Double Scattering of Protons in Helium

D. C. Dodder
University of Minnesota, Minneapolis, Minnesota
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THE phase-shift analysis[1] of the recent Minnesota proton-alpha-scattering experiments[2] reduces the data to three parameters, an s-wave phase shift and two p-wave phase shifts for partial waves of total angular momentum $\frac{1}{2}\hbar$ and $3\hbar/2$. The knowledge of the differential cross section as a function of angle is not sufficient, however, to remove an ambiguity which results in a case of the p-wave phase shifts. The two sets of p-wave phase shifts which satisfy the data correspond to normal and inverted p-doublets in the Li^5 nucleus. For each set, however, the two phase shifts pass through 90° at considerably different energies, so that there is a large range of energies over which the scattered protons are partially polarized. The polarizations at a scattering angle of 90° in the center of mass coordinate system corresponding to the two possible doublets are given as functions of energy in reference 1. Recently Wolfenstein[3] has discussed the possibility of the detection of such scattered polarized protons by means of a second scattering, as had been proposed for the similar case of neutrons and helium by Schwinger.[4] In order to determine the feasibility of performing the experiment of the double scattering with protons, the angular distribution of yield from the second scattering again at 90° in the center of mass system has been calculated as a function of energy for each of the possible doublets.

The amplitudes of scattered waves arising from a single scattering expressed in terms of eigenfunctions of spin-component in the direction of the initial beam may be re-expressed in terms of eigenfunctions of spin-component in the direction of the secondary beam. These amplitudes can now be considered as the amplitudes of plane waves moving in the direction of the secondary beam. They will then be refracted in the same fashion in the second scattering. The final intensity will be the average of the intensities of the waves corresponding to the two initial spin directions.

The final intensity can be written:

$$I=\frac{1}{k^2\bar{k}^2}\{|a\bar{a}|^2+|b\bar{a}|^2+|a\bar{b}|^2+|b\bar{b}|^2+\cos\Phi(\bar{b}^*\bar{a}-\bar{a}^*\bar{b})(b^*a-a^*b)\}$$

where

$$|a|^2=(\tfrac{1}{2}\sin 2\delta_0-\eta\cos\eta\ln 2)^2+(\sin^2\delta_0-\eta\sin\eta\ln 2)^2$$
$$|b|^2=\sin^2(\delta_1^--\delta_1^+)$$
$$b^*a-a^*b=2i\sin(\delta_1^--\delta_1^+)\{\sin\delta_0\sin(\delta_0-\phi_1-\delta_1^+-\delta_1^-)-\eta\sin(\eta\ln 2-\phi_1-\delta_1^+-\delta_1^-)\}$$

and where barred quantities are evaluated at the energy, in the laboratory system, of the protons in the secondary beam. Φ is the azimuthal angle about the secondary beam measured from the direction of the initial beam. Other notation is the same as in reference 1.

The product of the differential cross section of the two scatterings in the laboratory system may be written $M+N\cos\Phi$. If M and N are measured in square barns and E is the laboratory energy in Mev of the initial protons in the laboratory system, the results are as given in Table I.

It is seen that, as expected, a measurement of the asymmetry as a function of energy would conclusively decide which doublet exists. Indeed a single measurement at $E=2.8$ Mev would be decisive since for the normal doublet the scattered protons are almost entirely in the direction of the initial beam, while in the inverted doublet they are predominantly in the opposite direction.

Because of the low cross sections the intensity from the double scattering will not be high, but by using a large initial beam, large solid angles, and a high target density a reasonable yield seems feasible. At the 2.8 Mev energy, assuming original current 5 microamperes, solid angles of 0.05 steradians, target density corresponding to a pressure of 1 atmosphere, and effective length of each scattering volume of 1 cm, one obtains a yield of 1.2 scattered proton/sec. This value would seem to indicate that the measurement of the polarization effect by means of the double scattering might be quite possible. Instrumentation for performing the experiment is now under way in the University of Minnesota laboratory.

TABLE I. Angular dependence of products of cross sections for double scattering.

E	1.6	2.0	2.4	2.8	3.2
M	0.008	0.012	0.016	0.021	0.025
			Normal Doublet		
N	−0.001	−0.0055	0.001	0.017	0.011
			Inverted Doublet		
N	−0.001	−0.006	−0.011	−0.011	−0.007

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A Lower Limit for the Binding Energy of the Deuteron

G. R. Bishop, C. H. Collie, H. Halban, and R. Wilson
The Clarendon Laboratory, Oxford, England
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A SUMMARY of previous determinations of the binding energy of the deuteron has been given by Stevens[1] who found an average value of $-Q_\gamma=2.187\pm0.011$ Mev. However, recent measurements[2,3] give $-Q_\gamma=2.237\pm0.005$ Mev, in closer agreement with earlier observations.[4,5] A lower limit can be placed upon Q_γ by making measurements on the photo-disintegration of deuterium by the γ-rays from radio-gallium, Ga^{72}, which emits two lines in good intensities with energies of 2.21 Mev, and 2.51 Mev, as well as lower energy lines.[6]

Since the 2.21-Mev line lies between the two values of Q_γ which are in question, it is of interest to observe whether it can produce photo-neutrons from deuterium. If it does, then $-Q_\gamma$ must lie below 2.21 Mev.

This has been investigated by comparing the photo-disintegration produced in deuterium by the γ-rays of radio-gallium and radio-thorium. This can be done in two ways, by measuring either the photo-protons or the photo-neutrons produced. The first is to use an ionization chamber filled with deuterium gas,[7] for which the discriminator bias is chosen to count the photo-protons produced by the 2.51-Mev line of Ga^{72} and the 2.62-Mev line of RdTh, but none of lower energy that may be produced by the 2.21-Mev line of Ga^{72}. The second is to measure the ratio of the rates of production of photo-neutrons by these sources in a sphere of heavy water, by the well-known method of integrating the slow neutron density in a water tank with manganese as detector. This method detects all photo-neutrons independently of their energy, and, in particular, if the 2.21-Mev γ-line of Ga^{72} produces photo-disintegration, the photo-neutron ratio will differ from the photo-proton ratio observed in the ionization chamber.

The radio-thorium source used was a 200-mc standard, enclosed in a platinum cylinder 5 mm in length and 5 mm diameter. The gallium source was a sphere 15 mm in diameter of a gallium-nickel alloy. The alloy contained 12 percent of nickel and remained hard up to 100°C. This was used in preference to pure gallium which melts at 30°C. The source was enclosed in a copper sphere and irradiated with neutrons in the pile at the Atomic Energy Research Establishment at Harwell. The total activity for a long irradiation was 0.6C.

TABLE I.

Ratio: $\frac{\text{RdTh neutrons}}{\text{Ga neutrons}}$	Ratio: $\frac{\text{RdTh protons}}{\text{Ga protons}}$	Ratio: $\frac{\text{Photo-proton ratio}}{\text{Photo-neutron ratio}}$
0.96 ± 0.05	0.98 ± 0.04	1.02 ± 0.08
1.41 ± 0.04	1.43 ± 0.04	$1.01_4 \pm 0.06$

For the photo-proton ratio, the number of counts from the ionization chamber was recorded at the same discriminator bias voltage for each source. A correction for "wall effect" in the chamber has to be considered. If the range of a proton were proportional to its energy, a constant fraction of the photo-protons would be unrecorded. As it is not, the wall effect corrections for the two sets of photo-protons (estimated to be 150 kv and 210 kv) are different. It is possible to adjust the deuterium pressure ($2\frac{1}{2}$ atmos.) so that the total correction is only 10 percent, in which case the uncertainty in the ratio of counts is unlikely to exceed 1 percent.

To integrate the photo-neutron flux, the sources were placed at the center of a sphere of heavy water of 8-cm radius, contained in a thin copper vessel. This was suspended at the center of a cylindrical tank, 60 cm in diameter and 60 cm deep, containing an aqueous solution of $MnSO_4$. After irradiation for a known time, the whole solution was well mixed by a circulating pump and the mean activity determined in an aliquot fraction with a pair of thin glass-walled immersion type Geiger-Müller counters.

Results of two series of measurements are shown in Table I. Clearly the low energy γ-quanta from gallium are not producing many photo-neutrons.

By combining the experimental value[6] of the ratio

$$\frac{\text{2.51-Mev quanta}}{\text{2.21-Mev quanta}}$$

with the theoretical variation in cross section for the photo-disintegration of deuterium near the threshold,[8,9] one can calculate the expected value of the ratio we have measured for any value of Q_γ. For $-Q_\gamma = 2.185$ Mev the value of the ratio would be 1:65; if, as an extreme case, one supposes our results are consistent with a value of 1:1 for the ratio, then Q_γ and the quantum energy of the Ga^{72} γ-ray cannot differ by more than 3 kev, and therefore the binding energy of the deuteron cannot be less than 2.207 Mev.

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The Magnetic Moments of Sn^{117}, Sn^{119}, and Pb^{207}*

W. G. PROCTOR
Department of Physics, Stanford University, Stanford, California
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USING the nuclear induction spectrometer described in a previous letter,[1] the magnetic moments of the nuclei Sn^{117}, Sn^{119}, and Pb^{207} have been remeasured. The results are in good agreement with those from hyperfine structure measurements through which they were previously known, and which indicated for both tin nuclei a spin of $\frac{1}{2}$[2] and a magnetic moment of -0.89 nuclear magneton,[3] and for the nucleus Pb^{207} a spin $\frac{1}{2}$[4] and a magnetic moment of $+0.6$ nuclear magneton.[5]

The tin resonances were observed in the neighborhood of 7 MC, with a magnetic field near 4500 gauss, using a sample consisting of an aqueous solution of 5.3 molar $SnCl_2$ with 1.0 molar $MnCl_2$. These frequencies were compared to that of the resonance of Na^{23} when the above sample was replaced with one consisting of an aqueous solution of 0.69 molar NaCl with 1.0 molar $MnCl_2$. The resonances of Sn^{117}, Sn^{119}, and Na^{23} were observed in that order until six of each had been recorded. After correcting for magnetic field drifts, the ratios of the resonant frequencies in the same field were found to be

$$\nu(Sn^{117})/\nu(Na^{23}) = 1.3464 \pm 0.0004,$$
$$\nu(Sn^{119})/\nu(Na^{23}) = 1.4090 \pm 0.0005,$$
$$\nu(Sn^{119})/\nu(Sn^{117}) = 1.0465 \pm 0.0003.$$

Making a correction for electronic diamagnetism,[6] which here has been assigned an error of 15 percent, and taking $\frac{3}{2}$ for the spin and $+2.217 \pm 0.002$ nuclear magnetons for the magnetic moment of Na^{23},[7] one obtains

$$\mu(Sn^{117}) = -(1.000 \pm 0.001)\mu_N,$$
$$\mu(Sn^{119}) = -(1.047 \pm 0.001)\mu_N.$$

The ratio of this magnetic moments is, of course, given by the ratio of their resonant frequencies. Sn^{117} and Sn^{119} have respectively a natural abundance of 9.1 percent and 9.8 percent,[8] or a ratio of abundance of 0.93; the relative signal amplitudes, when a correction for their different gyromagnetic ratios is made, was 0.86 ± 0.12, which served to identify the isotopes. By comparing the recorded tin signals with the Na^{23} signals, the sign of the magnetic moments of both tin nuclei was seen to be negative, and the signal amplitudes were consistent with spin $\frac{1}{2}$.

Using as a sample an aqueous solution of 1.0 molar $Pb(C_2H_3O_2)_2$ and 0.8 molar $Mn(C_2H_3O_2)_2$, the frequency of the lead resonance which occurred near 5.8 MC at 6600 gauss was repeatedly compared with that of the Na^{23} resonance from the sodium sample above. A resonance frequency ratio of

$$\nu(Pb^{207})/\nu(Na^{23}) = 0.7901 \pm 0.0002$$

was found which leads, in analogy to the above results from the magnetic moments of the tin nuclei, to

$$\mu(Pb^{207}) = +(0.588 \pm 0.001)\mu_N.$$

The sign of the magnetic moment was found to be positive, and the signal amplitudes indicate a spin of $\frac{1}{2}$.

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The Application of Dyson's Methods to Meson Interactions

P. T. MATTHEWS
Clare College, Cambridge, England
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BY developing a combination of the electrodynamic theories of Schwinger[1] and Feynman,[2] Dyson[3] has given comparatively simple rules whereby the matrix element of the Hamiltonian for an effect such as the Lamb-Retherford shift[4] can be written down directly in terms of invariant functions $D_F(x)$ and $S_F(x)$.