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Waves and Electrons Traveling Together—A Comparison between Traveling Wave Tubes and Linear Accelerators

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A general theory is needed that would include both extreme cases of traveling wave amplifiers and linear accelerators without introducing any restriction on the magnitudes of either fields or space charge. A special case is given as an example and represents a curious sort of electromagnetic shock waves.

I. GENERAL DISCUSSION

THERE is a variety of structures where the electronic engineer uses a flow of electrons and an electromagnetic wave traveling in the same direction with the same velocity. Such is the case in all the "traveling wave" tubes, and in many other structures with drifting electrons. The multicavity magnetron has many points in common with the traveling wave devices, but for the fact that the magnetic field bends the trajectories and the cavities slow down the propagation of electric fields.

On quite a different scale, waves and particles (ions) traveling together are found in most large scale modern accelerators. Whenever the word "synchro" is used in the title, it simply means some device of this type; synchro-betatron, synchro-cyclotron, etc., are well-known examples of the fact. As for the linear accelerators, they represent a structure exactly similar to the traveling wave amplifier tube, but for a different scale.

A comparison of these two classes of devices immediately leads to the question: how is it possible that they be used for exactly opposite purposes? (A) In the traveling wave amplifiers, or the magnetrons, the synchronization of waves and electrons results in an energy transfer from the electrons to the wave and a powerful amplification of the wave. (B) Linear accelerators and "synchro-" devices are built for the opposite purpose, namely, an energy transfer from the wave to the particles, resulting in an acceleration of the ions.

How is this apparent contradiction to be solved? The answer is double: traveling wave tubes work with high space charge and weak waves, the velocity of the wave being maintained constant. "Synchro-" devices, on the contrary, use very low space charge and high power waves, the velocity of the waves being progressively increased.

In the first case, theory and experiment show that the net result is a light bunching of the electrons in the beam and a large amplification of the wave. In the second case, a strong bunching of the particles is soon obtained, the ion bunches being afterward accelerated when the wave velocity is increased.

This confronts the theoretician with a very interesting task of discussing in general terms, without all the approximations used by the engineers, a problem of waves and charged particles traveling together, when both the wave and the space-charge density may be large.

When an attempt is made at such a theoretical discussion, two points appear clearly of importance: *I.* Means providing for the lateral stability of the beam are essential, in order to prevent electrons (or ions) from escaping sidewise. *II.* The whole problem with space charge is very strongly non-linear. The two extreme cases (A) and (B) correspond to simple approximations that allow for a reduction to linear equations.

Many non-linear problems have general characteristics in common; a final stage of the motion is reached that may be very unsensitive to the initial conditions. The question of whether such a stable final motion can be found in the present problem will now be discussed, and a composite stable wave with space charge suggested for that purpose.

2. SLOW GUIDED WAVE

Let us first consider a *TM* wave propagating along a suitable wave guide, with a velocity lower

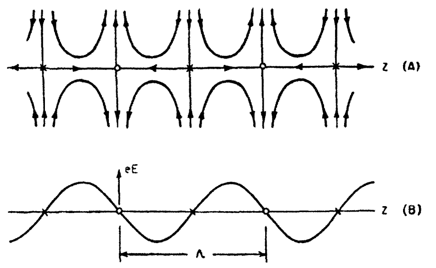

FIG. 1. ○—stable positions; ×—unstable positions along the Z axis.

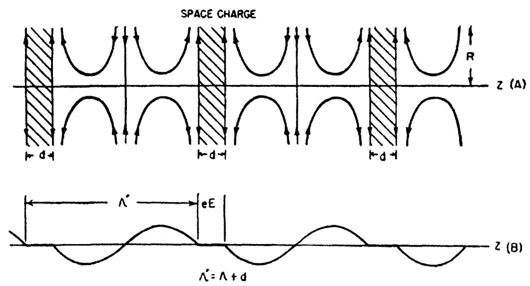

FIG. 2.

than c, and no space charge. The field distribution in space is shown on Fig. 1A, while Fig. 1B represents the intensity of the E_z field along the z axis, as a function of z. Let us now introduce charged particles e, and assume the arrows to show the direction of the force eE acting on the moving particles e of the beam. As long as longitudinal motions along the axis only are considered, the positions marked with a circle are stable positions and those marked with a cross are unstable. But if one looks at the upper drawing of Fig. 1A, he immediately notices that for transverse motions the crosses are stable and the circles unstable. This first discussion shows that a complete stability of the composite system (wave plus particles) cannot be achieved unless some special means are provided to insure transverse stability of the beam. Transverse stability may result from magnetic forces, as in the betatron or synchrotron; it can also be obtained in the case of a linear accelerator by using a longitudinal magnetic field or a system of electrostatic lenses. We shall consider the case of a stabilizing longitudinal magnetic field H as a workable theoretical problem (although in many practical problems the field strength needed for stability might be impossible to obtain!).

For a TM wave without space charge, propagating in the z direction with a velocity $W<c$, the fields are given by the following formulae

$$\begin{aligned}
&K^2=\omega^2[(1/W^2)-(1/c^2)]=4\pi^2[(1/\Lambda^2)-(1/\lambda^2)] \\
&\Lambda=2\pi(W/\omega)=\text{wave-length of the slow wave,} \\
&\lambda=2\pi(c/\omega), \\
&E_z=A\,J_0(jKr)\exp\{j\omega[t-(z/W)]\}, \\
&E_r=A(\omega/KW)J_1(jKr)\exp\{j\omega[t-(z/W)]\} \\
&\qquad\approx Aj(\omega/2W)r\exp\{j\omega[t-(z/W)]\} \\
&\qquad\text{for small } Kr\ (J_0J_1,\ \text{Bessel functions})
\end{aligned} \tag{1}$$

in the case of circular symmetry. In some instances it may be found convenient to refer the wave to a frame of reference moving with the velocity W, and with respect to which the wave is at rest. This can be obtained by a relativistic transformation, or by taking

$$\begin{aligned}
&\omega=0,\quad W_0=0,\quad \Lambda=\Lambda_0\ \text{finite},\quad \lambda=\infty, \\
&K=K_0=2\pi/\Lambda_0, \\
&E_{0z}=A_0J_0(jK_0r)\exp(-jK_0z), \\
&E_{0r}=A_0J_1(jK_0r)\exp(-jK_0z) \\
&\qquad\approx\tfrac{1}{2}A_0jK_0r\exp(-jK_0z)
\end{aligned} \tag{2}$$

in our general formulae (1). The radial field E_r (or E_{0r}) is proportional to the radial distance r from the z axis, so long as Kr remains small. For larger distances, the J_1 Bessel function is needed.

We shall discuss in the next section the properties of a composite wave, carrying a certain amount of space charge, and we shall limit our problem to the case of a wave of small radius R, where the preceding approximation can be used without practical error.

According to the preceding remarks, a constant longitudinal magnetic field shall be used to stabilize the space-charge.

3. SPACE-CHARGE DISTRIBUTION IN THE BEAM WITH MAGNETIC STABILIZATION

This problem has a well-known solution;[1] with a uniform space-charge density ρ rotating about the axis with a constant angular velocity ω_H, and giving a radial distribution of electric field E_r,

$$\begin{aligned}
\omega_H=-\tfrac{1}{2}\mu_0(e/m)H,\quad \rho=2\epsilon_0(m/e)\omega_H^2, \\
E_r=(m/e)\omega_H^2r=\tfrac{1}{4}(\mu_0H)^2(e/m)r.
\end{aligned} \tag{3}$$

The radial force eE_r on the moving particles plus

[1] L. Brillouin, Phys. Rev. 67, 260 (1945).

centrifugal force $m\omega_H^2 r$, is balanced by the Lorentz force $+\mu_0 e\omega_H rH$.

Now let us take a slice of thickness d in this beam, and insert it between the fields of our previous chargeless wave, making the whole structure move with a velocity W along the z axis, as shown on Fig. 2. We obtain continuity of the radial field in the boundary planes (at $t=0$, these planes are located at $z=0$ and $z=d$) when we take

$$\tfrac{1}{2}Aj(\omega/W)=\tfrac{1}{4}(\mu_0 II)^2(e/m), \qquad (4)$$
$$A=-\tfrac{1}{2}(eW/m\omega)j(\mu_0 II)^2.$$

The radial fields are matched across the boundary planes, when the beam is fine enough, and $KR\ll 1$.

The *composite wave* represented in Fig. 2 is *stable*, and propagates along the z axis with the velocity W. Its wave-length is

$$\Lambda'=\Lambda+d, \qquad (5)$$

and its total current averages

$$I=\rho W(d/\Lambda')\pi R^2. \qquad (6)$$

The magnitude II of the stabilizing magnetic field determines the value of the electric field in the chargeless region and also the space-charge density in the charged slices. The current can be adjusted by changing the thickness d of the space-charge slices. Its maximum value is, of course,

$$I_{\max}=\rho W\pi R^2. \qquad (7)$$

The composite wave thus found has discontinuities in the derivative of the longitudinal field distribution (Fig. 2B) and represents a sort of *electromagnetic shock wave*. It yields a rigorous solution of the wave equation with space-charge and represents a generalization of the solutions obtained for the two extreme special cases of the traveling wave amplifier or the linear accelerator. The solution obtained here should correspond to the final stage of complete bunching of the particles by the wave, after oscillations of the particles about their equilibrium positions (the circles of Fig. 1) have died out, and the field rearrangements have been completed.

Of course, the solution is valid only at a certain distance of the boundary of the wave guide, and would be seriously distorted in the neighborhood of the boundary, according to the type of guide structure used to slow down the waves.

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On the Neutron-Proton Force*

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It has been pointed out[1] that neutron-proton scattering experiments at energies below about 10 Mev do not give any *detailed* information about the neutron-proton force. They only determine the strength and range of the force, not its exact distance dependence.

J. Schwinger[2] has developed a powerful variational approach to scattering theory. This method, applied to the case of the scattering of neutrons by protons, leads to great simplifications in the analysis of the experiment. Schwinger was able to prove rigorously that to a good approximation the shape of the well does not matter for the scattering. The well is described phenomenologically by the scattering length[3] to which it gives rise (evaluated at zero energy) and by an *equivalent range*, r. This equivalent range depends both upon the width and the depth of the well, becoming smaller as the well is made deeper. For a square well of range b, the equivalent range r equals b if the depth of the well is such as to give a resonance level in the scattering exactly at zero energy. For the triplet state, therefore, $r<b$, for the singlet state, $r>b$. It is the great advantage of this analysis, how-

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[1] J. Smorodinsky, J. Phys. U.S.S.R. **8**, 219 (1944); **11**, 195 (1947).

[2] J. Schwinger, Phys. Rev. **72**, 742 (1947).

[3] E. Fermi and L. Marshall, Phys. Rev. **71**, 66 (1947).