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Second Sound in Liquid Helium II

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With a resonance method, the velocity of the temperature waves (second sound) in liquid helium II has been determined as a function of temperature from about 1.4°K to the λ-point. The velocity is found to be zero at the λ-point, reaches a maximum of 20.46 m/sec. at 1.7°K and decreases to 19.80 m/sec. at 1.42°K. The accuracy of the measurements is at least ±0.5 percent. The results are in good agreement with the theoretical predictions of both Tisza and Landau but tend to favor the former's theory over the latter's. The fact that second sound generated in the superfluid is able to register on a microphone diaphragm immersed in the fluid is discussed.

INTRODUCTION

IN an attempt to explain the many anomalous properties of liquid helium at temperatures below 2.19°K (liquid helium II), F. London has proposed a model for this fluid which is based on an ideal gas obeying Bose-Einstein statistics. In these statistics a peculiar "condensation" effect takes place at a certain critical temperature T_c whereby a certain fraction of the atoms go into a highly degenerate ground state, the remainder remaining in excited states. In particular the number of degenerate atoms N_0 is given by

$$N_0=N\left\{1-\left(\frac{T}{T_c}\right)^{\frac{3}{2}}\right\},\quad T<T_c, \tag{1}$$

N being the total atoms present per unit volume and

$$T_c=\frac{h^2}{2\pi mk}\left(\frac{n}{2.6124}\right)^{\frac{2}{3}}. \tag{2}$$

By putting the appropriate values for helium into Eq. (2) London finds $T_c=3.14$°K which is surprisingly close to the λ-point (2.19°K) considering that we are dealing with an ideal gas and not a liquid. Below T_c the entropy of the system is caused solely by excited atoms, the degenerate ones have zero entropy.

Using this admittedly rough model, London has been able to show that a number of known properties of liquid helium II are theoretically derivable, at least qualitatively. L. Tisza has improved the London model in a number of particulars. He supposes liquid helium II to be a "mixture" of two kinds of atoms, superfluid of density ρ_s and normal of density ρ_n. These correspond to the degenerate and excited atoms in London's Bose-Einstein gas and again the entropy of ρ_s is zero. A further assumption made by Tisza, which appears somewhat difficult to justify, is that the interaction between ρ_s and ρ_n is negligible and indeed ρ_s atoms do not exchange momentum with either ρ_n or other atoms. Tisza further supposes that the density function for degenerate atoms is

$$\rho_s=\rho\left\{1-\left(\frac{T}{T_c}\right)^{\sigma}\right\},\quad T<T_c,$$

$$\rho=\rho_s+\rho_n,\quad T_c=2.19°K,$$

and $\sigma\simeq 5$ to 6 as against $\frac{3}{2}$ in the ideal Bose gas. The justification for this latter modification lies partly in the better fit obtained with the experimental results for the specific heat discontinuity at the λ-point and partly because of the fact that, as London had observed, it is probable that the introduction of interaction between the atoms would modify the density function in this direction.

From this modified theory and a consideration of the hydrodynamical equations of the two phases, Tisza in 1940[1] predicted that temperature differences in liquid helium II should be propagated as a wave motion with a characteristic, temperature dependent velocity.

The experimental proof of this prediction was made in 1944 by V. Peshkov,[2] and these temperature waves were named "second sound."

[1] L. Tisza, J. de phys. et rad. [8] 1, 164, 350 (1940).
[2] V. Peshkov, J. phys. et rad. U.S.S.R. 8, 381 (1944).

Peshkov generated these waves by means of a wire resistance coil in which a variable temperature was maintained by means of an alternating current, and detected them from the indications of a small phosphor bronze resistance thermometer. By measuring the wave-length of running or standing waves (produced by reflection from the liquid surface) and knowing the heat waves to be double the frequency of the exciting a.c. he could determine the velocity which turned out to be of the order of 20 m/sec., i.e., around 10 times less than the velocity of ordinary sonic waves.

In Tisza's theory the velocity of second sound is given by

$$c=\left[-\frac{\rho_n}{\rho_s}\frac{\partial}{\partial T}\left(\frac{1}{S}\right)\right]^{-\frac{1}{2}}$$

where S is the entropy of helium II, which has been accurately determined as a function of temperature from measurements on the specific heat. Accordingly measurements of C as a function of temperature would yield information on the temperature variation of the important ratio ρ_s/ρ_n. The purpose of the following experiment is therefore clear. In addition to this we must mention that the existence of second sound was predicted independently by L. Landau,[3] somewhat later than Tisza, by use of an entirely different kind of theory. We shall discuss the bearing of the experimental results on the two theories at the end of this paper.

EXPERIMENTAL DETAILS

Following discussions with L. Onsager, of this laboratory, we decided to attempt to measure second sound velocities with the following kind of arrangement. The second sound would be generated by an a.c. heater of the same general type as that used by Peshkov, and standing waves would be produced in a column of liquid helium by reflections from the free surface. If this free surface were an anti-node, the temperature of this surface would fluctuate at double the frequency of the a.c. source, giving rise to a fluctuating evaporation of the liquid at this surface. In consequence small periodic fluctuations in the pressure of the vapor above the liquid surface would occur (first sound). These fluctuations of pressure would register on a microphone immersed in the helium vapor, and hence second sound resonances in the liquid would be detectable.

Figure 1 is a schematic diagram of our apparatus. He and Ni are, respectively, the liquid helium and liquid nitrogen Dewars. The latter is of Pyrex with a highly evacuated interspace. The helium Dewar, on the other hand, is of soft glass and the interspace is filled with air at a pressure of about 2 millimeters mercury. This has the following purpose. Before liquid helium is admitted, the apparatus in this Dewar is cooled to the temperature of liquid nitrogen, the air in its interspace acting as the heat transfer medium. A large fraction of the heat content of this apparatus is thereby removed and the load

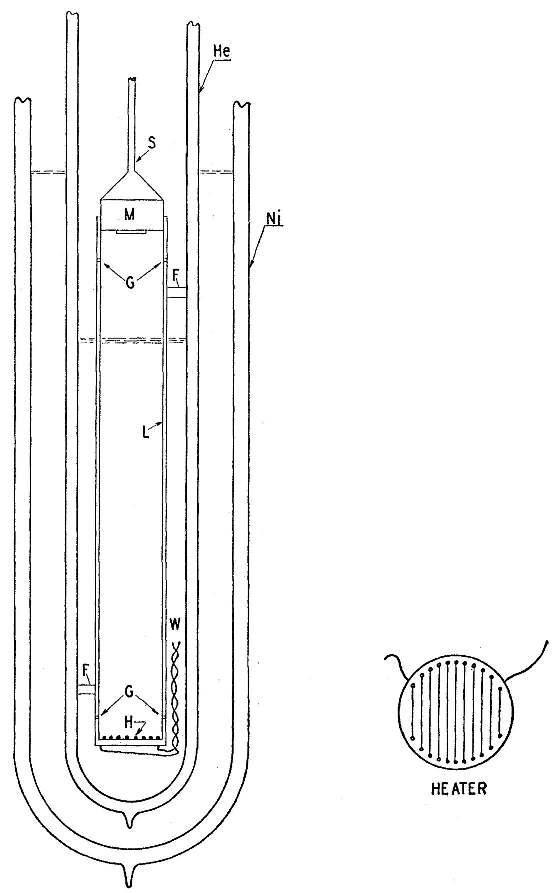

FIG. 1. Schematic drawing of the apparatus.

[3] L. Landau, J. Phys. U.S.S.R. 5, 71 (1941).

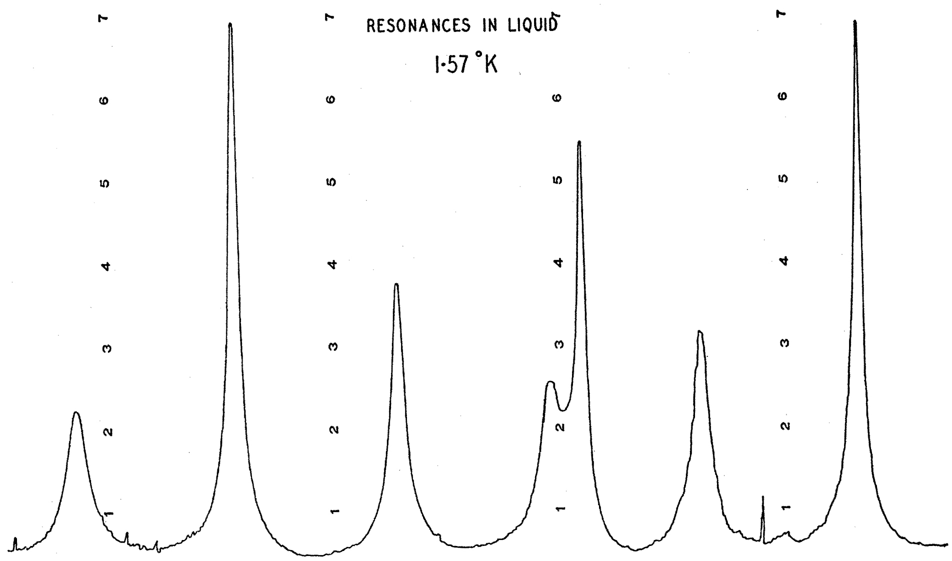

FIG. 2. Second sound resonances as registered on recording potentiometer at 1.57°K. The microphone is in the vapor above the liquid surface.

on the liquid helium accordingly much reduced. This is important in view of the very small latent heat of liquid helium. When the liquid helium is now admitted to the Dewar, the air in the interspace freezes and a very high vacuum is produced, thereby providing good thermal insulation for the liquid.[4] Both Dewars were fully silvered except for a pair of narrow longitudinal slits in each permitting visual observation of the the interior of the helium Dewar. The latter was about 18 inches long and 1 inch internal diameter. L is a thin walled Lucite tube approximately 15 cm long and 2 cm in diameter. It is provided with four small holes (G) to permit filling from the Dewar. H is a constantan heater, wound with 3-mil wire, a detail of which is shown separately. This heater is fed by number 40 copper wire (W) twisted to eliminate magnetic coupling into the receiver circuit. Several Lucite spacers (F) are used to center the cavity in the Dewar. Fitted tightly into the Lucite cavity is the microphone (M). The specifications for this device were severe. It had to be small, light in weight, and capable of operating at temperatures near absolute zero. The operating temperature ruled out the usual Rochelle salt-crystal type and we finally made use of the receiver of a Sonotone hearing aid used in "reverse" which is a magnetic type[5] instrument. The whole assembly is supported from the top of the Dewar by a small, thin-walled, stainless steel tube (S). This tube contains an insulated inner conductor and the pair form the shielded microphone leads.

The heater is fed continuously with 1000-cycle a.c., of good wave form, from a stable tuning fork oscillator. The power fed into the heater, which had a resistance of about 60 ohms at liquid helium temperature, was roughly 1/25 watt. The microphone output was amplified by an amplifier roughly tuned to 2000 cycles. This latter was provided with both a.c. and rectified output. The rectified output was fed into a Brown high speed recording potentiometer with a chart speed of 2 inches per minute. The Lucite cavity was illuminated through the slits in the Dewars by means of an external fluorescent lamp and the liquid helium level in the cavity was observed with a cathetometer. The vapor pressure in the cryostat was read on an absolute manometer filled with diffusion pump oil and was controlled manually by means of a needle valve. The 1937

[4] This technique originated at the Royal Society Mond Laboratory, Cambridge, and was communicated to us by Professor Onsager who learned of it on a recent visit to that laboratory. We found it to be immensely useful in all sorts of work with liquid helium and also liquid hydrogen.

[5] We are indebted to Dr. L. Grant Hector of Sonotone Corporation for his gift of several of these devices.

Leiden Scale was used in computing the corresponding absolute temperatures.

The procedure in taking data was as follows. As the liquid helium slowly evaporated and the level in the cavity fell, cathetometer readings were taken as a function of time. This clock was synchronized with the recorder chart, which recorded a series of resonance peaks. By plotting the liquid level as a function of time (this was always a straight line when the temperature was steady) the height of the liquid column in the cavity at each resonance was known. The difference in height between two successive resonance peaks being, of course, one-half wavelength. Usually about 6 to 8 peaks were taken at each temperature. Figure 2 shows a typical series of such resonance peaks at 1.57°K. One pair of plates of an oscilloscope were connected to the heater oscillator, the other pair to the a.c. output of the microphone amplifier. The resulting Lissajous figure confirmed that the sound frequency was double the electrical heater frequency.

RESULTS

Using the previously described method we determined the second sound velocity at a number of temperatures from about 1.4°K to the λ-point. Since we believe our accuracy is better than ±0.5 percent we present the data in both tabular (Table I) and graphical (Fig. 3) form. It will be noticed that the velocity extrapolates to zero, very accurately, at the λ-point as the theory requires.

It is apparent from our data that the liquid column, at resonance, is probably oscillating with a temperature anti-node at both the free surface and the heater surface. This fact enables us to compute the wave-length with great accuracy. Thus, at a given temperature, having roughly determined the half-wave-length from the distance between successive resonance peaks it is possible to assign a definite number of half-wave-lengths (n) to the whole liquid column at each resonance. If then successive resonant lengths of liquid column be designated by L_0, $L_1, \cdots L_s$ we found the following very accurately true:

$$\frac{L_0}{n}=\frac{L_1}{n-1}=\frac{L_2}{n-2}=\cdots=\frac{L_s}{n-s}$$

and therefore

$$\frac{\lambda}{2}=\frac{\sum_0^s L_i}{(s+1)(n-s/2)}.$$

In most of our velocity determinations S was of the order of 6 or 7 and n was usually about 30 or so and hence a very accurate determination of λ was possible.

In one run we held the temperature constant for about 23 successive resonances. The resonance peaks varied widely in height and width and appeared to be modulated by a much longer wave-length. We were able to determine this modulating wave-length roughly and the resulting velocity was of the order of the velocity of normal sound in helium vapor at the prevailing pressure and temperature in the vapor space of

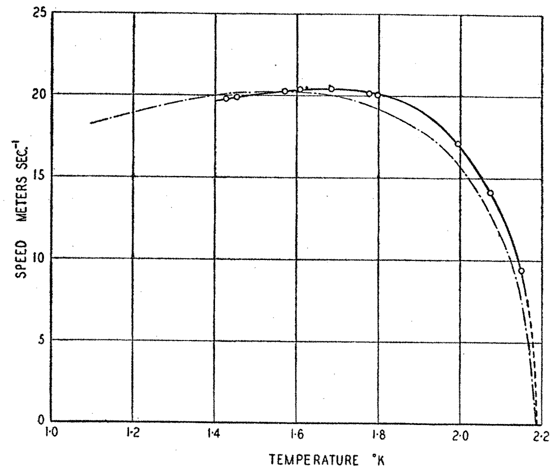

FIG. 3. Second sound velocity as a function of temperature. The heavy line with circles is the experimental curve. The lighter broken line is the theoretical curve computed from Tisza's theory of liquid helium II.

TABLE I. Second sound velocity *versus* temperature.

°K	meters/sec.
1.425	19.80
1.453	19.86
1.570	20.23
1.607	20.38
1.685	20.46
1.780	20.16
1.795	20.08
1.994	17.12
2.074	14.13
2.150	9.40

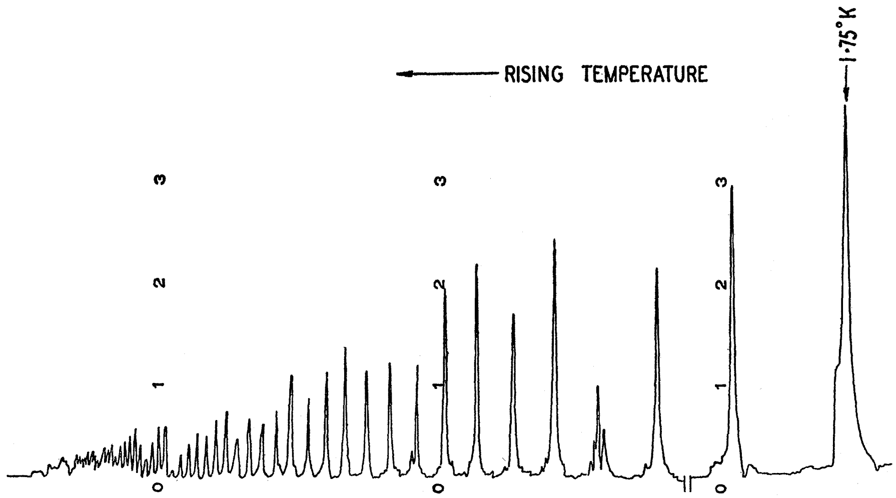

FIG. 4. Second sound resonances as registered on recording potentiometer with the microphone completely immersed in the liquid.

the Lucite cavity. All our runs showed this modulating feature to a greater or lesser extent.

Much more rarely we observed a double resonance peak. Such a one is shown on the *third peak* from the left in Fig. 2. We only observed two such twin peaks in all our work. In both cases only one of the twins fitted our second sound data. Assuming the spurious peak was resonance of ordinary sound in the liquid we could show that it was in accord with the known value of this velocity.

A rather curious effect was invariably observed when the liquid surface in the cavity was about even with the top of the heater wires. We always obtained a pair of second sound resonance peaks close together at this stage.

One run was made which was a departure from our standard technique. In this case the microphone was completely immersed in the liquid helium II and there was no observable vapor space in the Lucite cavity as observed by the cathetometer telescope.* The temperature was set at about 1.75°K and allowed to rise slowly and steadily up to the λ-point. Figure 4 shows the recorder chart record in this case. Examination of these data, with the curve of Fig. 3, leaves little doubt but that we are observing true second sound resonances in this case. Here, of course, both the frequency and the length of the resonant cavity are constant and a peak will be obtained whenever the temperature is such that the second sound velocity has a value capable of rendering this fixed system resonant. If L is the length of the cavity and f_0 the steady heater current frequency, then we shall observe a resonance peak when the second sound velocity $V(T)$ is given by

$$V(T)=\frac{4f_0L}{n},$$

where n is a whole number. Accordingly, starting at some known temperature (and velocity), near the maximum in the curve, and proceeding to the λ-point we should get a large number (theoretically an infinite number) of resonance peaks corresponding to velocities given by n, $n+1$, $n+2$, $\cdots$ in the above formula. As the temperature approaches the λ-point the peaks will be closer together due to the steepness of the velocity-temperature curve in this region until the speed of the recorder is no longer sufficient to record all the peaks. This is approximately what we observe.

* In view of the enormous heat conductivity of liquid helium II no "vapor lock" would be expected. Vapor locks in liquid helium I and liquid hydrogen always occurred in similar circumstances.

DISCUSSION

As mentioned previously, a theoretical relation for the second sound velocity *versus* temperature curve has been deduced by Tisza depending on (1) the experimentally determined relationship between the entropy and the temperature, (2) ρ_s/ρ_n as a function of temperature. If one takes, following London,[6] as an average in the range from 1°K to the λ-point,

$$S=2.1\times10^5T^{5.6}\ \mathrm{erg/^\circ K\ g},$$

$$\rho_n=\rho\left(\frac{T}{2.19}\right)^{5.6},$$

then we obtain

$$c=2.6\times10^3\left[\frac{T}{2.19}\left[1-\left(\frac{T}{2.19}\right)^{5.6}\right]\right]^{\frac{1}{2}}\ \mathrm{cm/sec}.$$

This function is plotted as a broken line in Fig. 3 and the agreement is seen to be very good. As mentioned in the introduction, the Landau theory predicts that the temperature variation of velocity shall follow a somewhat different course than that predicted by the Tisza theory. Thus at absolute zero the velocity according to Tisza should be zero, according to Landau it should be finite. The two theories, therefore, diverge in the neighborhood of 1°K, Landau showing a rising velocity, Tisza a falling one as the temperature decreases. Unfortunately we are unable to achieve a temperature below 1.4°K with our set-up but in view of the accuracy of our measurements and the fact of a distinct maximum in our curve, Tisza's theory seems to be somewhat favored over Landau's.

There remains the reason for the observance of second sound resonances in the case where the microphone is totally immersed in liquid helium II. There is no reason to expect, from the theory, that the temperature waves could give rise to pressure fluctuations in the liquid and therefore register an effect upon the microphone diaphragm. Professor Onsager has suggested to us that the mechanism may lie in the nature of the cavity which we are using. Referring back to Fig. 1 the four small filling holes G will be noted. The temperature fluctuations of the second sound inside the cavity will, according to Tisza's theory, produce a flow of ρ_s atoms into the cavity through the holes and a balancing flow of ρ_n atoms out through the holes. The flow of ρ_s atoms will be frictionless, but the ρ_n atoms, possessing viscosity, will probably experience a pressure drop in their passage through the holes. Since the flow of the ρ_s and ρ_n will be fluctuating we could by this mechanism have fluctuations of pressure in the cavity, induced by the second sound, which would be capable of registering on our microphone diaphragm.

It will be observed from Fig. 2 that many of the resonance peaks are extremely sharp, indicating a high "Q". This suggests that both the transmission coefficient across the liquid surface, and the dissipation in the liquid, are small. The absence of end effects in the resonant column also confirms the low transmission coefficient.

Finally we should like to express our thanks to Professor Lars Onsager for his many valuable discussions throughout the course of this work. We are also indebted to Mr. Robert T. Webber for his aid in taking data. This work was financially supported by the U. S. Navy under contract N6ori-44, Task Order III.

[6] We are greatly indebted to Professor Tisza for drawing our attention to his articles in *Journal de Physique*, copies of which had not reached us, and for making available to us an advance copy of F. London's paper before the Cambridge conference in 1946. The above formulas are drawn from this latter source. Mr. Tisza also communicated to us Peshkov's results, given at the same conference, which are in good agreement with ours.