paths mark directions for which there is least transfer of energy from excess electrons, which are accelerated by the applied field, to the lattice. The path directions are believed to depend upon the wave-length, and hence upon the velocity of excess electrons. The orientating mechanism proposed by him is electron wave interaction with the lattice that results in scattering into preferential lattice directions.

The experimental foundation for an exact theory, however, is not yet complete. We are working further on this problem.

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The System of Microstresses in Cold-Worked Metal

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Smith and Stickley have measured the x-ray line widths from specimens of cold-worked α-brass and tungsten as functions of Bragg angle and x-ray wave-length. Their results accord with the microstress theory of broadening. They find that with α-brass, which is elastically anisotropic, the line width depends upon crystallographic direction in a systematic manner, while with tungsten, which is elastically isotropic, it does not. For several very simple theoretical models of internal-stress systems the variation in line width with crystallographic direction is computed here. The one which fits the above results best, though poorly, is that which assumes that the homogeneously strained domains suffer uniform normal stress of random magnitude. A plausible model for the microstress system and a very good fit are obtained by superposing upon this model any of the other models considered and thus obtaining the proper amount of anisotropy.

INTRODUCTION

CONSIDERABLE attention has been given in the last twenty-five years to the generally observed broadening of the x-ray diffraction lines of polycrystalline metal by cold work. The microstress theory of x-ray line broadening assumes a system of internal stresses (microstresses) in a piece of cold-worked metal which are in some manner random in magnitude and direction and are constant only over domains very small compared with the sample. If we apply Bragg's law, $\lambda = 2d_{hkl} \sin \theta_{hkl}$, to one of these homogeneously stressed and, therefore, homogeneously strained domains, we find upon logarithmic differentiation, taking λ constant, the following relation between the microstrain $e_{hkl} = \Delta d_{hkl}/d_{hkl}$ in the direction normal to the (hkl) planes and the angular displacement of the hkl reflection to which it gives rise,

$$\Delta\theta_{hkl} = -e_{hkl} \tan \theta_{hkl}.$$

The displaced reflections coming from all the various crystallites which are in position to reflect combine to form a broadened line, in which the root-mean-square displacement of the Bragg angle is evidently $\tan \theta_{hkl}$ times the r.m.s. value of e_{hkl}, which we shall denote by E_{hkl}.

Smith and Stickley find $B_{hkl}/\tan \theta_{hkl}$ (B = corrected half-intensity breadth of line) to be independent of λ, in agreement with this prediction of the microstress theory.[1] With cold-worked tungsten, which is elastically isotropic, they found it also independent of crystallographic direction, but not with α-brass, which is elasti-

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[1] Charles S. Smith and E. E. Stickley, Phys. Rev. 64, 191 (1943).

cally anisotropic. Their observations with cold-worked α-brass are presented in Table I.

The half-intensity breadth B is the range of scattering angle (2θ) corresponding to that part of the line over which the intensity is at least half that at the center of the line. On the assumption that the microstresses obey a Gaussian law, the distribution of intensities over each line is Gaussian, and the corresponding r.m.s. microstrain is $E=(B/\tan\theta)/2^2\times 1.175=(B/\tan\theta)/4.70$, where the 2's are accounted for by the doubling of angles due to reflection and to taking both right and left halves of the lines. This E is also tabulated in Table I. It may be in error by a constant factor close to one if the microstress distribution is not Gaussian, but that is of little consequence for the following arguments.

The table includes the orientation factor $\Gamma=(k^2l^2+l^2h^2+h^2k^2)/(h^2+k^2+l^2)^2$, upon which Young's modulus for the various directions depends.[2]

For cold-worked tungsten, Smith and Stickley obtain the value 14.00×10^{-3} radian for $B/\tan\theta$, whence $E=2.980\times10^{-3}$.

It is the purpose of this paper to determine the implications of these values of E concerning the system of internal stresses existing in the cold-worked brass and tungsten. The method used will be to postulate various theoretical models of internal-stress systems, to compute the corresponding values of E with particular reference to metals of the cubic system, to which both α-brass and tungsten belong, and to compare these values with the empirical data.

FIRST MODEL. UNIFORM NORMAL STRESS

The first model to be considered is the microstress system in which all parts of the cold-worked sample are assumed to suffer uniform normal stress whose magnitude is random, of mean value zero, and constant throughout only small domains.

For materials of the cubic system the inverse of Hooke's law, reduced by symmetry considerations, gives the relations[3]

$$\left.\begin{aligned} e_{xx}&=s_{11}X_x+s_{23}Y_y+s_{23}Z_z,\\ e_{yy}&=s_{23}X_x+s_{11}Y_y+s_{23}Z_z,\\ e_{zz}&=s_{23}X_x+s_{23}Y_y+s_{11}Z_z,\\ e_{yz}&=s_{44}Y_z,\\ e_{zx}&=s_{44}Z_x,\\ e_{xy}&=s_{44}X_y,\end{aligned}\right\} \tag{1}$$

between the stress and strain existing at any point, where e_{xx}, e_{yy}, e_{zz} are the normal components of strain in the directions of the crystallographic axes, being positive for extension, and e_{yz}, e_{zx}, e_{xy} are the shearing components of strain associated with the subscripted axes; e_{yz} is, for example, approximately the change in angle between two lines originally parallel to the y and z axes, and is positive for a decrease in that angle. X_x, Y_y, Z_z are the normal components of stress in the directions of the crystallographic axes, being positive for tension, and Y_z ($=Z_y$ because there is no resultant torque), for example, is the shearing component of stress acting in the direction of the z axis on a plane whose directed normal is the positive y axis. The s's are called the elastic moduli for the material with respect to the crystallographic axes.

For uniform normal stress of magnitude n, $X_x=Y_y=Z_z=n$, $Y_z=Z_x=X_y=0$. Substitution into Eqs. (1) yields $e_{xx}=e_{yy}=e_{zz}=(s_{11}+2s_{23})n$,

TABLE I. The results of Smith and Stickley with α-brass.

Line	Radiation	$B/\tan\theta$	Direction	E	Γ
200	Fe	13.09×10^{-3} radian	[100]		0.0000
400	Cu	11.65			
	Ni	12.72			
	Co	13.49			
Average		12.74		2.714×10^{-3}	
311	Cu	11.05	[311]		0.1570
	Ni	10.55			
	Co	10.72			
	Fe	11.41			
Average		10.93		2.328	
420	Cu	10.91	[210]	2.322	0.1600
220	Ni	9.40	[110]		0.2500
	Co	10.00			
	Fe	10.45			
Average		9.95		2.106	
331	Cu	9.23	[331]		0.2741
	Ni	9.65			
Average		9.44		2.008	
222	Cu	7.44	[111]		0.3333
	Ni	8.15			
	Co	7.55			
	Fe	8.07			
Average		7.80		1.660	

[2] a. W. Voigt, *Lehrbuch der Kristallphysik* (B. G. Teubner, Leipzig, 1910), pp. 589, 739; b. W. A. Wooster, *A Textbook of Crystal Physics* (Cambridge University Press, London, 1938), p. 252; c. C. S. Barrett, *Structure of Metals* (McGraw-Hill Book Company, Inc., New York, 1943), p. 455.

[3] a. Reference 2a, page 586ff.; b. Reference 26, page 248; c. Reference 2c, page 454.

$e_{yz}=e_{zx}=e_{xy}=0$. Thus, the resultant strain is a uniform dilatation. Hence, the normal strain in any direction $[hkl]$ is independent of h, k, l, and its r.m.s. value must be

$$E=(s_{11}+2s_{23})N, \tag{2}$$

where N is the r.m.s. value of n. This result and those below hold whether or not the distributions are Gaussian.

Since its right side is independent of crystallographic direction, Eq. (2) fits the measurements on tungsten within the experimental error, but it yields a 40 percent error at best for α-brass. For tungsten $s_{11}=2.573\times10^{-13}$ cm^2/dyne, $s_{23}=-0.729\times10^{-13}$ cm^2/dyne, $s_{44}=6.604\times10^{-13}$ cm^2/dyne.[4] Putting these values into Eq. (2) and setting $E=2.980\times10^{-3}$, one finds $N=2.675\times10^{10}$ dynes/cm^2. For α-brass $s_{11}=19.4\times10^{-13}$ cm^2/dyne, $s_{23}=-8.35\times10^{-13}$ cm^2/dyne, $s_{44}=13.9\times10^{-13}$ cm^2/dyne.[4] Taking E as 2.12×10^{-3}, one gets an order of magnitude $N=7.80\times10^{9}$ dynes/cm^2.

SECOND MODEL. RANDOM PRINCIPAL STRESSES

In the second model it is assumed that the principal stresses[5] existing at any point of cold-worked metal are random in orientation and random and independent in magnitude, being constant throughout only small domains, and each on the average zero. Because of the linearity of Eqs. (1), the normal strain in the $[hkl]$ direction is the sum of the $[hkl]$ strains produced by each of the three principal stresses separately. Its mean-square value is, thus, the sum of the mean-square values of these three components, since, because of the independence, the cross-products average to zero. Suppose that these three principal stresses all have the same r.m.s. value T. Because of their randomness of orientation, each one on the average produces the same r.m.s. $[hkl]$ strain. Thus, E^2 is three times the mean-square $[hkl]$ strain produced by a single normal stress of random magnitude and random direction.

If the magnitude of such a single normal stress is t, and its orientation with respect to the crystallographic axes is specified by the direction cosines l, m, n, then[6]

$$\left.\begin{aligned} X_x&=l^2t, & Y_z&=mnt,\\ Y_y&=m^2t, & Z_x&=nlt,\\ Z_z&=n^2t, & X_y&=lmt. \end{aligned}\right\} \tag{3}$$

The normal strain in the $[hkl]$ direction due to this stress is[7]

$$e=H^2e_{xx}+K^2e_{yy}+L^2e_{zz}+KLe_{yz}+LHe_{zx}+HKe_{xy}, \tag{4}$$

where H, K, L are the direction cosines for this direction, $H=h/(h^2+k^2+l^2)^{\frac{1}{2}}$, etc. Combining Eqs. (1), (3), and (4), one obtains

$$e=[s_{23}+(s_{11}-s_{23})(H^2l^2+K^2m^2+L^2n^2)+s_{44}(KLmn+LHnl+HKlm)]t,$$

which is to be squared and averaged over all directions l, m, n. This averaging is performed by transforming to spherical coordinates, taking $l=\sin\theta\cos\phi$, $m=\sin\theta\sin\phi$, $n=\cos\theta$. The probability that θ lies between θ and $\theta+d\theta$ is $\frac{1}{2}\sin\theta d\theta$; the probability that ϕ lies between ϕ and $\phi+d\phi$ is $d\phi/2\pi$. The result is

$$(e^2)_{\text{Av}}=(1/15)t^2\{[3s_{11}^2+4s_{11}s_{23}+8s_{23}^2]+[s_{44}^2-4(s_{11}-s_{23})^2]\Gamma\},$$

where

$$\Gamma=K^2L^2+L^2H^2+H^2K^2=\tfrac{1}{2}-\tfrac{1}{2}(H^4+K^4+L^4).$$

Thus,

$$E=T(1/5)^{\frac{1}{2}}\{[3s_{11}^2+4s_{11}s_{23}+8s_{23}^2]+[s_{44}^2-4(s_{11}-s_{23})^2]\Gamma\}^{\frac{1}{2}}. \tag{5}$$

Because for an elastically isotropic metal[8] $s_{44}=2(s_{11}-s_{23})$, this E is independent of direction for such a metal, in accord with the observations on tungsten. To fit the observed value of E for tungsten, Eq. (5) requires that $T=3.34\times10^{9}$ dynes/cm^2. For α-brass (5) becomes

$$E=T(208-577\Gamma)^{\frac{1}{2}}\times10^{-13}\text{ cm}^2/\text{dyne}.$$

Γ lies between 0 for [100] and $\frac{1}{3}$ for [111]. Thus, $E_{100}/E_{111}=3.7$, which considerably exceeds the variation exhibited in Table I.

[4] a. E. Schmid and W. Boas, *Kristallplastizität* (Verlagsbuchhandlung, Julius Springer, Berlin, 1935), p. 200; b. F. Seitz and T. A. Read, J. App. Phys. **12**, 100 (1941).

[5] A. E. H. Love, *Mathematical Theory of Elasticity* (Cambridge University Press, London, 1927), p. 81.

[6] Reference 5, p. 80.

[7] Reference 5, p. 43.

[8] Reference 2a, p. 589.

THIRD MODEL. RANDOM SHEARING STRESSES

In the next model there is at each point a set of orthogonal coordinate axes with respect to which the normal components of stress all vanish. In such a model there is nowhere any dilatation. Suppose, as before, that the orientation of these axes is random and that the magnitudes of the three shearing components of stress are random and independent, all having the same r.m.s. value S. It is again only necessary to consider the effect of one of these three shearing stresses. Its magnitude will be designated by s and the direction cosines with respect to the crystallographic axes of the two axes with which it is associated by l_1, m_1, n_1; l_2, m_2, n_2, with $l_1l_2+m_1m_2+n_1n_2=0$, i.e., the two axes perpendicular. Thus,[6]

$$\left.\begin{array}{ll} X_x=2l_1l_2s, & Y_z=(m_1n_2+n_1m_2)s, \\ Y_y=2m_1m_2s, & Z_x=(n_1l_2+l_1n_2)s, \\ Z_z=2n_1n_2s, & X_y=(l_1m_2+m_1l_2)s. \end{array}\right\} \quad (6)$$

Combining Eqs. (1), (4), and (6), one finds

$$e=\{2(s_{11}-s_{23})(H^2l_1l_2+K^2m_1m_2+L^2n_1n_2) +s_{44}[KL(m_1n_2+n_1m_2)+LH(n_1l_2+l_1n_2) +HK(l_1m_2+m_1l_2)]\}S.$$

To average the square of this e with respect to direction, one may transform the direction l_1, m_1, n_1 to spherical coordinates and add another angular coordinate ψ to specify the position of direction l_2, m_2, n_2 about the direction l_1, m_1, n_1. The probability that ψ lies between ψ and $\psi+d\psi$ is $d\psi/2\pi$. We write $l_1=\sin\theta\cos\phi$, $m_1=\sin\theta\sin\phi$, $n_1=\cos\theta$; $l_2=\sin\psi\sin\phi-\cos\psi\cos\theta\cos\phi$, $m_2=-\sin\psi\cos\phi-\cos\psi\cos\theta\sin\phi$, $n_2=\cos\psi\sin\theta$. Thus, the square root of three times the mean-square value of the above e is

$$E=S(1/5)^{\frac{1}{2}}\{4(s_{11}-s_{23})^2 +3[s_{44}^2-4(s_{11}-s_{23})^2]\Gamma\}^{\frac{1}{2}}. \quad (7)$$

Like (5), (7) fits the observations on tungsten for which one obtains $S=10.10\times10^9$ dynes/cm^2. For α-brass (7) becomes

$$E=S(716-732\Gamma)^{\frac{1}{2}}\times10^{-13}\ \text{cm}^2/\text{dyne}.$$

This yields $E_{100}/E_{111}=4.0$, which again far exceeds the observed ratio, 1.635.

SLIP

The next two models include the phenomenon of slip.[9] The shearing stress in a crystal of α-brass or any other face-centered-cubic metal on a (111) plane in a $[10\bar{1}]$ direction cannot exceed a certain critical value, which probably is less than $\frac{1}{2}\times10^8$ dynes/cm^2,[10] without the crystal's slipping on that plane in that direction. This stress is small compared to the stresses that have been found in this work. Thus, it will be assumed that slip relieves essentially all of the shearing stress on the slip plane in the slip direction. For tungsten and other body-centered-cubic metals the slip direction is [111], but the slip plane is indefinite.

Fourth Model. Random Principal Stresses

The fourth model will be like the second except that it will be assumed that in each domain slip occurs once, relieving all of the corresponding shearing stress. As the mechanism for this relief one may assume that the previously existing stress system is modified so as to leave no resultant shearing stress on the plane of slip in the slip direction but not to change any other of the six stress components referred to axes coinciding with the slip. It will be assumed that this slip occurs in a random one of the 24 possible ways. However, the crystallographic axes can be chosen in each domain so that the normal to the slip plane and the slip direction always have the same direction cosines, say a, b, c, and A, B, C, respectively, where on account of the perpendicularity of the slip plane and direction $aA+bB+cC=0$. The normal $[hkl]$ strain due to the original principal stress t acting in the direction with cosines l, m, n, plus that caused by a set of stresses, which relieves the shearing stress related to the directions with cosines a, b, c, and A, B, C, but does not alter the other stress components as determined with respect to a set of orthogonal axes of which two have these directions, is

[9] See references 4a; 2b, p. 39; 2c, p. 288.

[10] Compare E. Schmid, *International Conference on Physics, Vol. II, The Solid State of Matter* (Phys. Soc., London, 1935).

$$e=\{s_{23}+(s_{11}-s_{23})(H^2l^2+K^2m^2+L^2n^2)+s_{44}(KLmn+LHnl+HKlm)$$
$$-[2(s_{11}-s_{23})(H^2aA+K^2bB+L^2cC)+s_{44}(KL[bC+cB]+LH[cA+aC]+HK[aB+bA])]$$
$$\cdot[aAl^2+bBm^2+cCn^2+(bC+cB)mn+(cA+aC)nl+(aB+bA)lm]\}t.$$

Because the crystallographic axes have been oriented with respect to the slip rather than the reflecting plane, the latter is equally likely to be any plane of the form $\{hkl\}$. Thus, it is necessary to average not only over l, m, n, but also over all permutations and changes of sign of H, K, L. Squaring the above e, taking these two averages, and averaging over t, whose r.m.s. value we write as T', tripling, and extracting the square root, one obtains

$$E=T'(1/15)^{\frac{1}{2}}\{[9s_{11}^2+12s_{11}s_{23}+24s_{23}^2]$$
$$+[2s_{44}^2-12(s_{11}-s_{23})^2]\Gamma$$
$$-(a^2A^2+b^2B^2+c^2C^2)[4(s_{11}-s_{23})^2$$
$$-(2s_{44}^2+12[s_{11}-s_{23}]^2)\Gamma]\}^{\frac{1}{2}}. \quad (8)$$

For a metal which slips on (111), $a^2A^2+b^2B^2+c^2C^2=\frac{1}{3}(A^2+B^2+C^2)=\frac{1}{3}$. Similarly for one which slips in direction [111] this quantity is $\frac{1}{3}$. Thus, the indefiniteness of the slip plane in tungsten is of no consequence. Under these circumstances Eq. (8) becomes

$$E=T'(1/45)^{\frac{1}{2}}\{[23s_{11}^2+44s_{11}s_{23}+68s_{23}^2]$$
$$+[8s_{44}^2-24(s_{11}-s_{23})^2]\Gamma\}^{\frac{1}{2}}. \quad (9)$$

This E is seen to depend on Γ even for isotropic materials. For tungsten (9) becomes

$$E=T'(2.35+1.93\Gamma)^{\frac{1}{2}}\times10^{-13}\ \text{cm}^2/\text{dyne},$$

giving $E_{100}/E_{111}=0.89$ instead of 1.00. For α-brass (9) is

$$E=T'(139-377\Gamma)^{\frac{1}{2}}\times10^{-13}\ \text{cm}^2/\text{dyne},$$

whence $E_{100}/E_{111}=3.2$ as compared with the empirical value 1.635.

Fifth Model. Random Shearing Stresses

If slip is added to the third model just as it was above to the second, one gets by a process similar to the above

$$E=S'(1/15)^{\frac{1}{2}}\{12(s_{11}-s_{23})^2$$
$$+6[s_{44}^2-6(s_{11}-s_{23})^2]\Gamma$$
$$+(a^2A^2+b^2B^2+c^2C^2)[-4(s_{11}-s_{23})^2$$
$$+6(s_{44}^2+2[s_{11}-s_{23}]^2)\Gamma]\}^{\frac{1}{2}}, \quad (10)$$

where S' is the r.m.s. value of s here. If $a^2A^2+b^2B^2+c^2C^2=\frac{1}{3}$,

$$E=S'(8/45)^{\frac{1}{2}}\{4(s_{11}-s_{23})^2$$
$$+3[s_{44}^2-4(s_{11}-s_{23})^2]\Gamma\}^{\frac{1}{2}}. \quad (11)$$

Although the E in Eq. (10) is not in general independent of Γ for isotropic materials, in the particular case of (11) it is. For tungsten (11) is $E=S'\times2.786\times10^{-13}$ cm^2/dyne, which requires $S'=10.70\times10^9$ dynes/cm^2 to fit the observations. For α-brass (11) is

$$E=S'(676-1540\Gamma)^{\frac{1}{2}}\times10^{-13}\ \text{cm}^2/\text{dyne},$$

from which $E_{100}/E_{111}=4.06$ where 1.635 is observed.

SIXTH MODEL. RANDOM CRYSTALLOGRAPHIC STRESSES

In this model the magnitudes of the six stress components determined with respect to the crystallographic axes in each domain are assumed random, independent, and of mean value zero. If the r.m.s. values of the three normal components are all P, and those of the three shearing components are all Q, we have from (1) and (4)

$$E=\{(s_{11}+2s_{23})^2P^2+[s_{44}^2Q^2$$
$$-2(s_{11}-s_{23})^2P^2]\Gamma\}^{\frac{1}{2}}. \quad (12)$$

This E is independent of Γ for isotropic materials only if $P^2=2Q^2$. Applying it to tungsten, for $E=2.980\times10^{-3}$, one must have $P=10.8\times10^9$ dynes/cm^2, $Q=7.6\times10^9$ dynes/cm^2. For brass a considerable variety of results can be obtained, depending upon P/Q. This point is considered at greater length below.

SYNTHESIS

It is not an accident that Eqs. (2), (5), (7)–(9), and (12) all exhibit E as a function of Γ of the same form,

$$E=E_0(1-R\Gamma)^{\frac{1}{2}}. \quad (13)$$

Any theory involving stresses which on the average act symmetrically with respect to the six crystallographic axes (three positive and three

negative) necessarily yields such a result. This is a consequence of the fact that the strain in the direction $[hkl]$ is a quadratic function of H, K, L. Its mean-square value is, therefore, an even symmetric quartic function of H, K, L. Thus, it can contain no odd powers of these quantities. It can contain no quadratic terms, for all even, symmetric, homogeneous, quadratic functions of H, K, L are multiples of $H^2+K^2+L^2=1$. And all even, symmetric, homogeneous, quartic functions of H, K, L are made up of multiples of $K^2L^2+L^2H^2+H^2K^2=\Gamma$ and $H^4+K^4+L^4=1-2\Gamma$. Thus, E^2 must be a linear function of Γ, and E must be of the foregoing form.

One should, therefore, determine the E_0 and R which fit the observations best. For tungsten R is obviously zero and $E_0=2.980\times10^{-3}$. In the case of α-brass we want the R for which the quantity $E_{obs}/(1-R\Gamma)^{\frac{1}{2}}$ is most nearly constant, E_{obs} being the E of Table I. This quantity is tabulated for several values of R in Table II. It is seen to be very nearly constant for R between 1.50 and 1.65. The [111] value does not seem to fit well, possibly because the low intensity of 222 reflection resulted in poor breadth measurements for this line. In what follows it will be assumed that the best fit is given by $R=1.58$, $E_0=2.70\times10^{-3}$, for which the corresponding values of (13),

$$E=2.70\times10^{-3}(1-1.58\Gamma)^{\frac{1}{2}},$$

have been tabulated in Table II.

For the case of α-brass, only one of the foregoing models can, when the corresponding E is written in the form of (13), take on the value of 1.58 for R, namely, the sixth. Eq. (12) is transformed into Eq. (14) for $P=9.8\times10^8$ dynes/cm^2, $Q=6.0\times10^9$ dynes/cm^2. For the first model, $R=0$, while for the second to fifth, inclusive, R is at least 2.7. (Note that R cannot exceed 3.) However, one can readily get four more models by superposing a stress system for which R exceeds 1.58 upon a system (the first model) for which R is less than 1.58. On the assumption that the superposed internal-stress systems are independent, the squares of the corresponding E's add directly, i.e., the radicands of the E's themselves add directly, and the resultant E is, of course, still of the form (13).

TABLE II. Determination of E_0 and R for α-brass.

		$\frac{E}{(1-R\Gamma)^{\frac{1}{2}}}$					
Γ	E	$R=1.50$	1.55	1.60	1.65	1.70	$(1-1.58\Gamma)\times2.70\times10^{-3}$
0.0000	2.714×10^{-3}	2.714×10^{-3}	2.714	2.714	2.714	2.714	2.70×10^{-3}
0.1570	2.328	2.662	2.678	2.692	2.704	2.720	2.34
0.1600	2.322	2.668	2.680	2.692	2.708	2.724	2.32
0.2500	2.106	2.668	2.694	2.720	2.748	2.780	2.10
0.2741	2.008	2.620	2.652	2.684	2.716	2.750	2.02
0.3333	1.660	2.346	2.386	2.426	2.472	2.522	1.86

Two parameters are then available, and the desired values for both E_0 and R can be obtained.

One of these parameters is N; the other is T, S, T', or S'. It may be added that three or more different internal-stress systems can be superposed, but the amount of each required then becomes indeterminate. The following combinations are found to yield the desired result of $R=1.58$ and $E_0=2.70\times10^{-3}$:

$N=6.6\times10^9$ dynes/cm^2, $T=14.2\times10^8$ dynes/cm^2;
$N=5.8\times10^9$ dynes/cm^2, $S=8.2\times10^8$ dynes/cm^2;
$N=6.5\times10^9$ dynes/cm^2, $T'=17.6\times10^8$ dynes/cm^2;
$N=5.6\times10^9$ dynes/cm^2, $S'=8.6\times10^8$ dynes/cm^2.

CONCLUSIONS

The fact that the fit shown in Table II is rather good, being based on Eq. (13), which applies to any system of microstresses, constitutes strong evidence in favor of the microstress theory. It not only accounts for the observed constancy of $B/\tan\theta$ with respect to wavelength, as Smith and Stickley concluded,[1] but also for its observed variation with direction.

Although the sixth model and the several superposition models all fit the data, they must be examined with regard to their reasonableness. On this basis the sixth model is immediately eliminated, since it is not reasonable that the three normal stresses in the directions of the crystallographic axes should be independent of each other. The combination of the first and fourth models might be thrown out because the latter does not fit the observations on tungsten. However, with as small a ratio of T' to N as has been found here for α-brass, the predicted variation of E with direction would be less than four percent, which may well be smaller than the experimental error.

The combinations with the models involving random shearing stress are quite plausible, since

any system of stresses can be resolved into the sum of three shearing stresses acting on mutually orthogonal planes and a uniform normal stress.[11] The models in which uniform normal stress and random principal stresses are combined (superposition of first and second and superposition of first and fourth) are quite reasonable. It is implausible that the principal stresses should be either all equal, as in the first model, or all independent, as in the second. Something intermediate is rather to be expected, because when one of the principal stresses is positive, for example, the Poisson contraction prejudices the others toward being positive also. Superposition of the two systems (second or fourth on first) yields a model in which the coefficient of correlation between any two principal stresses is $N^2/(N^2+T^2)$. It is evident that no one simple model by itself appears reasonable.

One may object to our treatment of slip, for actually which of the 24 possible slip mechanisms comes into play is not a matter of random choice but of the one on which the greatest shearing component acts. Moreover, there may be multiple slip and even twinning in a single domain. Had these considerations been taken into account, however, the calculation would have become very difficult. The point may be raised that Table I lists $B/\tan\theta$ only for those members of each family of planes which were in position to reflect. For the other planes E may be different from the tabulated values. However, no satisfactory way of taking this into account has been devised except to assume that the tabulated values are typical.

It is to be noted that the values of N listed in the preceding section are all close to 6.0×10^9 dynes/cm^2, while those of T, S, T', and S' are all much smaller. We may conclude that the microstress system in the α-brass is indeed essentially like that of the first model but with the admixture of a small amount of anisotropic stress of any variety, the exact form of the description of the latter being possibly immaterial.

The problem of the internal energy change associated with cold-work and its relation to the x-ray diffraction effects has been reviewed by others.[12,13] The present microstress systems lead to the same order of r.m.s. strain and hence to the same order of energy as was reported by Hayworth. This energy value, 0.05 cal./g, is far less than the only measured values of 1 cal./g, and the generally accepted implication is that the difference is associated with distortions on an atomic scale, e.g., Taylor dislocations.

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[11] Reference 5, p. 83.

[12] F. Seitz and T. A. Read, J. App. Phys. **12**, 177–178 (1941).

[13] F. E. Hayworth, Phys. Rev. **52**, 618 (1937).