tators of the 15 elements except for numerical constants consist of one of the original 15 elements. Assuming that ($\eta^2>0$), it can be shown that the spectrum of B_0 is discrete and the spectrum of the B_j is continuous.[2] For $\eta^2<0$, the alternative operators

$$\bar{B}_0=i\bar{b}\left((-\eta^2)^{\frac{1}{2}}\frac{\partial}{\partial\eta_0}-\eta_0/(-\eta^2)^{\frac{1}{2}}\eta_\alpha\frac{\partial}{\partial\eta_\alpha}\right),$$
$$\bar{B}_j=i\bar{b}\left((-\eta^2)^{\frac{1}{2}}\frac{\partial}{\partial\eta_j}+\eta_j/(-\eta^2)^{\frac{1}{2}}\eta_\alpha\frac{\partial}{\partial\eta_\alpha}\right), \tag{3}$$

possess the same properties as the B's with the exception that the spectrum of the $\bar{B}_0$ is continuous and the $\bar{B}_j$ discrete.

The above considerations may be extended to include the hyperquadratic form of n dimensions. The commutators of the n operators of type B generate the "rotational" operators. The character of the spectra of the "rotational" and B operators will depend only on the signs of the square terms in the hyperquadratic form and constant multipliers. It is well known that a Riemannian space of m dimensions may be considered as embedded in a flat space of $n=m(m+1)/2$ dimensions. It is quite possible, then, that the quantization of a Riemannian m space will feature the $m(m+1)/2$ operators of type B and their commutators.

The physical significance of the operators defined in (2) or (3) has not been definitely found. It seems desirable, however, to impute to the spatio-temporal parts the character of momentum-energy operators. The commutators of the coordinates with B_j or $\bar{B}_j$ are given by

$$[x_j,B_k]=ia\delta_{jk}B_4,$$
or
$$[x_j,\bar{B}_k]=ia\delta_{jk}B_4, \tag{4}$$

respectively. Equation (4) is similar to the usual commutation relationship between coordinates and momentum with $\hbar$ replaced by the operator iaB_4 or $ia\bar{B}_4$. It is seen that such an identification necessitates the concept of a Planck's "constant" operator. There may be more plausible identifications.

[1] H. S. Snyder, Phys. Rev. **71**, 38 (1947).
[2] It is assumed that the spectra b_α of B_α is such as to render the solution of $B_\alpha 4=b_\alpha 4$ single valued.

Anomalous Effects in Hysteresis Loops of a Single Crystal of $BaTiO_3$

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February 16, 1948

ANOMALOUS effects for a barium titanate single crystal were obtained by photographing hysteresis loops[1] on a cathode-ray oscillograph with a 16-mm motion picture camera. Interesting configurations occur as shown in the accompanying figures.

The crystal was 1.1 mm by 0.5 mm by 0.1 mm thick and was silvered on the 0.5-mm by 0.1-mm faces and mounted between aluminum foil electrodes on a glass microscope slide. The whole system was inserted in a Kofler hot stage of a microscope and held at a temperature about 90°C for Fig. 2 and 85°C for Fig. 1. The crystal was detwinned by the use of electric fields and connected into the circuit with the oscillograph. The naked eye perceives on the screen of the oscilloscope transient jumps of a step-like configuration on the slope of the hysteresis loops.

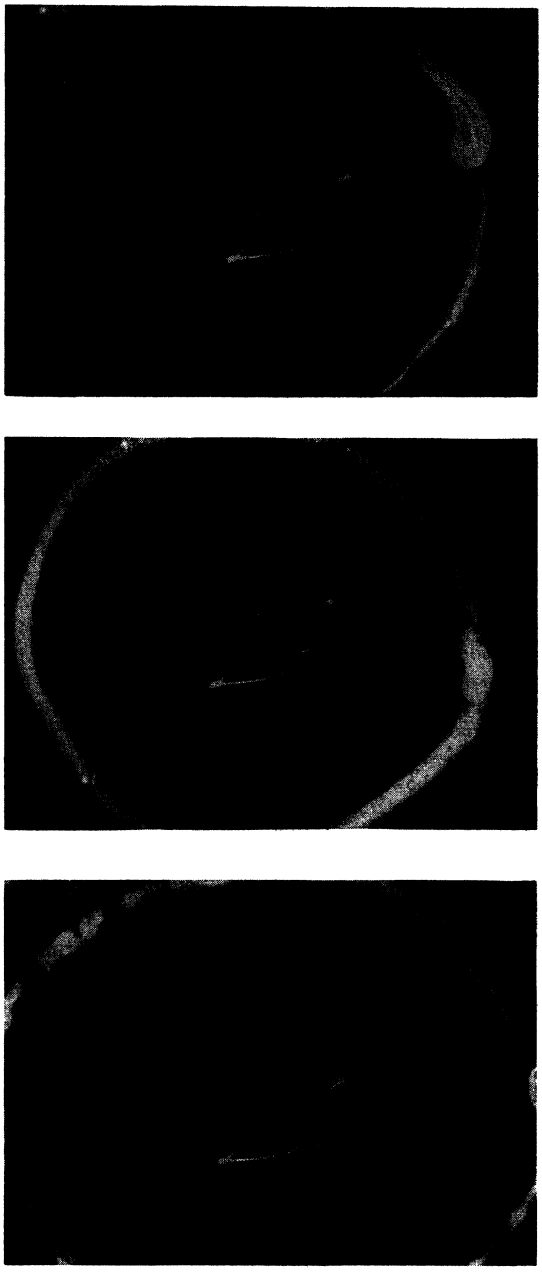

FIG. 1. Hysteresis loops taken with a 16-mm Ciné Kodak motion picture film at 24 frames per second exposure approximately 0.01 sec. at f:2.8 with tri X Aero film. A type 108 Sylvania 3-inch cathode-ray oscillograph was used. The maximum field strength on the single crystal of $BaTiO_3$ was 2570 volts/cm peak a.c. at 60 cycles at a temperature of 85°C.

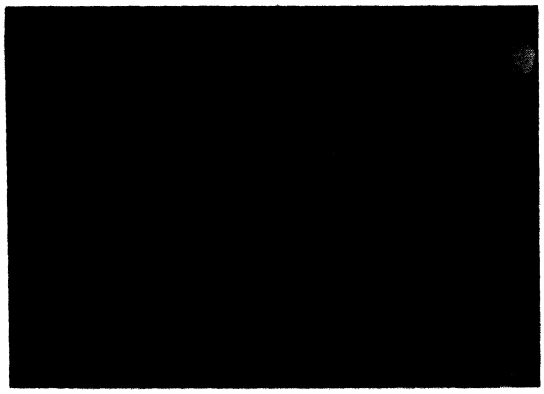

FIG. 2. Same conditions as in Fig. 1 with a temperature of about 90°C.

Figures 1 and 2 show the hysteresis loops which are so oriented that the charge or polarization axis is vertical, while the voltage or field-strength axis is horizontal.

Actual jumps in the polarization are shown as if a domain which contributes a charge ΔQ were aligned only after a finite voltage increases ΔV. In Fig. 2 apparently the whole crystal is acting as a single domain which snapped into alignment for practically the maximum field strength of 2570 volts/cm. Jumps of the order of magnitude as shown in Fig. 1 occur in about 10 percent of the frames, while a jump of the type shown in Fig. 2 was observed only once in about a thousand frames. Examination of the motion picture oscilloscope records and visual examination of the oscilloscope screen shows that the jumps in slope on the hysteresis loop occur in random phase.

Phenomenologically the Barkhausen effect in the case of ferromagnetic materials is analogous to the above effect for ferro-electrics. The domains for the ferro-electric case are much larger, however.

[1] C. B. Sawyer and C. H. Tower, Phys. Rev. **35**, 269–273 (1930). A. de Bretteville, J. Am. Ceramic Society **29** (11), 303 (1946); Phys. Rev. **69**, 687 (1946).

A Possible New Type of Spin-Spin Interaction

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February 11, 1948

IN view of the hyperfine structure difficulties brought to light by the experiments of Nafe, Nelson, Rabi[1] and others, I would like to advance a theory that might possibly have a bearing on these questions. Instead of introducing a new interaction between the electron and the electromagnetic field as Breit[2] has tried, we bring in an interaction between the electron and a new type of field. While the sources of the Maxwell field are the charges and the current densities, the sources of the new field are the spin densities. Thus the interaction between elementary particles, instead of being described by the usual charges and magnetic moments: coulomb, magnetic moment—orbit and magnetic moment—magnetic moment interaction (we avoid the term spin to keep it for the new field) contains now another term of pure spin-spin interaction. For non-relativistic velocities, spin density and density of magnetic moment are parallel, and thus the effect of the new field is to change the usual spin-spin interaction by an arbitrary factor that experiment alone can determine.

The non-relativistic equations for the new field (it has vectorial components Q and R) are

$$\begin{aligned} \text{curl}Q &= q\,\text{curl}S, & \text{div}Q &= 0, \\ \text{div}R &= -p\,\text{div}S, & \text{curl}R &= 0, \end{aligned}$$

where S is the spin density and q an arbitrary coupling factor. They can be compared with Maxwell's equations for B and E;

$$\begin{aligned} \text{curl}B &= J + \text{curl}M, & \text{div}B &= 0, \\ \text{div}E &= \rho - \text{div}P, & \text{curl}E &= 0, \end{aligned}$$

and the symmetry is at once obvious except for non-existence of a convection charge current in the spin field.

I had arrived previously at this conception from the point of view of general relativity when applied to Dirac particles. Generalized relativity has, so far, only utilized space curvature, omitting a possible torsion of space. The reason for that is that a classical particle has its four momentum parallel to its four velocity, or in other words, its transported momentum is in the direction of transport. A Dirac particle, on the other hand, behaves differently because the momentum vector is different in direction from the velocity vector (and the consequence of this is that one talks about spin). Torsion of space as a field acts differently from curvature of space (gravitation) only if the direction of transport is different from the transported vector. The Q and R field introduced above is the field of this torsion of space.

[1] Nafe, Nelson, and Rabi, Phys. Rev. **71**, 914 (1947).
[2] G. Breit, Phys. Rev. **72**, 984 (1947).

The Growth of Barium Titanate Crystals

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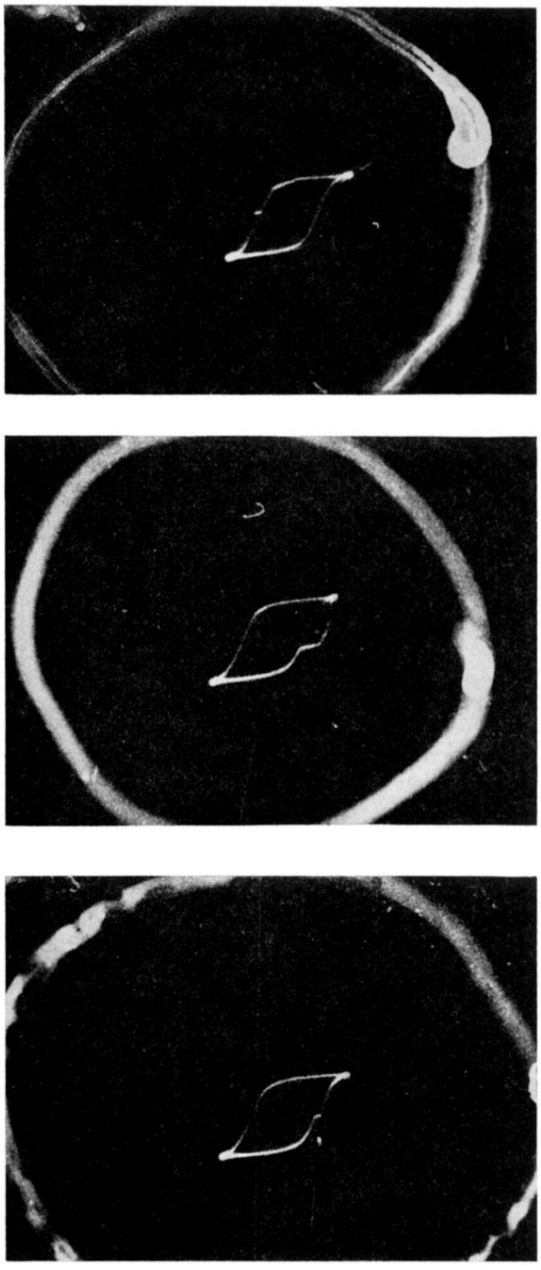
February 19, 1948

DURING the past year, the interest in the ferroelectric barium titanate has increased and this laboratory has received a number of inquiries concerning the growth of single crystals.

Bourgeois[1] describes a process by which he obtained microscopic crystals similar to perovskite with a stoichiometrical composition supposedly $2BaO \cdot 3TiO_2$. He used an excess of $BaCl_2$ as flux and equimolar parts of $BaCO_3$ and TiO_2 to form the compound. On repeating this process no well defined crystals resulted.

By adding an appreciable excess of $BaCO_3$, Blattner, Matthias, and Merz[2] were able to obtain well formed crystals which corresponded (except for their optical behavior) to what had to be anticipated from the polycrystalline $BaTiO_3$.



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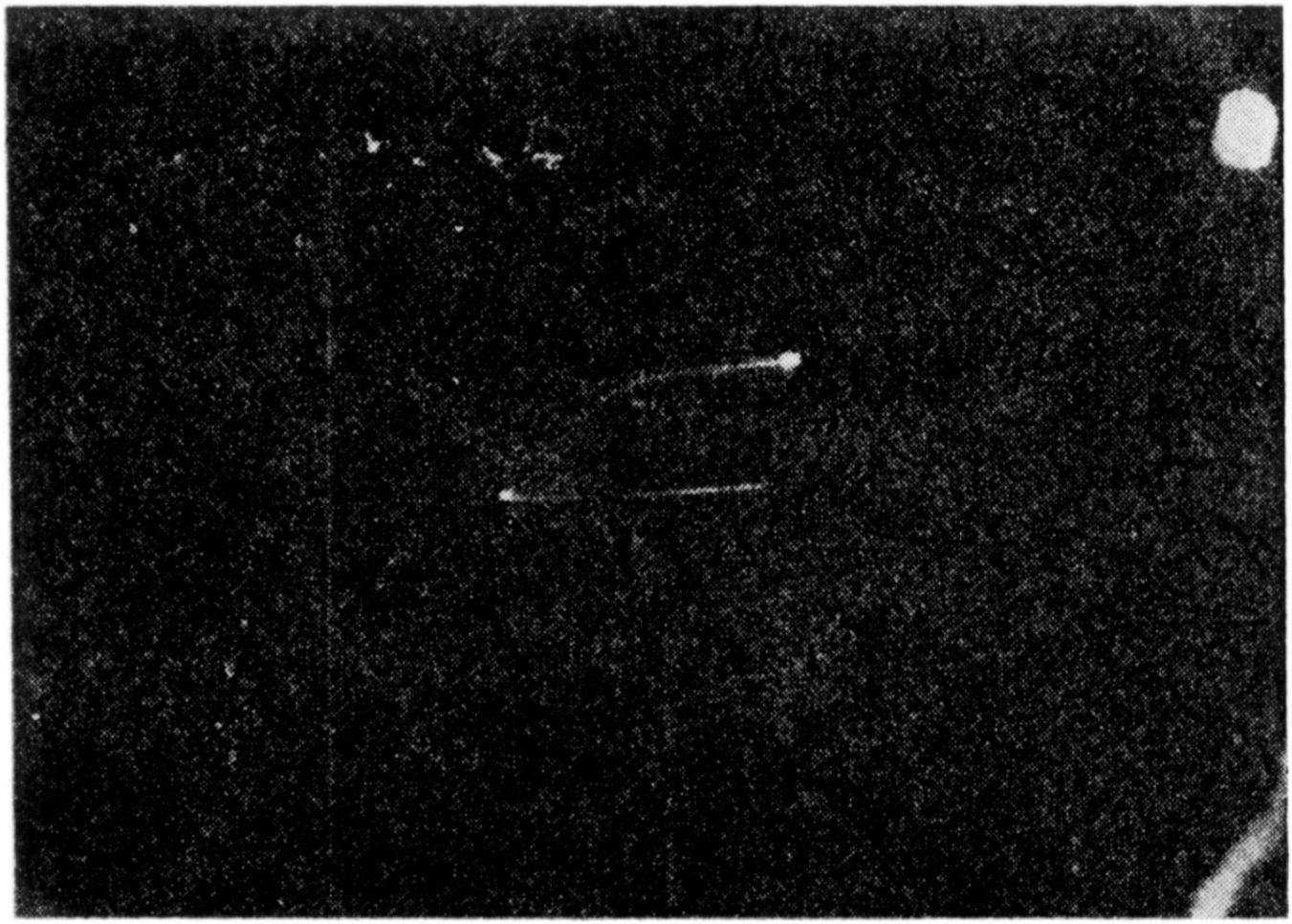

FIG. 2. Same conditions as in Fig. 1 with a temperature of about 90°C.