since it makes use of a Lorentz transformation in going over to an accelerated frame of reference. It therefore seems desirable to try a different method.

One way of solving the problem is first to generalize the definition of angular velocity in relativity theory and then to use that definition to get the motion of the fluid directly in an inertial coordinate system. In classical mechanics the angular velocity of a fluid is given by

$$\boldsymbol{\omega}=\tfrac{1}{2}\nabla\times\mathbf{v},$$

where $\mathbf{v}$ is the velocity vector. In general relativity theory, the obvious generalization is to define angular velocity as an antisymmetric tensor

$$\omega_{ik}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x^k}-\frac{\partial u_k}{\partial x^i}\right),\quad (i, k=1, 2, 3, 4),$$

where u_i is the covariant velocity vector, the contravariant vector for a moving point being given by

$$u^i=dx^i/d\tau,$$

with $d\tau$ the proper-time element.

Consider a Galilean coordinate system with coordinates $x^1=x$, $x^2=y$, $x^3=z$, $x^4=ct$, so that the metric is taken as -1, -1, -1, $+1$, and $d\tau=(1-\beta^2)^{\frac{1}{2}}dt$ $(\beta=v/c)$. For small velocities one finds that ω_{23}, ω_{31}, ω_{12} reduce to the classical expressions for the x, y, z components of $\boldsymbol{\omega}$, while ω_{41}, ω_{42}, ω_{43} are proportional to the components of acceleration. If we take $v=v(R)$, with $R^2=x^2+y^2$, and

$$v=-vy/R,\quad v_y=vx/R,\quad v_2=0,$$

calculate ω_{ik} and set

$$\omega_{12}=\omega=\text{const.},$$

we find

$$v(R)=\frac{\omega R}{(1+\omega^2R^2/c^2)^{\frac{1}{2}}}.$$

This velocity v is in general different from that obtained by Hill, although it has nearly the same behavior for small and large values of R.

This question will be treated more fully in a forthcoming paper.

[1] E. L. Hill, Phys. Rev. 69, 488 (1946).

Solar and Lunar Effects on Cosmic Rays

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DUPERIER[1] has from a harmonic analysis of the hourly records made in London, reported a semi-diurnal variation of cosmic-ray intensity of amplitude 0.18 percent and nearly in opposite phase to the semidiurnal oscillation of the barometer. He explains this variation in terms of the finite lifetime of mesons and the up-and-down motion of the meson-producing layer in the atmosphere, caused by the semidiurnal solar atmospheric tides; and using certain calculations of atmospheric oscillations[2] he showed that one could entirely account in this manner for the observed semidiurnal variation of cosmic-ray intensity.

Independently of Duperier, one of us[3] has applied the theory of forced oscillations, magnified by resonance, of the earth's atmosphere with a period of 12 hours as worked out by Pekeris[2] to the oscillation of the meson-producing layer in the atmosphere and obtained, taking into account the finite lifetime of mesons, for the amplitude of the semidiurnal variation of the meson intensity 0.14 percent. This is in fair agreement with the value deduced by Duperier from his harmonic analysis of the London records.

We have continued our calculations with a view to seeing to what extent the tide-producing properties of the moon affect the meson intensity as observed at the earth's surface. For this purpose we have used with slight modifications, the theory of atmospheric tides developed by Pekeris.[2] We also use the stratification of the atmosphere employed by him. It is, of course, to be expected that the lunar effect will be smaller than the solar effect, in view of the fact that the atmosphere has free periods of oscillation, *viz.* 10.5 and 12 hr., neither of which is sufficiently close to 12 hr. 25 min. which is the semiperiod of the motion of the moon round the earth.

It can be shown that, when the period of forced oscillation of the atmosphere is 12 hr. 25 min. (mean solar time), the atmosphere oscillates like an ocean of depth of 7.04 km. Assuming that the moon moves in the equatorial plane, the semidiurnal component of the tidal potential Ω at a place on the equator is

$$-2.63\times10^4\cos(\sigma t+2\varphi+\epsilon)$$

where $2\pi/\sigma$ and φ are the period of oscillation and the longitude of the place, respectively, and ϵ is a constant. The amplitude of the lunar semidiurnal pressure variation at the equator is taken to be 72 microbars. Substituting these values of Ω and p in Eq. (47) of Pekeris,[2] the constant B/A is calculated. In the case of forced oscillations the vertical velocity in the region (0–1) of the atmosphere is given by the equation

$$w=-\frac{\gamma H}{2(n+1)x^n}\left\{A\left[J_n(x)-\frac{\gamma x}{0.8n-2}J_{n+1}(x)\right]+B\left[Y_n(x)-\frac{\gamma x}{0.8n-2}Y_{n+1}(x)\right]\right\}-\frac{i\sigma\Omega}{g}.$$

Since B/A is known, we obtain the values of A and B by means of this equation. In the region (1–2) χ satisfies the differential equation,

$$\frac{d^2\chi}{dy^2}+\frac{1}{h_1}\frac{d\chi}{dy}+\frac{\gamma-1}{\gamma}\frac{\chi}{h_1H}=0.$$

The solution of this equation is

$$\chi=C\exp(-y/2h_1)\cos\frac{Vy}{2h_1}+D\exp(-y/2h_1)\sin\frac{Vy}{2h_1},$$

where

$$V=\left|\left(1-\frac{8h_1}{7H}\right)^{\frac{1}{2}}\right|.$$

We have also

$$w=\gamma H\exp(-y/2h_1)\left\{C\left[\left(\frac{h_1}{H}-\frac{1}{2}\right)\cos\frac{Vy}{2h_1}+\frac{V}{2}\sin\frac{Vy}{2h_1}\right]+D\left[\left(\frac{h_1}{H}-\frac{1}{2}\right)\sin\frac{Vy}{2h_1}-\frac{V}{2}\cos\frac{Vy}{2h_1}\right]\right\}-\frac{i\sigma\Omega}{g}.$$

$$i\sigma\frac{p}{\rho_0}=\frac{1}{2}gH\left[(C+VD)\exp(-y/2h_1)\cos\frac{Vy}{2h_1}+(D-VC)\exp(-y/2h_1)\sin\frac{Vy}{2h_1}\right]-i\sigma\Omega.$$

The continuity of χ and w gives us C and D. By substituting the appropriate value of y, we find the pressure variation at 16 km, which is taken to be the bottom of the meson-producing layer. If p_0 is the pressure at 16 km when the atmosphere is in equilibrium, and $p+p_0$ the pressure when it is disturbed, and if h is the displacement of the isobar, on which the pressure is p_0, from the equilibrium position, then considering the equilibrium of the layer between 16 km and $16+h$ km, $p=g\rho_0 h$, i.e., $h=p/g\rho_0$, neglecting the variation of density ρ_0 in the layer.

We thus find that h varies harmonically with an amplitude of 101.3 cm. Taking the mean free path (decay) of mesons to be 8.5 km,[4] we obtain in the usual way for the amplitude of the semidiurnal variation in meson intensity at the earth's surface due to lunar effects 0.012 percent, the amplitude decreasing with increasing latitude. This may be compared with the result reported recently by Duperier,[5] who obtains from his records (0.023 ± 0.011) percent for the lunar semidiurnal amplitude of the cosmic-ray intensity variation.

We hope to publish elsewhere in the near future a more comprehensive theory of solar and lunar effects on cosmic rays.

[1] A. Duperier, Proc. Phys. Soc. **57**, 464 (1945).
[2] C. L. Pekeris, Proc. Roy. Soc. **A158**, 650 (1937).
[3] A. W. Mailvaganam, Proc. Phys. Soc. to appear shortly.
[4] B. Rossi, H. V. N. Hilberry and J. B. Hoag, Phys. Rev. **56**, 837 (1939).
[5] A. Duperier, Nature **157**, 296 (1946).

Removal of the Electron Beam from the Betatron*

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THE electron beam has been extracted from the University of Illinois 22-Mev betatron by means of a simple laminated iron channel. The iron structure is about 10 cm long and 0.7 cm high with a 1.75-mm by 5-mm channel in the edge facing the orbit. It is placed just outside the point where the magnetic field decreases as $1/r$ and a few centimeters outside of the equilibrium orbit. At the entrance end the channel is tangent to the orbits so that electrons entering it proceed in approximately a straight line to the other end where they emerge in a very weak magnetic field.

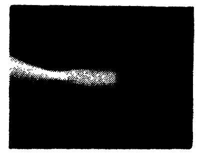

Fig. 1.

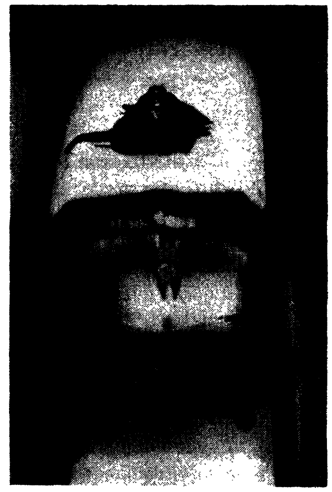

Fig. 2.

When an electron orbit is expanded into the region where the field falls off faster than $1/r$, the radial focusing forces disappear and the orbits spiral outward automatically. Spirals having a pitch of 1 mm to 5 mm have been observed. Electrons can be caused to enter the channel by the spiralling action. They are forced out to the spiralling region by the orbit expander.

An injector above the plane of the orbit was used so that the outward motion of the electrons was not obstructed. Tests with injectors placed inside the equilibrium orbit show that this type of injection should also be suitable.

A short time after the extraction scheme began to operate, the glass window 2 cm beyond the exit of the channel showed a sharp brown image of the channel with slight spreading of the beam. Multiple scattering in the $\frac{1}{16}''$ glass window spread the beam beyond. At one meter from the channel an ionization yield of over 100 r per minute was obtained.

When 4 inches of Pressdwood were used to absorb the beam, the ionization intensity dropped to about 1 percent, indicating that the x-ray intensity is very small. Photographic effects behind various thicknesses of Pressdwood gave ranges in agreement with the electrical calibration of the betatron's energy. The abrupt drop of photographic density at the end of the range is good evidence that the effects are caused by electrons.

Figure 1 shows the beam cross section as it emerges from the glass window and passes through a photographic film. The rectangular end is about the size of the channel and the fan-shaped end is composed of electrons not quite captured in the channel.

Figure 2 shows the window of the porcelain vacuum tube. The end of the iron channel is visible and the dark spot produced by the beam penetrating the glass can be seen at the inside edge of the window.

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