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High Energy Neutron-Proton Scattering and the Meson Theory of Nuclear Forces with Strong Coupling*

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In the first part of this paper the anisotropy of the elastic scattering of 14-Mev neutrons by protons is investigated in the meson field theory of nuclear forces with the scheme proposed by Møller and Rosenfeld in the strong coupling limit and in the symmetrical form. In the second Born's approximation, the virtual isobars decrease the ratio R of the differential cross sections in the backward and perpendicular directions (in the center of mass system), and R decreases with the isobar energy. R is still larger than 1; nevertheless, our results suggest the possibility that the combined effects of the virtual isobars and the tensor force might give a forward scattering in agreement with Amaldi's experiments. In the second part, the cross section of the inelastic scattering of 100–200-Mev neutrons by protons with production of isobars is evaluated in the Møller-Rosenfeld, pseudoscalar and Schwinger theories. The ratio of the total inelastic cross section and the elastic one is of the order of 0.03, for 100-Mev neutrons and for an isobar energy equal to 45 Mev. An experiment to detect the isobars would be important for a check of the fundamental ideas of the strong coupling formalism.

INTRODUCTION

In this paper we investigate the two following problems: (1) the anisotropy of the elastic scattering of 14-Mev neutrons by protons in the symmetrical meson field theory of nuclear forces in the form proposed by Møller and Rosenfeld[1]; (2) the total cross section of the inelastic scattering of 100–200 Mev neutrons by protons with production of isobars, in the Møller-Rosenfeld, pseudoscalar[2] and Schwinger[3] forms of the meson theory, both problems in the limiting case of a strong coupling[4] between the meson field and the nucleons.

The angular distribution of the elastic scattering of high energy neutrons by protons was studied experimentally by Amaldi, Bocciarelli, Trabacchi, and Ferretti[5] and they found a preferentially forward scattering for neutrons of energies 12.5, 13.5, and 14 Mev. Their results are in contradiction with the assumption of an exchange force between the nucleons[6] and so cannot be explained by the meson theory in the symmetrical form[7] and in the weak coupling limit, as was shown by Hulthén[8] and Jauch.[9]

This is essentially due to the fact that in a theory which assumes a charge-exchange force between nucleons, the potential of states with an even angular momentum has a sign opposite to that of the potential of states with an odd angular momentum.[10] As the P-waves are the most important for determining the anisotropy of the scattering of 14-Mev neutrons by protons, the repulsive P-potential is essentially responsible for the theoretical discrepancies.

Recently, Fierz and Wentzel[11] showed that in

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[1] C. Møller and L. Rosenfeld, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **17**, No. 8 (1940).

[2] N. Kemmer, Proc. Roy. Soc. **166A**, 127 (1938).

[3] J. Schwinger, Phys. Rev. **61**, 387 (1942).

[4] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); **14**, 633 (1941); **16**, 222, 551 (1943); J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1941); W. Pauli and S.M. Dancoff, Phys. Rev. **62**, 85 (1942); R. Serber and S. M. Dancoff, Phys. Rev. **63**, 143 (1943); W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943).

[5] E. Amaldi *et. al.*, Naturwiss. **30**, 582 (1942).

[6] G. Wick, Zeits. f. Physik **84**, 799 (1933).

[7] The neutral meson theory (H. A. Bethe, Phys. Rev. **57**, 260, 390 (1940)) gives a satisfactory angular distribution for the scattering, as would be expected, since in this theory there is no charge exchange between the nucleons. This theory, however, does not take into account charged mesons, which were observed in the cosmic radiation, gives no saturation for the nuclear forces, and spoils the explanation of the β-decay, which was already given by Yukawa by assuming charged mesons alone. The theory in the symmetrical form, which considers both neutral and charged mesons in a symmetrical way, is the only one which satisfies the above requirements and accounts for the equality of the forces between like and unlike particles in corresponding states.

[8] L. Hulthén, Arkiv. f. Mat. Astr. Fysik **29A**, No. 33 (1943); **30A**, No. 9 (1943); **31A**, No. 15 (1944); also B. Ferretti, Ricerca Scient. **12**, 843,993 (1941).

[9] J. M. Jauch, Phys. Rev. **67**, 125 (1945).

[10] Cf. W. Pauli, "Meson theory of nuclear forces," Lectures at the Massachusetts Institute of Technology, 1944, p. 43.

[11] M. Fierz and G. Wentzel, Helv. Phys. Acta **17**, 215 (1944).

the meson theory with strong coupling the P-potential changes its sign for small distances of the interacting nucleons and for certain values of the isobar energy. And they already pointed out that such a behavior of the potential energy in the strong coupling theory might well give an anisotropy for the high energy neutron-proton scattering in agreement with Amaldi's experiments.

In the first part of this paper we investigate this possibility. We shall study the anisotropy of 14-Mev neutrons scattered by protons in the Møller-Rosenfeld theory in the strong coupling limit (and in the symmetrical form) and in the second Born's approximation. The use of this approximation is justified as far as the energy of the particles is concerned. It is not yet clear whether Born's collision method is good for the present problem, since the coupling constants, in rising powers of which we develop the wave function, are bounded from below in the strong coupling theory. The coupling constants that we chose, however, are still less than 1 and the convergence of the above development is assured in the Møller-Rosenfeld theory even if we replace the source function by the δ-function. As the first Born's approximation gives a cross section which essentially coincides with the one calculated in the weak coupling limit theory, we have treated the problem in the second approximation. It is only from this approximation on that the virtual excited states (isobars) of the nucleons make a contribution to the scattering cross section. Our task was to investigate whether this contribution is such as to decrease the value of R computed[12] with the first approximation and to see the dependence of R on the isobar energy. Our calculations lead to an affirmative conclusion and show that R decreases with the isobar energy. We believe that this result will remain if one treats the problem with a better approximation method.

In the second part of this paper we evaluate the cross section of the inelastic scattering of very high energy neutrons by protons with production of isobars. This problem has a great interest for the following reasons: if the strong coupling theory is basically true one should be able to detect experimentally the excited states of the nucleons. The fact that such excited particles have not yet been observed might be due to the high value of the isobar energy—energy of excitation of a nucleon into a higher spin and charge state—as compared to the experimentally available energies in the laboratory up to now. An estimation of the order of magnitude of the isobar energy can be obtained from the value of this energy which gives a satisfactory angular distribution of 14-Mev neutrons elastically scattered by protons. The results presented in Table I indicate that a rather too low value of the isobar energy would be expected for an agreement of the Møller-Rosenfeld theory with the Amaldi's experiments, if such an agreement is attained at all. This is highly improbable since a value of the isobar energy of the order of 5 Mev would give rise to an inelastic scattering of 14-Mev neutrons which has not been observed. One possibility, of course, is that the lifetime of such isobars might be very small so that the excited particles would fall down to their ground state before detection. Even so, one would expect to detect the slower unexcited particles as well as the remaining energy which would be carried out by electrons and neutrinos or by γ-rays.

On the other hand, the tensor force, which does not occur in the Møller-Rosenfeld potential, was shown by Jauch[9] to decrease appreciably the ratio R as compared to the value obtained with the Møller-Rosenfeld theory, both cases in weak coupling approximation. One may therefore expect that the combined effects of the tensor force and the isobar states of the nucleons will give a value for R in agreement with experiment and for a reasonably high isobar energy. In a paper just published, Wentzel[13] indicates that the isobar energy must be of the order of 45 Mev, if the tensor force is taken into account. This is the value that we used in our calculations.* Such isobar energy would require nucleons with a

[12] $R=\sigma(\pi)/\sigma(\pi/2)$ is the ratio of the differential cross sections in the backward and perpendicular directions in the center of mass system.

[13] G. Wentzel, Helv. Phys. Acta **18**, 430 (1945).

* I am indebted to Professor Pauli and Professor Jauch for communicating Wentzel's results to me in advance of their publication. Wentzel (reference 13) was able to show that the strong coupling theory with a tensor force (Schwinger's mixture) gives a value for R in agreement with Amaldi's experiments. This result shows the importance of the tensor force for the scattering problem (besides its quadrupole moment effect) and, together with the present calculations (Part I), is a decisive argument against the original Møller-Rosenfeld formalism.

minimum energy of the order of 100 Mev—in the laboratory system—for the effect to be detected experimentally.

PART I. THE ANISOTROPY OF THE SCATTERING OF 14-MEV NEUTRONS BY PROTONS IN THE MØLLER-ROSENFELD THEORY IN THE SYMMETRICAL FORM AND IN THE STRONG COUPLING LIMIT

1. The Wave Equation of the Two-Body Problem

The Hamiltonian of two nucleons in the symmetrical theory with strong coupling has the following form:*

$$H=\frac{1}{2M}(\Delta_I+\Delta_{II})+\frac{\epsilon}{2}\sum_{k=1}^{3}(d_{Ik}^2+d_{IIk}^2)+V. \quad (1)$$

The first and third terms are the usual kinetic and potential operators; the second term is the isobar energy of the two nucleons, where ϵ is a constant depending on the extension of the source and on the coupling constants, and the d's are the operators of the spin of the particles. The spin d_k and the isotopic spin h_k have the eigenvalues:

$$\sum_{k=1}^{3} d_k^2=\sum_{k=1}^{3} h_k^2=j(j+1); \quad j=\tfrac{1}{2},\, \tfrac{3}{2},\, \cdots$$

$$d_3=m; \quad h_3=n,$$

$$j\geqslant m,\, n\geqslant -j$$

and $n+\frac{1}{2}$ represents the nucleon charge in e-units.

In the Møller-Rosenfeld theory the potential energy of two nucleons I and II is:

$$\begin{aligned} V&=\Omega W(r); \quad \Omega=\textstyle\sum_\alpha(\mathbf{e}_\alpha{}^I\cdot\mathbf{e}_\alpha{}^{II});\\ W(r)&=(f\mu)^2e^{-\mu r}/r, \qquad (2)\\ r&\equiv|\mathbf{x}|=|\mathbf{x}_I-\mathbf{x}_{II}|\end{aligned}$$

where f is the coupling constant of the meson fields with the nucleons and μ is the meson mass; the matrix $e_{\alpha i}$ satisfies the commutation rules:

$$[d_1,\, e_{i2}]=[e_{i1},\, d_2]=ie_{i3},$$

$$[h_1,\, e_{2\alpha}]=[e_{1\alpha},\, h_2]=ie_{3\alpha},$$

and corresponds to the matrix $\tau_\alpha\sigma_i$ of the weak coupling theory.

The expression (2) for the potential operator holds only for distances r considerably larger than the source size a. It gives reasonable nuclear forces only if we assume that a is small compared to the forces range μ^{-1}:

$$a\mu\ll 1.$$

Besides this inequality, another condition has to be fulfilled in the present theory, namely:

$$(f\mu)^2\gg(a\mu)^2$$

which defines the strong coupling approximation.[4]

Because of the occurrence of the term Ω in (2) the spin of each nucleon does not commute with the Hamiltonian (1). It is therefore more convenient to replace the set of quantum numbers $j_I,\, m_I,\, n_I;\, j_{II},\, m_{II},\, n_{II}$ of two nucleons by the equivalent set $J,\, M,\, K,\, N;\, j_I,\, j_{II}$, where $J,\, M,\, K,\, N$ are the total spin and isotopic spin quantum numbers:

$$\left.\begin{aligned} \textstyle\sum_k (d_k{}^I+d_k{}^{II})^2&=J(J+1),\\ \textstyle\sum_k (h_k{}^I+h_k{}^{II})^2&=K(K+1),\\ d_3{}^I+d_3{}^{II}=M;\quad & h_3{}^I+h_3{}^{II}=N,\end{aligned}\right\} \quad (3)$$

$$|j_I-j_{II}|\leqslant J,\, K\leqslant j_I+j_{II} \quad (4)$$

which are integrals of motion.

Eliminating the motion of the center of mass and the spin-isotopic spin wave functions, the equation of two nucleons can be written in the following form:

$$\begin{aligned}&\{\Delta/M+E-E_j\}F(j_I,\, j_{II};\,\mathbf{x})\\ &=W(r)\sum_{j_I'j_{II}'}(j_I,\, j_{II};\, J,\, M,\, K,\, N|\Omega|j_I',\, j_{II}';\\ &\qquad J,\, M,\, K,\, N)F(j_I',\, j_{II}';\,\mathbf{x}), \quad (5)\end{aligned}$$

where F is the wave function and:[14]

$$E_j=\tfrac{1}{2}\epsilon\{(j_I+\tfrac{1}{2})^2+(j_{II}+\tfrac{1}{2})^2-2\}. \quad (6)$$

$(j_I,\, j_{II};\, J,\, M,\, K,\, N|\Omega|j_I',\, j_{II}';\, J,\, M,\, K,\, N)$ is the matrix of Ω with respect to the adopted set of quantum numbers. This matrix, which was calculated by Fierz[15] in a recent paper, is diagonal with respect to the quantum numbers $J,\, M,\, K,\, N$ and obeys the selection rule:

$$\Delta j_A=\pm 1; \quad A=I,\, II. \quad (7)$$

2. The Cross Section in the Second Born's approximation

In order to solve Eq. (5) by successive approximations, we develop the function F in a series of powers of the coupling constant $(f\mu)^2$; assuming

* In the system of units in which $\hbar=c=1$.

[14] The isobar energy in this form vanishes for the ground state $j_I=j_{II}=\frac{1}{2}$ of the nucleons.

[15] M. Fierz, Helv. Phys. Acta 17, 181 (1944).

these powers already included in the functions f_n we obtain from:

$$F(j_I, j_{II}; \mathbf{x}) = \sum_n f_n(j_I, j_{II}; \mathbf{x})$$

the sequence of equations:

$$\left\{\frac{\Delta}{M}+E-E_j\right\} f_n(j_I, j_{II}; \mathbf{x}) = W(r) \sum_{j_I' j_{II}'} (j_I j_{II}|\Omega|j_I', j_{II}') \times f_{n-1}(j_I', j_{II}'; \mathbf{x}) \quad (8)$$

where, for abbreviation, we have omitted the indices J, M, K, N of the matrix. These equations refer, of course, to a given set of values of these indices.

For our purpose, it is more convenient to work in momentum space and use Dirac's theory of scattering.[16] Let $\varphi_n(j_I, j_{II}; \mathbf{k})$ and $(\mathbf{k}|W|\mathbf{k}')$ be the transformed functions of f_n and $MW(r)$ in this space. The Eqs. (8) will go over to:

$$\{k^2_{j_Ij_{II}}-k^2\}\varphi_n(j_I, j_{II}; \mathbf{k}) = \sum_{j_I' j_{II}'} (j_I j_{II}|\Omega|j_I' j_{II}') \times \int (\mathbf{k}|W|\mathbf{k}')\varphi_{n-1}(j_I' j_{II}'; \mathbf{k}')d^3k', \quad (9)$$

$$k^2_{j_Ij_{II}} = M(E-E_j).$$

Born's approximation consists in taking a plane wave as the zeroth approximation. The normalized plane waves which obey the Pauli principle are, in configuration space:

$$u_a \cos(\mathbf{k}\cdot\mathbf{x}) \quad \text{and} \quad u_s \sin(\mathbf{k}\cdot\mathbf{x}), \quad (10)$$

where u_a and u_s are functions, respectively, antisymmetric and symmetric in the spin and isotopic spin variables. In the elastic scattering problem the only possible values of J and K are 0 and 1. The spin-isotopic spin wave function will be antisymmetric for the states $J=1$, $K=0$ and $J=0$, $K=1$, while it will be symmetric for $J=K=0$ and $J=K=1$; thus the functions (10) give the following plane waves in momentum space:

$$\left.\begin{array}{l} \text{for } J=1,\ K=0 \text{ and } J=0,\ K=1: \\ \qquad \dfrac{(2\pi)^{\frac{3}{2}}}{2}\{\delta(\mathbf{k}-\mathbf{k}_0)+\delta(\mathbf{k}+\mathbf{k}_0)\}, \\ \text{for } J=1,\ K=1, \text{ and } J=0,\ K=0: \\ \qquad \dfrac{(2\pi)^{\frac{3}{2}}}{2}\{\delta(\mathbf{k}-\mathbf{k}_0)-\delta(\mathbf{k}+\mathbf{k}_0)\}. \end{array}\right\} \quad (11)$$

$\mathbf{k}_0$ is the wave vector of the initial state in the center of mass system.

If we impose the condition that the particles be initially in their ground state, $j_0=(\frac{1}{2}, \frac{1}{2})$, we have to multiply the functions (10) by δ_{jj_0}, where we indicate with a single letter j the pair j_I, j_{II}.

As we see from (11), each spin state is composed of four different states, a singlet ($K=0$) and the triplet states ($K=1$) in the isotopic spin variables. N, however, is an integral of motion and as we require that the two particles be a proton and a neutron in the initial and final states we can fix $N=0$ (if we remember that $N+1$ = charge of the two nucleons in e-units) and disregard the two other states corresponding to $N=\pm 1$ for $K=1$.

From the plane waves (11) multiplied by δ_{jj_0} we obtain the following first approximation:

for $J=1$, $K=0$:

$$\varphi_1(j; \mathbf{k}) = \frac{(2\pi)^{\frac{3}{2}}}{2}\left[\frac{1}{k_j^2-k^2} - i\pi\delta(k_j^2-k^2)\right] \cdot (j; 1, 0|\Omega|j_0; 1, 0)[(\mathbf{k}|W|\mathbf{k}_0) + (\mathbf{k}|W|-\mathbf{k}_0)] \quad (12)$$

and analogous formulas for $J=0$, $K=1$; $J=K=1$; $J=K=0$. We have now indicated in the matrix of Ω the values of J and K. The expression which contains the δ-term in (12) corresponds to the condition of only outgoing waves in configuration space.

The second approximation will be:

for $J=1$, $K=0$:

$$\varphi_2(j; \mathbf{k}) = \frac{(2\pi)^{\frac{3}{2}}}{2}\left[\frac{1}{k_j^2-k^2} - i\pi\delta(k_j^2-k^2)\right] \cdot \sum_{j'} (j; 1, 0|\Omega|j'; 1, 0)(j'; 1, 0|\Omega|j_0; 1, 0) \times \left\{\int \frac{(\mathbf{k}|W|\mathbf{k}')(\mathbf{k}'|W|\mathbf{k}_0)}{k_{j'}^2-k'^2}d^3k' + \int \frac{(\mathbf{k}|W|\mathbf{k}')(\mathbf{k}'|W|-\mathbf{k}_0)}{k_{j'}^2-k'^2}d^3k'\right\}$$

and analogous expressions for the remaining states.

[16] P. A. M. Dirac, *Quantum Mechanics* (Clarendon Press, Oxford, 1935), second edition, p. 195.

Finally the differential cross section is:

$$\sigma(\vartheta)=\tfrac{1}{4}\sum_{J,M}\{\text{ampl. }\textstyle\sum_K(\varphi_1+\varphi_2)\}^2 \tag{13}$$

or

$$\sigma(\vartheta)=\tfrac{1}{4}\sigma_S(\vartheta)+\tfrac{3}{4}\sigma_{\mathrm{tr}}(\vartheta),$$

where

$$\begin{aligned}\sigma_S(\vartheta)=4\pi^4\Big\{&[(j;0,0|\Omega|j_0;0,0)+(j;0,1|\Omega|j_0;0,1)](\mathbf{k}|W|\mathbf{k}_0)+[(j;0,1|\Omega|j_0;0,1)\\&-(j;0,0|\Omega|j_0;0,0)](\mathbf{k}|W|-\mathbf{k}_0)+\textstyle\sum_{j'}\Big([(j;0,0|\Omega|j';0,0)(j';0,0|\Omega|j_0;0,0)\\&+(j;0,1|\Omega|j';0,1)\cdot(j';0,1|\Omega|j_0;0,1)]\displaystyle\int\frac{(\mathbf{k}|W|\mathbf{k}')(\mathbf{k}'|W|\mathbf{k}_0)}{k_0^2-ME_{j'}-k'^2}d^3k'\\&+[(j;0,1|\Omega|j';0,1)(j';0,1|\Omega|j_0;0,1)-(j;0,0|\Omega|j';0,0)\cdot(j';0,0|\Omega|j_0;0,0)]\\&\int\frac{(\mathbf{k}|W|\mathbf{k}')(\mathbf{k}'|W|-\mathbf{k}_0)}{k_0^2-ME_{j'}-k'^2}d^3k'\Big)\Big\}^2\end{aligned} \tag{14}$$

for the ordinary spin singlet, $J=0$. An analogous expression is easily derived for $\sigma_{\mathrm{tr}}(\vartheta)$, the cross section of the triplet states $J=1$.

The denominator which occurs in the integrals of (13) comes from the conservation of energy:

$$E=E_{j0}+k_0^2/M,$$

where E is the total energy of the system in the final state. This relation gives, together with the definition (9) of k_j^2:

$$k_j^2=k_0^2+M(E_{j0}-E_j),$$
$$E_{j0}=0 \text{ for } j_0=(\tfrac{1}{2},\tfrac{1}{2}).$$

As in our problem the initial and final spins of the two nucleons must be $\frac{1}{2}$, only the matrix elements with $j=j_0=\frac{1}{2}$ will occur in the first two terms of σ_s and σ_{tr}. In the terms with integrals the following elements will be present:

$$(\tfrac{1}{2},\tfrac{1}{2};J,K|\Omega|\tfrac{1}{2},\tfrac{1}{2};J,K)\times(\tfrac{1}{2},\tfrac{1}{2};J,K|\Omega|\tfrac{1}{2},\tfrac{1}{2};J,K), \tag{15}$$

$$\left.\begin{aligned}&(\tfrac{1}{2},\tfrac{1}{2};J,K|\Omega|\tfrac{1}{2},\tfrac{3}{2};J,K)\times(\tfrac{1}{2},\tfrac{3}{2};J,K|\Omega|\tfrac{1}{2},\tfrac{1}{2};J,K),\\&(\tfrac{1}{2},\tfrac{1}{2};J,K|\Omega|\tfrac{3}{2},\tfrac{1}{2};J,K)\times(\tfrac{3}{2},\tfrac{1}{2};J,K|\Omega|\tfrac{1}{2},\tfrac{1}{2};J,K),\end{aligned}\right\} \tag{16}$$

$$(\tfrac{1}{2},\tfrac{1}{2};J,K|\Omega|\tfrac{3}{2},\tfrac{3}{2};J,K)\times(\tfrac{3}{2},\tfrac{3}{2};J,K|\Omega|\tfrac{1}{2},\tfrac{1}{2};J,K). \tag{17}$$

Terms with values of j_I, j_{II}, higher than $\frac{3}{2}$ are excluded in the second approximation because of the selection rule (7) and the elastic condition. The elements (16) and (17) represent the contribution of the virtual isobars to the cross section in that approximation. Furthermore, the indices j_I, j_{II} of the matrix Ω must obey the vector model inequalities (4). These exclude the two terms (16) from the singlet cross section ($J=0$) as well as from the terms with $K=0$.

The final expression of the cross section is easily obtained by means of Fierz's formulas[15] for the matrix Ω and of the momentum transformed of the Yukawa potential:

$$(\mathbf{k}|W|\mathbf{k}')=\frac{g^2}{\mu^2+(\mathbf{k}'-\mathbf{k})^2};\quad g^2=\frac{(f\mu)^2M}{2\pi^2}.$$

3. Numerical Results and Conclusions

The integrals which occur in the cross section can be reduced to a simpler form, suitable for numerical evaluation; we shall not give here the details of this reduction.[17] The evaluation of these integrals leads to two different expressions for the cross section according to whether we have:

$$k_0^2<ME_j, \tag{18}$$

or:

$$k_0^2>ME_j. \tag{19}$$

In this work we evaluated the ratio $R=\sigma(\pi)/\sigma(\pi/2)$ only for the condition (18). This inequality means physically that the kinetic energy of the bombarding particles must be less than the isobar energy and this leads to only elastic scattering. The inequality (19) would imply inelastic scattering, the outgoing nucleon or the scatterer or both being able to make a transition into an

[17] Cf. J. Leite Lopes, Ph.D. Thesis, Princeton University, 1945; also J. Leite Lopes, An. Acad. Bras. Ci. (to be published).

TABLE I. Angular distribution of the scattering of 14-Mev neutrons by protons in the symmetrical Møller-Rosenfeld theory with strong coupling (second Born approximation). $R=8.04$ for $(f\mu)^2=0.59$ in the first Born approximation; the theory does not include longitudinal mesons.

E_i (in Mev)	$(f\mu)^2=0.375$	R $(f\mu)^2=0.46$	$(f\mu)^2=0.59$
9	5.73	5.25	4.85
15	5.81	5.65	5.13
30	6.71	6.56	6.01

excited state at the cost of the kinetic energy (initial). The problem of inelastic scattering will be considered in the second part of this paper.

The inequality (18) imposes an additional condition upon the second order terms of the cross section (13): it excludes the terms with the intermediate values $\frac{1}{2}$, $\frac{1}{2}$ of j_I and j_{II} (reality of the kinetic energy of the incident neutrons).

The numerical evaluation of $\sigma(\vartheta)$ was made for $\vartheta=\pi$ and $\vartheta=\pi/2$ with the following data: $E_0=14$ Mev (in the laboratory system); $\mu=177m$, where m is the electron rest mass. The ratio R depends on ϵ, the isobar energy constant and on the coupling constant. We give the results in Table I. The numerical evaluation of the integrals was made by taking a sum of the integrands over five equidistant points in the interval (0, 1). The values of $(f\mu)^2$ that we chose are those given by Pauli and Kusaka,[4] Jauch and Hu,[18] and Hulthén.[8] Hulthén's coupling constant is perhaps the most reliable for our problem since he determined it with the Møller-Rosenfeld scheme.

One sees from Table I that the ratio R decreases with the isobar energy. This is in agreement with Fierz and Wentzel finding that the potential energy of the P-waves becomes attractive for values of ϵ less than about 30 Mev. A comparison of the values of R given in the table with the value of R calculated in the first Born's approximation, namely $R=8.04$,* shows that the virtual isobars make a contribution to R in the direction of decreasing the backward anisotropy of the scattering. As to the values of R themselves, they depend, of course, on the approximation used and on the rough evaluation of the integral terms of the cross section. Since the Born's approximation underestimates the S-waves we expect the exact values of R to be smaller than those given in Table I. On the other hand, as was already pointed out in the Introduction, a possibility which the results of our calculations suggests is that the combined effects of the tensor force and of the isobars give a result in final agreement with the experimental angular distribution (see reference 13).

We must also have in mind that in the Møller and Rosenfeld theory here investigated, only one coupling constant of the vector field was taken into account, namely that of the transverse mesons. For the sake of simplicity, the vector theory so far studied in the strong coupling limit excludes the interaction of the nucleons with the longitudinal mesons. This results in the fact that the mixed theory in second Born's approximation disposes of two constants, the isobar energy and the coupling constant $(f\mu)^2$. With regard to the values of R the theory is thus equivalent to the same theory in weak coupling limit and in first approximation, which also disposes of two constants, the longitudinal and transverse mesons coupling constants. We do not know yet the effect, in strong coupling limit, of the longitudinal mesons on the anisotropy of the neutron-proton scattering.

PART II. INELASTIC SCATTERING OF HIGH ENERGY NEUTRONS BY PROTONS WITH PRODUCTION OF ISOBARS

4. Introduction

In the preceding part of this paper, it was shown that the virtual excited states of the nucleons give a contribution to the cross section of the elastic scattering of high energy neutrons by protons in such a way as to decrease the backward anisotropy which is obtained with the weak coupling theory in the symmetrical form. This result is of interest because it suggests that the wrong angular distribution given by the thus far investigated meson theories in the symmetrical form is due not only to the exchange character of the potential operator but also to the weak coupling approximation. Or, in other words, in the theory with strong coupling the symmetrical form may be maintained, as it is desirable for the saturation requirements of the nuclear forces, and the observed anisotropy of the

[18] J. M. Jauch and N. Hu, Phys. Rev. 65, 289 (1944).

* This value of R is obtained with only the interaction nucleons-transverse mesons. If one includes longitudinal mesons one gets $R=4.8$.

neutron-proton scattering may be attributed to the virtual excited states of the nucleons. If this is so, a question which immediately arises is whether it is possible to detect experimentally the nucleons in an excited state.* If the theory predicts the excitation of heavy particles in a certain range of energy then, at least in principle, an experiment to detect the isobars would furnish a crucial test for the strong coupling assumptions. If one could thus decide between the two alternative approximations—the weak and the strong coupling approximations—a new light would certainly be shed on the nature of the nuclear particles.

In this part, we evaluate, for a certain range of energy of the bombarding particles, the cross section of the inelastic scattering of neutrons by protons, with excitation of one of the particles into a higher spin and charge state. In a paper just published by Wentzel,[13] it is shown that one obtains a reasonable forward elastic scattering in a strong coupling theory with a tensor force, for values of the isobar energy of the order of 45 Mev. This is the value that we assumed in this section. We limited our evaluation of the inelastic cross section to the first Born approximation. This is probably not too bad an approximation within the high energy region to which we restricted ourselves. The computations were also made for the three main forms of the meson field theory, namely the Møller-Rosenfeld, the pseudoscalar, and the Schwinger theories. The ratio of the inelastic to the elastic cross section is found to be of the order of 3 percent for 100-Mev bombarding neutrons.

5. Inelastic Cross Section in the Møller-Rosenfeld Theory

From (13), Section 2, we obtain the differential cross section in the first approximation by simply cancelling the second-order terms. In the inelastic problem and in this approximation the dependence of σ_{inel} on the isobar energy E_j arises from the energy equation:

$$k^2/M+E_j=k_0^2/M.$$

The process which we investigate is the following: two particles collide against each other, each having an initial spin $\frac{1}{2}$; after collision one of the nucleons is left with spin $\frac{3}{2}$: transition $\frac{1}{2}, \frac{1}{2} \rightarrow \frac{1}{2}, \frac{3}{2}$. From the vector model inequalities (4) we see that the nucleons cannot be in a singlet state $J=0$, $K=0$.

Fierz's formulas[15] for the matrix Ω give:

$$(\tfrac{1}{2}, \tfrac{1}{2}; 1, 1|\Omega|\tfrac{1}{2}, \tfrac{3}{2}; 1, 1)=4\sqrt{2}/9$$

and the differential cross section has the following form:

$$\sigma_{\text{in}}{}^{MR}(\vartheta)=\frac{32}{27}\frac{k}{k_0}(f\mu)^4M^2\frac{k^2k_0{}^2\cos^2\vartheta}{\{(\mu^2+k^2+k_0{}^2)^2-4k^2k_0{}^2\cos^2\vartheta\}^2}.$$

The total cross section is:

$$\sigma_{\text{in}}{}^{MR}=2\pi\int_0^\pi \sigma_{\text{in}}{}^{MR}(\vartheta)\sin\vartheta d\vartheta$$

$$=\frac{4\pi}{27}(f\mu)^4M^2\frac{1}{k_0{}^2}\left\{\frac{4kk_0}{(\mu^2+k^2+k_0{}^2)^2-4k^2k_0{}^2}-\frac{1}{\mu^2+k^2+k_0{}^2}\log\frac{\mu^2+(k+k_0)^2}{\mu^2+(k-k_0)^2}\right\}.$$

6. Inelastic Cross Section in the Pseudoscalar Theory

In this theory, the component M of the total ordinary spin of two nucleons is not a constant of motion. The cross section is accordingly:

$$\sigma(\vartheta)=\tfrac{1}{4}\sum_{J,M,M_0}\{\textstyle\sum_K \text{ampl. } \varphi_1\}^2. \tag{13'}$$

* A question which is intimately connected with the observation of these excited states is the lifetime of such states and the possible processes of unexcitation or disintegration. A nucleon with spin $\frac{3}{2}$ can have any one of the following charges (degeneracy in the symmetrical theory): $(n+\frac{1}{2})e=-e$, 0, e, $2e$. For a nucleon with spin $j=\frac{3}{2}$ and charge 0 or e, Jauch, in recent calculations, has found a lifetime of the order of 10^{-18} sec. for a transition into the ground state $(j=\frac{1}{2})$ with emission of γ-rays. A nucleon which has undergone such intermediate processes of excitation and emission of photons will have a final kinetic energy much smaller than the initial one and this can be easily detected in a cloud-chamber picture. A nucleon with excited charge $-e$ or $2e$ would probably undergo β-decay.

Using the matrix elements given in Appendix I we obtain:

$$\sigma_{\text{in}}{}^{P}(\vartheta)=\frac{1}{81}f^4\frac{k}{k_0}M^2\left\{\frac{12k^2k_0{}^2\kappa^4\cos^2\vartheta}{\{(\kappa^2+k^2+k_0{}^2)^2-4k^2k_0{}^2\cos^2\vartheta\}^2}+\frac{k^2k_0{}^2\sin^2\vartheta}{(\kappa^2+k^2+k_0{}^2)^2-4k^2k_0{}^2\cos^2\vartheta}\right\}.$$

The tensor force singularities cancel out in first Born's approximation.

The total cross section is:

$$\sigma_{\text{in}}{}^{P}=\frac{\pi}{162}\frac{f^4M^2}{k_0{}^2}\left\{\frac{12kk_0\kappa^4}{(\kappa^2+k^2+k_0{}^2)^2-4k^2k_0{}^2}-\frac{6\kappa^4+(\kappa^2+k^2+k_0{}^2)^2-4k^2k_0{}^2}{2(\kappa^2+k^2+k_0{}^2)}\log\frac{\kappa^2+(k+k_0)^2}{\kappa^2+(k-k_0)^2}+8kk_0\right\}.$$

7. Inelastic Cross Section in the Schwinger Theory

The differential cross section in Schwinger's theory is obtained in the same way as in the pseudoscalar case. With the help of the matrices given in Appendix II we find:

$$\begin{aligned}\sigma_{\text{in}}{}^{S}(\vartheta)=\frac{1}{324}f^4M^2\frac{k}{k_0}\Bigg\{&3(\mu^2-\kappa^2)^2\left[\frac{(\mathbf{k}-\mathbf{k}_0)^2}{[\kappa^2+(\mathbf{k}-\mathbf{k}_0)^2][\mu^2+(\mathbf{k}-\mathbf{k}_0)^2]}-\frac{(\mathbf{k}+\mathbf{k}_0)^2}{[\kappa^2+(\mathbf{k}+\mathbf{k}_0)^2][\mu^2+(\mathbf{k}+\mathbf{k}_0)^2]}\right]^2\\
&+4(\mu^2-\kappa^2)^2\frac{|\mathbf{k}\times\mathbf{k}_0|^2}{[\kappa^2+(\mathbf{k}-\mathbf{k}_0)^2][\mu^2+(\mathbf{k}-\mathbf{k}_0)^2][\kappa^2+(\mathbf{k}+\mathbf{k}_0)^2][\mu^2+(\mathbf{k}+\mathbf{k}_0)^2]}\\
&+8\left[\frac{3\mu^4[(k+k_0)^2-(\mathbf{k}-\mathbf{k}_0)^2]}{[\mu^2+(\mathbf{k}-\mathbf{k}_0)^2][\mu^2+(\mathbf{k}+\mathbf{k}_0)^2]}-\frac{2\mu^2(\mu^2-\kappa^2)[(\mathbf{k}+\mathbf{k}_0)^2-(\mathbf{k}-\mathbf{k}_0)^2]}{[\mu^2+(\mathbf{k}-\mathbf{k}_0)^2][\mu^2+(k+k_0)^2]}\right.\\
&\left.\times\left(\frac{(\mathbf{k}-\mathbf{k}_0)^2}{[\kappa^2+(\mathbf{k}-\mathbf{k}_0)^2][\mu^2+(\mathbf{k}-\mathbf{k}_0)^2]}-\frac{(\mathbf{k}+\mathbf{k}_0)^2}{[\kappa^2+(\mathbf{k}+\mathbf{k}_0)^2][\mu^2+(\mathbf{k}+\mathbf{k}_0)^2]}\right)\right]\Bigg\}.\end{aligned}$$

The total cross section is easily obtained from this formula and has a long expression which we do not need to write here.

8. Differential Cross Section of the Elastic Scattering

Møller-Rosenfeld theory:

$$\sigma_S(\vartheta)=\frac{M^2(f\mu)^4}{9}\left\{\frac{1}{\mu^2+4k_0{}^2\sin^2(\vartheta/2)}-\frac{2}{\mu^2+4k_0{}^2\cos^2(\vartheta/2)}\right\}^2$$

$$\sigma_{\text{tr}}(\vartheta)=\frac{M^2(f\mu)^4}{81}\left\{\frac{1}{\mu^2+4k_0{}^2\sin^2(\vartheta/2)}+\frac{2}{\mu^2+4k_0{}^2\cos^2(\vartheta/2)}\right\}^2.$$

Pseudoscalar theory:*

$$\sigma_S(\vartheta)=\frac{M^2(f\kappa)^4}{81}\left\{\frac{1}{\kappa^2+4k_0{}^2\sin^2(\vartheta/2)}-\frac{2}{\kappa^2+4k_0{}^2\cos^2(\vartheta/2)}\right\}^2,$$

$$\begin{aligned}\sigma_{\text{tr}}(\vartheta)=\frac{M^2}{81}f^4\Bigg\{&48\left[\frac{k_0{}^2\sin^2(\vartheta/2)}{\kappa^2+4k_0{}^2\sin^2(\vartheta/2)}+\frac{2k_0{}^2\cos^2(\vartheta/2)}{\kappa^2+4k_0{}^2\cos^2(\vartheta/2)}\right]^2-8\left[\frac{k_0{}^2\sin^2(\vartheta/2)}{\kappa^2+4k_0{}^2\sin^2(\vartheta/2)}\right.\\
&\left.+\frac{2k_0{}^2\cos^2(\vartheta/2)}{\kappa^2+4k_0{}^2\cos^2(\vartheta/2)}+\frac{8k_0{}^4\sin^2\vartheta}{(\kappa^2+4k_0{}^2\sin^2(\vartheta/2))(\kappa^2+4k_0{}^2\cos^2(\vartheta/2))}\right]+3\Bigg\}.\end{aligned}$$

* The formula given by Hulthén (Arkiv. Mat. Astr. Fysik **31A**, No. 15 (1944)) for the triplet cross section in the pseudoscalar theory and in first Born's approximation is not correct. The above formula was derived in the well-known way from potential (20) or (21) of Appendix I. The tensor force is responsible for the occurrence of k_0 in the numerator of the triplet cross section and this makes the total cross section larger than the one given by the Møller-Rosenfeld theory; the constant term of the triplet cross section is due to the δ-term of (20).

9. Numerical Results and Conclusions

In Table II the numerical values of the total cross section of the inelastic scattering are given in each theory. E_0 is the energy of the bombarding neutrons in the laboratory system and the value assumed for the isobar energy was 45 Mev. The coupling constant was taken equal to 0.375, as given by Pauli and Kusaka.[4] It is seen that the tensor force increases considerably the cross section, its effect being smaller, as would be expected, in the Schwinger theory than in the pseudoscalar theory. The total elastic cross section (which is easily obtained from the formulas given in Section 8 by integration over the angles) is of the order of $0.06\cdot10^{-24}$ cm² for 100-Mev particles and Møller-Rosenfeld potential while it is of the order of $0.4\cdot10^{-24}$ cm² for the pseudoscalar theory and the same energy. A comparison between these values and those of the inelastic cross section shows that the ratio $\sigma_{\text{in}}/\sigma_{\text{el}}$ is of the order of 0.03 for the M.R. theory and of the order of 0.05 for the pseudoscalar theory. This ratio is probably inside the experimental range and gives a test for the theory as soon as 100-Mev heavy particles are available in the laboratory.

TABLE II. Cross section of the inelastic scattering of neutrons by protons with production of isobars.

	σ_{in} (in cm²)		
E_0 (in Mev)	Møller-Rosenfeld	Pseudosc.	Schwinger
100	$0.18\cdot10^{-26}$	$0.26\cdot10^{-25}$	$0.9\cdot10^{-26}$
110	$0.32\cdot10^{-26}$	$0.32\cdot10^{-25}$	$1.1\cdot10^{-26}$
150	$0.81\cdot10^{-26}$	$0.58\cdot10^{-25}$	$2.6\cdot10^{-26}$
180	$0.88\cdot10^{-26}$	$0.64\cdot10^{-25}$	
	$\mu=177m$	$\kappa=177m$	$\kappa=177m$; $\mu=2\kappa$

In conclusion, I wish to express my gratitude to Professor W. Pauli and Professor J. M. Jauch for the suggestion of this problem and many valuable discussions.

APPENDIX I. THE POTENTIAL MATRIX IN THE PSEUDOSCALAR THEORY

In the pseudoscalar theory with strong coupling the potential energy of two nucleons I and II is (in the small source assumption):

$$V=f^2\sum_\alpha\left\{(\mathbf{e}_\alpha{}^I\cdot\boldsymbol{\nabla})(\mathbf{e}_\alpha{}^{II}\cdot\boldsymbol{\nabla})\frac{e^{-\kappa r}}{r}+\frac{4\pi}{3}(\mathbf{e}_\alpha{}^I\cdot\mathbf{e}_\alpha{}^{II})\delta(\mathbf{x})\right\},\tag{20}$$

where κ is the pseudoscalar meson mass.

In the current literature only the first term of V is written down but it is easily seen that this first term alone and the differentiated form of V, namely:

$$\left.\begin{aligned}
V&=\frac{f^2}{3}\left\{\Omega\frac{\kappa^2}{r}e^{-\kappa r}+\Lambda\frac{1}{r^3}(3+3\kappa r+\kappa^2r^2)e^{-\kappa r}\right\},\\
\Omega&=\textstyle\sum_\alpha(\mathbf{e}_\alpha{}^I\cdot\mathbf{e}_\alpha{}^{II}),\\
\Lambda&=\sum_\alpha\left\{\frac{3}{r^2}(\mathbf{e}_\alpha{}^I\cdot\mathbf{x})(\mathbf{e}_\alpha{}^{II}\cdot\mathbf{x})-(\mathbf{e}_\alpha{}^I\cdot\mathbf{e}_\alpha{}^{II})\right\},
\end{aligned}\right\}\tag{21}$$

do not give the same transformed in momentum space. This is essentially due to the small source assumption which substitutes for the potential energy:

$$V=-f^2\textstyle\sum_\alpha\sum_{ij}e_{\alpha i}{}^Ie_{\alpha j}{}^{II}J_{ij}$$

$$J_{ij}=-\int\int U^I(x)U^{II}(x')\frac{\partial^2}{\partial x_i\partial x_j}\frac{e^{-\kappa R}}{R}d^3xd^3x';\quad R=|\mathbf{x}-\mathbf{x}'|$$

its value for point sources.* The first term of (20) thus obtained gives rise to (21) and to an additional δ-term, analogous to the right-hand side of Yukawa's equation:

$$\{-\nabla^2+\kappa^2\}X=4\pi\delta(\mathbf{x})$$

for point sources. If this singular term is subtracted from the potential energy—because it gives no

* $U^A(x)$ is the source function of nucleon A; for a point-nucleon $U(x)=\delta(x)$. See Pauli and Dancoff, reference 4,

finite binding energy for the deuteron nor any scattering cross section*—we get (21) and (20), whose momentum transformed is:

$$(\mathbf{k}|\,V|\mathbf{k}') = -G^2\left\{\frac{\Gamma}{\kappa^2+\mathbf{K}^2} - \frac{\Omega}{3}\right\},$$

$$G^2 = f^2/2\pi^2;\quad \mathbf{K} = \mathbf{k}' - \mathbf{k};\quad \Gamma = \textstyle\sum_\alpha (\mathbf{e}_\alpha{}^I\cdot\mathbf{K})(\mathbf{e}_\alpha{}^{II}\cdot\mathbf{K}).$$

Because of the occurrence of the tensor force Λ or Γ in the potential operator, the component M of the total (ordinary) spin is not an integral of motion. The potential matrix has the form:

$$\begin{aligned}(J,\,M,\,K,\,N;\,j_I,\,j_{II};\,\mathbf{k}|\,V|\,J,\,M',\,K,\,N;\,j_I',\,j_{II}';\,\mathbf{k}')\\ = -G^2\Bigg\{\frac{1}{\kappa^2+\mathbf{K}^2}(J,\,M,\,K,\,N;\,j_I,\,j_{II};\,\mathbf{k}|\,\Gamma|\,J,\,M',\,K,\,N;\,j_I',\,j_{II}';\,\mathbf{k}')\\ -\tfrac{1}{3}(J,\,M,\,K,\,N;\,j_Ij_{II};\,\mathbf{k}|\,\Omega|\,J,\,M',\,K,\,N;\,j_I',\,j_{II}';\,\mathbf{k}')\Bigg\}.\end{aligned}$$

The matrix of Ω was already calculated by Fierz and the computation of the matrix of Γ is straightforward if we use Fierz's formulas for the $e_{\alpha i}$'s.[15] The calculation is a little long and we shall not give its details here as they do not involve anything new.

The final matrix elements for the excitation of only one nucleon are the following:

$J=K=1$

$$(1;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|-1;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = G^2\frac{\sqrt{2}}{18}\frac{(K_1-iK_2)^2}{\kappa^2+\mathbf{K}^2}, \tag{A}$$

$$(1;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|0;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = \frac{G^2}{9}\frac{(K_1-iK_2)K_3}{\kappa^2+\mathbf{K}^2}, \tag{B}$$

$$(\pm1;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|\pm1;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = -\frac{G^2\sqrt{2}}{9}\left\{\frac{3(K_1{}^2+K_2{}^2)+2K_3{}^2}{2(\kappa^2+\mathbf{K}^2)} - \frac{4}{3}\right\}, \tag{C}$$

$$(0;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|0;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = \frac{2G^2\sqrt{2}}{9}\left\{\frac{K_1{}^2+K_2{}^2+2K_3{}^2}{2(\kappa^2+\mathbf{K}^2)} - \frac{2}{3}\right\},$$

$$(-1;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|1;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = \text{c.c. of (A)},$$

$$(-1;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|0;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = -\text{c.c. of (B)}$$

$$(0;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|-1;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = -\text{(B)},$$

$$(0;\,\tfrac{1}{2},\,\tfrac{1}{2};\,\mathbf{k}|\,V|1;\,\tfrac{1}{2},\,\tfrac{3}{2};\,\mathbf{k}') = \text{c.c. of (B)}.$$

The Eqs. (9), Section 2, are now of the form:

$$\{k_j{}^2-k^2\}\varphi_n(j;\mathbf{k}) = \sum_{j'}\int(M;\,j;\,\mathbf{k}|\,V|\,M_0;\,j';\,\mathbf{k}')\varphi_{n-1}(j';\,\mathbf{k}')d^3k',$$

where we have omitted the constant indices of the matrix and put: $j=(j_I,\,j_{II})$. These equations hold for a given set of values of J, K, N, M, M_0. From this equation an expression for (13′) is easily derived, as in Section 2. The term δ_{jj0} does not multiply (11) any more.

APPENDIX II. THE POTENTIAL MATRIX IN THE SCHWINGER THEORY

The potential energy of two nucleons (small sources) in this theory is

$$V = f^2\sum_\alpha\left\{(\mathbf{e}_\alpha{}^I\cdot\nabla)(\mathbf{e}_\alpha{}^{II}\cdot\nabla)\frac{e^{-\kappa r}-e^{-\mu r}}{r} + (\mathbf{e}_\alpha{}^I\cdot\mathbf{e}_\alpha{}^{II})\mu^2\frac{e^{-\mu r}}{r}\right\},\quad \mu>\kappa$$

which gives in momentum space

$$(\mathbf{k}|\,V|\mathbf{k}') = G^2\sum_\alpha\left\{(\mathbf{e}_\alpha{}^I\cdot\mathbf{K})(\mathbf{e}_\alpha{}^{II}\cdot\mathbf{K})\left[\frac{1}{\mu^2+\mathbf{K}^2} - \frac{1}{\kappa^2+\mathbf{K}^2}\right] + (\mathbf{e}_\alpha{}^I\cdot\mathbf{e}_\alpha{}^{II})\frac{\mu^2}{\mu^2+\mathbf{K}^2}\right\},$$

* N. Kemmer, Helv. Phys. Acta **10**, 47 (1937) (for the deuteron binding energy); Professor Pauli has shown, in unpublished calculations, that a δ-potential gives no scattering cross section.

where again

$$G^2=\frac{f^2}{2\pi^2};\quad \mathbf{K}=\mathbf{k}'-\mathbf{k}.$$

The matrix

$$(J,\ M,\ K,\ N;\ j_I,\ j_{II};\ \mathbf{k}|\ V|\ J,\ M',\ K,\ N;\ j_I',\ j_{II}';\ \mathbf{k}')$$

is obtained in the same way as was indicated in Appendix I. The non-diagonal elements, with respect to M, are easily derived from the pseudoscalar case since the Ω-term does not contribute to these elements. The diagonal terms are

$$(\pm 1;\tfrac{1}{2},\tfrac{1}{2};\mathbf{k}|\ V|\pm 1;\tfrac{1}{2},\tfrac{3}{2};\mathbf{k}')=\frac{-G^2\sqrt{2}}{9}\left\{\frac{(\mu^2-\kappa^2)[3(K_1{}^2+K_2{}^2)+2K_3{}^2]}{2[\kappa^2+\mathbf{K}^2][\mu^2+\mathbf{K}^2]}-\frac{4\mu^2}{\mu^2+\mathbf{K}^3}\right\},$$

$$(0;\tfrac{1}{2},\tfrac{1}{2};\mathbf{k}|\ V|0;\tfrac{1}{2},\tfrac{3}{2};\mathbf{k}')=-\frac{G^2\sqrt{2}}{9}\left\{\frac{(\mu^2-\kappa^2)[K_1{}^2+K_2{}^2+2K_3{}^2]}{[\kappa^2+\mathbf{K}^2][\mu^2+\mathbf{K}^2]}-\frac{4\mu^2}{\mu^2+K^2}\right\}.$$

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Resonance Reactions and Anomalous Scattering

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The purpose of the present paper is to give a derivation of the resonance formula for nuclear reactions which is free from artificial assumptions. Mathematically, the method used amounts to a Taylor series development of the wave function with respect to the energy. It is assumed that the first (energy independent) term in this development is, within a region of configuration space where all particles are close together, the same, no matter in which way the compound state is formed and that this is, in that region of configuration space, already a good approximation. The second term in the development of the wave function with respect to the energy difference from the resonance energy can then be calculated very easily and this calculation is carried out in Section II for resonance scattering, in Sections III and IV for resonance reactions. It is assumed in both calculations that the colliding particles have zero orbital angular momentum around their center of mass. The third term in the same expansion is estimated for resonance scattering in Section VI and it is shown that, if there were no other resonances in the neighborhood, the effect of the third term would be negligible over a very wide energy range (several hundred kilovolt). The formulae for the cross sections, as obtained, are of greater generality than the customary ones inasmuch as they contain extra terms which could be interpreted as potential scattering and potential reaction. The existence of such terms has been noticed already by Bethe. However, as discussed in Section V, the extra terms are, particularly in the neighborhood of the resonance, much smaller than the resonance terms so that one is led back, in practice, to the ordinary resonance formulae, as given, e.g., by Bethe. In particular, the disintegration probability is, as function of energy, proportional to the velocity with which the reaction products separate if the orbital angular momentum of the separating particles vanishes. It may be worth while to remark that the resonance part of the collision matrix has a particularly simple form and is, e.g., of rank 1. The case of orbital angular momentum $1\hbar$ is discussed in Section VII. In this case, the disintegration probability of the compound state is proportional to the third power of the velocity of the separating particles so that the scattering is, at very low energies, proportional to the square of the energy. The same holds, in this case, of the "potential scattering" also. Section VII also contains an investigation of the region of validity of the formulae in case of angular momentum $1\hbar$ between the colliding particles and shows that this region will extend to the neighboring resonances.

I. INTRODUCTION

ALTHOUGH there is no experimental evidence available to corroborate the resonance formula[1] with any great accuracy, a number of reformulations of the underlying picture[2] have appeared in the literature.[3] All these formula-

[1] The first four sections of the present article are based on a report, dated April 23, 1945, which the writer prepared for the Uranium Project. He is indebted to the Army Engineers for clearing that report for publication.

[2] G. Breit and E. P. Wigner, Phys. Rev. **49**, 519, 642 (1936); N. Bohr, Nature **137**, 344 (1936).

[3] H. A. Bethe and G. Placzek, Phys. Rev. **51**, 450 (1937); F. Kalckar, J. R. Oppenheimer, and R. Serber, Phys. Rev. **52**, 273 (1937); H. A. Bethe, Rev. Mod. Phys. **9**, 71 (1937); P. L. Kapur and R. Peierls, Proc. Roy. Soc. **166**, 277 (1938); and, in particular, A. J. F. Siegert, Phys. Rev. **56**, 750 (1939).