{\substack{k,\,l=1\cdots 4\\ \lambda=1,\,2}}\int\int[a_l^*(\mathbf{p}')U_l^+(\mathbf{p}')^*+b_l(\mathbf{p}')U_l^-(\mathbf{p}')^*]B_4^2\times\{q_{\mathbf{p}'-\mathbf{p},\lambda}[(\mathbf{e}_{\mathbf{p}'-\mathbf{p},\,\lambda}\cdot\mathbf{B})M^{-1}(\mathbf{p}\cdot\mathbf{B}+g\hbar D)+(\mathbf{p}'\cdot\mathbf{B}+g\hbar D)M^{-1}(\mathbf{e}_{\mathbf{p}'-\mathbf{p},\,\lambda}\cdot\mathbf{B})]+q^*_{\mathbf{p}-\mathbf{p}',\lambda}[(\mathbf{e}_{\mathbf{p}-\mathbf{p}',\,\lambda}\cdot\mathbf{B})M^{-1}(\mathbf{p}\cdot\mathbf{B}+g\hbar D)+(\mathbf{p}'\cdot\mathbf{B}+g\hbar D)M^{-1}(\mathbf{e}_{\mathbf{p}-\mathbf{p},'\lambda}\cdot\mathbf{B})]\}\times B_4^2[a_k(\mathbf{p})U_k^+(\mathbf{p})+b_k^*(\mathbf{p})U_k^-(\mathbf{p})]d\mathbf{p}d\mathbf{p}'. \quad (20b)$$

$$\bar{H}_2=\frac{1}{4\pi^2}\frac{e^2}{\hbar c}(1/kk')^{\frac{1}{2}}\sum_{\substack{l,\,m=1\cdots 4\\ \lambda,\lambda'=1,2}}\int\int\int\int[a_l^*(\mathbf{p}')U_l^+(\mathbf{p}')^*+b_l(\mathbf{p}')U_l^-(\mathbf{p}')^*]B_4^2[M^{-1}(\mathbf{B}\cdot\mathbf{e}_{\mathbf{k},\,\lambda})(\mathbf{B}\cdot\mathbf{e}_{\mathbf{k}',\boldsymbol{\lambda}'})]\times[q_{\mathbf{k},\,\lambda}q_{\mathbf{k}',\,\lambda'}\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}+\mathbf{k}')+q_{\mathbf{k},\,\lambda}q^*_{\mathbf{k}',\,\lambda'}\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}-\mathbf{k}')+q^*_{\mathbf{k},\,\lambda}q_{\mathbf{k}'\lambda'}\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}+\mathbf{k}')+q^*_{\mathbf{k},\,\lambda}q^*_{\mathbf{k}',\,\lambda'}\times\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}-\mathbf{k}')]\times B_4^2[a_m(\mathbf{p})U_m^+(\mathbf{p})+b_m^*(\mathbf{p})U_m^-(\mathbf{p})]d\mathbf{k}d\mathbf{k}'d\mathbf{p}d\mathbf{p}'. \quad (20c)$$

In addition if there is an external potential ϕ we have a term:

$$\bar{H}_{\text{static}}=\int\rho\phi d\mathbf{r}=e\sum_{k,\,l=1\cdots 4}\int\int[a_l^*(\mathbf{p}')U_l^+(\mathbf{p}')^*+b_l(\mathbf{p}')U_l^-(\mathbf{p}')^*]B_4^2B_4\phi(\mathbf{p}'-\mathbf{p})B_4^2\times[a_k(\mathbf{p})U_k^+(\mathbf{p})+b_k^*(\mathbf{p})U_k^-(\mathbf{p})]d\mathbf{p}'d\mathbf{p}; \quad (20d)$$

where

$$\phi(\mathbf{p}'-\mathbf{p})=\int\exp(-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r})\phi(\mathbf{r})d\mathbf{r}.$$

F. Calculational Techniques

With the Hamiltonian (20) we may select matrix elements for any desired process and carry out the calculations in a manner entirely analogous to that used in the usual Dirac electron theory. As in the Dirac theory one eventually reduces the calculation to a summation of the matrix element of some operator over all eight states corresponding to a given momentum, and then uses the rule derived from (19):

$$\sum_{k=1\cdots 8}U_k^*B_4^2\theta B_4^2U_k\rho_{kk}=\operatorname{Spur}(U^*B_4^2)\theta(B_4^2U)\rho=\operatorname{Spur}B_4\theta.$$

As in the Dirac theory it is necessary to introduce annihilation operators so that a summation over a small number of states may be replaced by a summation over all eight states.

In our case it is necessary to introduce two types of annihilation operators, one to separate vector and scalar states, the other to distinguish between positive and negative mesons. We first discuss the annihilation operator for distinguishing between spin states.

Consider a solution U_k of the wave Eq. (4). If we can find a transformation matrix S such that SU_k has only $1\cdots 10$ components if k is a vector state, and $11\cdots 15$ components if k is a scalar state, then:

$$B_4^2S^{-1}ZSU_k=B_4^2U_k \text{ if } k \text{ is a vector state},$$
$$=0 \text{ if } k \text{ is a scalar state}, \quad (21)$$

where Z is the matrix

$$Z=\left(\begin{array}{ccc|ccc} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ \hline & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{array}\right). \tag{22}$$

We shall show that

$$S=\exp\left(igK^{-1}\left(\frac{K_vK_s}{g^2+K_vK_s}\right)^{\frac{1}{2}}D\right) \tag{23}$$

is such a matrix. The wave Eq. (4) tells us that for a plane wave solution U_k we have

$$[-E_kB_4+ic\mathbf{p}\cdot\mathbf{B}+Mc^2+ig\hbar cD]U_k{}^+=0.$$

(We have assumed that U_k is a positive state. This in no way affects the argument below.) Then

$$[S(-E_kB_4+ic\mathbf{p}\cdot\mathbf{B}+Mc^2+ig\hbar cD)S^{-1}]SU_k{}^+=0.$$

Using (23) for S and the commutation rules (1), (7), and (9), we arrive after some algebra at the equation:

$$\left\{-E_kB_4+ic\mathbf{p}\cdot\mathbf{B}+M_c{}^2\left(\frac{g^2+K_vK_s}{K_vK_s}\right)^{\frac{1}{2}}\right.$$
$$\left.+ig\left(\frac{K_vK_s}{g^2+K_vK_s}\right)^{\frac{1}{2}}K^{-1}(-E_kB_4+ic\mathbf{p}\cdot\mathbf{B})D\right\}SU_k{}^+=0.$$

We now prove that the term containing D is separately equal to zero. From the last equation we have:

$$SU_k{}^+=-\left(\frac{K_vK_s}{g^2+K_vK_s}\right)^{\frac{1}{2}}\frac{1}{c^2}M^{-1}\{-E_kB_4+ic\mathbf{p}\cdot\mathbf{B}\}$$
$$\times\left\{1+ig\left(\frac{K_vK_s}{g^2+K_vK_s}\right)^{\frac{1}{2}}K^{-1}D\right\}SU_k{}^+.$$

$$\therefore\quad (-E_kB_4+ic\mathbf{p}\cdot\mathbf{B})DSU_k{}^+=0,\ \text{since}\ B_\mu DB_\mu=0.$$

So we finally obtain

$$\left[-E_kB_4+ic\mathbf{p}\cdot\mathbf{B}+Mc^2\left(\frac{g^2+K_vK_s}{K_vK_s}\right)^{\frac{1}{2}}\right]SU_k{}^+=0. \tag{24}$$

If we now consider the special representation (3) we see that in Eq. (24) the ten-component wave function and the five-component wave function have been uncoupled so that the wave Eq. (24) for SU_k is simply the sum of the vector and scalar wave equations. If SU_k is to represent a vector state it must satisfy the vector wave equation, hence have only $1\cdots 10$ components. Thus we have proven that (23) is the desired transformation matrix.

Using (23) and remembering that $ZD+DZ=D$ while Z commutes with all other matrices we find after algebra:

$$\begin{aligned} B_4{}^2S^{-1}ZSU_k &= B_4{}^2[Z+(2Z-1)\\ &\quad\times(g/g^2+K_vK_s)D(\mathbf{p}\cdot\mathbf{B}/h)]B_4{}^2U_k,\\ &= B_4{}^2U_k \text{ if } k \text{ is a vector state,}\\ &= 0 \text{ if } k \text{ is a scalar state.}\end{aligned} \tag{25}$$

Thus the quantity in square brackets in (25) is the desired annihilation operator.

In order to separate positive and negative states we use (16) which tells us:

$$\begin{aligned}&(1/2E_k)B_4[E_kB_4+Mc^2+(\mathbf{p}\cdot\mathbf{B}+g\hbar D)M^{-1}\\ &\quad\times(\mathbf{p}\cdot\mathbf{B}+g\hbar D)]B_4{}^2U_k\\ &\qquad= B_4{}^2U_k \text{ if } U_k \text{ is a positive state,}\\ &\qquad= 0 \text{ if } U_k \text{ is a negative state.}\end{aligned} \tag{26}$$

Equipped thus with the commutation rules (1) and (9), the inverse orthogonality relation (18), and the annihilation operators (25) and (26) we are able to reduce all calculations to taking the spur of a matrix. This implies that we always sum or average over the different vector polarizations. Using a special representation like (3) it can be shown that if $D=B_5$, then

$$\begin{aligned} &S_pB_i{}^2=8;\\ &S_pB_i{}^2B_j{}^2=4(i\neq j);\\ &S_pB_i{}^2B_j{}^2B_k{}^2=2(i\neq j\neq k);\\ &S_pB_i{}^2B_j{}^2B_k{}^2B_l{}^2=1(i\neq j\neq k\neq l);\\ &S_pB_i{}^2B_j{}^2B_k{}^2B_l{}^2B_m{}^2=0(i\neq j\neq k\neq l\neq m).\end{aligned} \tag{27}$$

Also the spur of any matrix containing an odd power of one of the B's must vanish.

G. Physical Interpretation of $\bar{H}_0$

We now apply these techniques to the evaulation of the Hamiltonian $\bar{H}_0$ (20a) and find after some calculation:

$$\bar{H}_0=\sum_{k=1\cdots 4}\int E_k(\mathbf{p})[a_k{}^*a_k+b_k{}^*b_k+1]d\mathbf{p}, \tag{20e}$$

where

$$\begin{aligned} E_k(\mathbf{p}) &= (M_v{}^2c^4+g^2h^2c^2(M_v/M_s)+p^2c^2)^{\frac{1}{2}}\\ &\qquad \text{if } k \text{ is a vector state;}\\ E_k(\mathbf{p}) &= (M_s{}^2c^4+g^2h^2c^2(M_s/M_v)+p^2c^2)^{\frac{1}{2}}\\ &\qquad \text{if } k \text{ is a scalar state.}\end{aligned} \tag{28}$$

Thus we see that the coupling has given a "mesomatic" mass to the states. The mass as defined by (28) [(i.e., from $E=(\mu^2c^4+p^2c^2)^{\frac{1}{2}}$] is clearly to be considered as the physical mass, with M merely being an auxiliary constant without obvious physical significance.

It should be noted that we have effectively uncoupled the vector and scalar states by writing the Hamiltonian in the form (20e). That is, we would have obtained the identical Hamiltonian if we had assumed uncoupled vector and scalar states of the appropriate masses. The term $\bar{H}_1$, however, cannot be separated in this way, so that we have not merely made a trivially different formalism. The physical meaning of this is clear. There can be no transition from a vector to a scalar meson unless we introduce another particle so as to conserve angular momentum. Such a particle is the photon, and it is precisely this effect of the excited state on electromagnetic interactions that we are seeking.

III. ELECTROMAGNETIC INTERACTIONS

A. Lifetime of Excited State

From the expression (20b) for the Hamiltonian $\bar{H}_1$ we see that it is possible for a meson in an excited state to decay with emission of a photon to the ground state. If we assume, for example, that the scalar of mass μ_s is the excited state, then a positive scalar meson at rest can decay into a vector meson of mass μ_v and momentum $\mathbf{p}$, and a photon of momentum $-\mathbf{p}$. Conservation of energy tells us:

$$\mu_s c^2 = (\mu_v^2 c^4 + p^2 c^2)^{\frac{1}{2}} + pc.$$

The lifetime for such a transition is given by:

$$1/\tau = (2\pi/h)|\bar{H}_{vs}|^2 \rho_E,$$

with the matrix element:

$$\bar{H}_{vs} = -\frac{1}{2\pi}\frac{e}{(\hbar c)^{\frac{1}{2}}}\frac{1}{(p)^{\frac{1}{2}}} U_v^+(\mathbf{p})^* B_4^2[(\mathbf{e}_{-\mathbf{p},\lambda}\cdot\mathbf{B})M^{-1}g\hbar D + (\mathbf{p}\cdot\mathbf{B} + g\hbar D)M^{-1}(\mathbf{e}_{-\mathbf{p},\lambda}\cdot\mathbf{B})]B_4^2 U_s^+(0).$$

Using the spur techniques discussed previously we may evaluate the lifetime and find:

$$1/\tau = \tfrac{1}{4}(e^2/\hbar c)K^2(\mu_v c^2/h)(\mu_s/\mu_v - \mu_v/\mu_s)^3, \tag{29}$$

where we have written $K^2 = (g^2\hbar^2/M_v M_s c^2)$, a dimensionless constant which represents the strength of the coupling between vector and scalar states. We see that for any "strong-coupled" case, i.e., a case where $K^2 \approx 1$, the lifetime of the excited state will be very short unless the energies are very close together. For example, if $\mu_v c^2 = 200$ Mev; $\mu_s c^2 = 400$ Mev; $K^2 = 1$, then $\tau = 5\times 10^{-22}$ sec. Hence the excited state is unobservable directly.

The photons which have been observed at Berkeley might possibly be such decay photons. Further experimental evidence is needed to decide this point. The coincidence measurements to determine whether the photons are produced singly or in pairs are of crucial importance in this connection.

B. Rutherford Scattering

An experiment by which the existence of such an excited state might be detected is scattering by a Coulomb field. Let us image a vector (ground state) meson of mass μ_v, charge e, and momentum $\mathbf{p}$ impinging on a fixed scattering center of charge Ze. Then, using the static Hamiltonian (20d), we find for the Born approximation scattering cross section:

$$\sigma(\theta, \varphi) = \sigma_v(\theta, \varphi) + \sigma_s(\theta, \varphi),$$

where $\sigma_v(\theta, \varphi)$ represents the cross section for the final state being vector, $\sigma_s(\theta, \varphi)$ for it being scalar. $\sigma_v(\theta, \varphi)$ is the usual vector cross section:

$$\sigma_v(\theta, \varphi) = \left(\frac{Ze^2}{2\mu_v v^2}\right)^2\left[\left(1 - \frac{v^2}{c^2}\right) + \tfrac{1}{6}\sin^2\theta\frac{v^4}{c^4}\right]\frac{1}{\sin^4(\theta/2)}d\Omega,$$

while

$$\sigma_s(\theta, \varphi) = \tfrac{1}{3}K^2\left(\frac{Ze^2}{\mu_v c^2}\right)^2\left[\frac{pp'[(c^2/v^2) - 1]}{p^2 + p'^2 - 2pp'\cos\theta} + \frac{pp'^3\sin^2\theta}{(p^2 + p'^2 - 2pp'\cos\theta)^2}\right]d\Omega.$$

Here p' is the momentum of the outgoing scalar meson as given by the conservation of energy:

$$p'^2 = p^2 + \mu_v^2 c^2 - \mu_s^2 c^2.$$

$\sigma_s(\theta, \varphi)$ is of course zero for $p^2 + \mu_v^2 c^2 - \mu_s^2 c^2 < 0$.

This cross section for production of excited-state mesons is proportional to K^2, hence will be appreciable only for a "strong-coupled" theory. As we have already seen, in such a theory the excited state decays almost instantaneously if the mass difference is at all appreciable. Hence the total cross section σ_s represents the cross section for production of decay photons. We find for this cross section:

$$\sigma_s = \int \sigma_s(\theta, \varphi)d\Omega = (\pi/3)K^2(Ze^2/\mu_v c^2)^2 \times \{[2(c^2/v^2) - 1 + (p'^2/p^2)] \times \ln(p + p'/p - p') - 2(p'/p)\}. \tag{30}$$

In the case where the scalar is the ground state, we have exactly the same matrix element for the transition, and we find for the cross section for excited-state production;

$$\sigma_{s\to v} = \pi K^2(Ze^2/\mu_s c^2)^2\{2[(c^2/v^2) - 1]\ln(p + p'/p - p') + (\mu_s/\mu_v)^2[(1 + (p'^2/p^2))\ln(p + p'/p - p') - 2(p'/p)]\}.$$

As in the previous case, p' refers to the momentum of the excited state meson.

In the non-relativistic region this is appreciable, and is considerably larger than the bremsstrahlung cross section, so that the existence of a low lying strongly coupled excited state would manifest itself by the production of many photons on passage of a meson beam through a heavy target.

In the relativistic limit the ratio of the cross section (30) to the bremsstrahlung cross section for a vector meson is a factor of the order of magnitude $K^2(\hbar c/e^2)(\mu_v e^2/E)$. Thus for $E \approx 137\mu_v c^2$, the upper limit of reliability of the theory, a strongly coupled scalar excited state would lead by Rutherford scattering alone to burst production too great to be compatible with the experimental evidence on μ-mesons. This would not be true, however, if the mass difference of the states either were great enough so that the excited state could not be produced, or small enough so that the decay photons had very low energy.

For the degenerate case where $\mu_v=\mu_s$ we obtain for the Rutherford cross section:

$$\sigma_{\mu_v=\mu_s}=(Ze^2/2\mu v^2)^2[(1-v^2/c^2)+\tfrac{1}{4}K^2(v^2/c^2)(1-\cos\theta) +\tfrac{1}{8}(1+K^2)(v^4/c^4)\sin^2\theta](1/\sin^4\theta/2)d\Omega. \quad (31)$$

For this case both initial and final states may be either vector or scalar, with the average over the initial state taken such that the vector is given weight three, the scalar weight one.

C. High Energy Compton Effect

As is well known, the total Compton effect cross section for vector mesons behaves like E^2 at high energies in the barycentric system.[6] It has been shown by Wilson[7] that this result is not cut down by radiation damping until $E>(137)^{\frac{1}{2}}\mu c^2$, so that the Christy and Kusaka argument against the μ-meson having spin 1 is still valid.

We will inquire whether the assumption of a scalar excited state allows for destructive interference and cancellation of the E^2 terms as discussed in the Introduction. The calculation is very tedious and has only been carried out for the case $E\gg\mu_sc^2\geq\mu_vc^2$.

The scattering may occur directly through the two-quantum Hamiltonian $\bar{H}_2$, or via any of four possible intermediate states, through $\bar{H}_1$. These are:

(1) Absorption of the incident photon by the meson, followed by emission of the scattered photon;
(2) Emission of the scattered photon followed by absorption of the incident photon;
(3) Absorption of the incident photon, accompanied by creation of a meson pair, followed by annihilation of one member of the pair with the original meson, accompanied by emission of the scattered photon;
(4) Emission of the scattered photon accompanied by creation of a meson pair, followed by annihilation of one member of the pair with the original meson and absorption of the incident photon.

Initially the meson is assumed to be in the ground (vector) state, while the intermediate and final states may be either vector or scalar.

After a very long calculation we find for the total cross section in the case $E\gg\mu_sc^2\geq\mu_vc^2$:

$$\sigma=(5\pi/9)(e^2/\mu_vc^2)^2(E/\mu_vc^2)^2(1+\tfrac{3}{2}K^2+\tfrac{1}{2}K^4) =(1+\tfrac{3}{2}K^2+\tfrac{1}{2}K^4)\sigma_{\text{vector}}, \quad (32)$$

where σ_{vector} is the ordinary vector meson cross section.

For the degenerate case $\mu_v=\mu_s$,

$$\sigma=\tfrac{3}{4}\sigma_{\text{vector}}(1+2K^2+K^4). \quad (33)$$

The results of this special case thus tend to indicate that the high-energy divergences of the electromagnetic cross sections of higher spin particles are made worse rather than alleviated by the assumption of excited states.

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[6] W. Pauli, Rev. Mod. Phys. 13, 230 (1941).
[7] A. H. Wilson, Proc. Camb. Phil. Soc. 37, 311 (1941).