THE CORRECTION FOR THERMOELECTRIC CURRENT TO BE APPLIED TO THE THROW OF A BALLISTIC MOVING-COIL GALVANOMETER.

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THEORETICAL.

ON account of the simplicity and convenience of the condenser method of determining the ballistic constant of a moving-coil galvanometer, one is likely to use it in preference to the more complicated inductance-coil methods in making an accurate determination of the constant. This is especially true when the instrument is used on open circuit, in which case there are no particular difficulties to be overcome in making a precise measurement. When the discharge to be measured takes place in the closed galvanometer circuit,[1] the constant should be determined under similar conditions, because of the dependence of the ballistic throw upon damping. When using a standard condenser for obtaining the constant on closed circuit it is necessary, immediately after the condenser has been discharged, to connect the galvanometer through a resistance such that the total resistance is the same as it is to be in the circuit with which the measurements are to be made. This is easily accomplished by means of special keys or switches.

Upon changing from open to closed circuit one is likely to encounter a thermoelectric current[2] which deflects the coil either in the same direction as or opposite to that of the throw caused by the impulsive discharge. The time of reaching the maximum elongation from the null point is not the same in the case of a current of very short duration as in the case of a deflection due to a steadily impressed electromotive force. Were these time intervals alike, the correction could be made by algebraic addition of the maxima. The relation between them depends upon the amount of damping in the circuit, and is therefore different for different values of the logarithmic decrement. Zeleny[3] has given an experimental curve for correcting the observed throw and obtaining from it

[1] Zeleny and Erikson, Manual of Physical Measurements, 3d ed., p. 172; Smith, Electrical Measurements, p. 195.

[2] A. Zeleny, PHYS. REV., O.S., 23, 1906, 414. Another method of determining the constant, not yet published, obviates the necessity for making the correction for thermoelectric effect.

[3] Loc. cit.

the magnitude of the throw which would have resulted had there been no thermo-electromotive force in the circuit.

It is the purpose of this paper to obtain a relation between the quantities involved, by means of which the correction may be made. The following notation will be used:

I_0 = moment of inertia of the coil;
$2f$ = proportionality constant between damping moment and angular velocity of the coil;
$M = nAH$, where n is the number of turns and A the mean area of the coil, and H the field intensity;
q^2 = elastic torque constant of the suspensions;
e = impressed electromotive force;
r = total resistance in the circuit;
i = current at instant t;
θ = angle of deflection;
Λ = logarithmic decrement of coil;
ρ = ratio between successive elongations;
T = complete period of coil.

When a steady electromotive force exists in the galvanometer circuit, the resulting motion of the coil may be expressed by the differential equation of moments

$$I_0\frac{d^2\theta}{dt^2} + 2f\frac{d\theta}{dt} + q^2\theta = Mi, \tag{1}$$

one solution of which is

$$\theta = \epsilon^{-at}(A\cos bt + B\sin bt) + \frac{Mi}{q^2}, \tag{2}$$

the condition for this solution being $I_0q^2 > f^2$. A and B are the constants of integration, $a = f/I_0$ and $b = \sqrt{(q^2/I_0) - (f^2/I_0^2)}$. Imposing the initial conditions, $t = 0$, $\theta = 0$, $d\theta/dt = 0$, we find

$$\theta = \frac{Mi}{q^2}\left[1 - \epsilon^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right)\right]. \tag{3}$$

This equation represents a damped motion of period $T = 2\pi/b = 2\Lambda/a$, θ having the value Mi/q^2 after a sufficiently long time has elapsed. The first maximum elongation is reached at the instant

$$t = \frac{\pi}{b} = \frac{T}{2}, \tag{4}$$

found by equating $d\theta/dt$ obtained from (3) to zero and solving for t.

Putting this value of t in (3) and solving for θ_d', the first maximum elongation,

$$\theta_d' = \frac{Mi}{q^2}(1 + \epsilon^{-\Lambda}). \tag{5}$$

The equation of motion of the coil when there is no torque tending to produce a definite displacement of the coil from its null position is

$$I_0 \frac{d^2\theta}{dt^2} + 2f\frac{d\theta}{dt} + q^2\theta = 0, \tag{6}$$

the solution of which, for the condition $I_0q^2 > f^2$, is the same as eq. (2) with the exception of the last term, which in this case is zero. Using the initial conditions $t = 0$, $\theta = 0$, $d\theta/dt = MQ/I_0$ to evaluate the constants of integration, the result is

$$\theta = \frac{MQ}{I_0}\epsilon^{-at}\left(\frac{1}{b}\sin bt\right). \tag{7}$$

This is also a damped vibratory motion of the same period as that given by eq. (3), provided a and b have the same values, respectively. Its first maximum elongation occurs at the instant

$$t = \frac{1}{b}\tan^{-1}\frac{b}{a}. \tag{8}$$

Suppose, now, that a quantity of electricity Q in a condenser is discharged through the galvanometer and that the circuit is immediately closed; and that, upon closing, a thermoelectric current flows. If the direction of the steady current is the same as that of the discharge, the resulting maximum is greater than it would be in the absence of the thermo-electromotive force; if the direction of the current is opposite to that of the discharge, the resulting throw is smaller. Clearly, the correction to be applied to the observed throw is the angle attained in a steady deflection[1] during the time interval required for a ballistic maximum, obtainable with the aid of equations (8) and (3). Designating this angle by ϕ, we find

$$\phi = \frac{Mi}{q^2}\left[1 - \epsilon^{-\frac{a}{b}\tan^{-1}\frac{b}{a}}\left(\cos\tan^{-1}\frac{b}{a} + \frac{a}{b}\sin\tan^{-1}\frac{b}{a}\right)\right],$$

or, in terms of the logarithmic decrement,

$$\phi = \frac{Mi}{q^2}\left[1 - \rho^{-\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}} \cdot \frac{2\cdot\frac{\Lambda}{\pi}}{\sqrt{1+\frac{\Lambda^2}{\pi^2}}}\right]. \tag{9}$$

[1] Assuming steady deflection small in comparison with throw. Zeleny mentions 1/3 as he maximum permissible ratio. For further discussion see below.

In order to make this equation for the correction applicable to any galvanometer of the type under discussion, it is convenient to express ϕ in terms of either of two ratios: $R_1 = \phi/d$, giving its value as a fraction of the steady deflection; or, $R_2 = \phi/\theta_d'$, which gives the value as a fraction of the *first maximum due to the steady current.* The value of d, from eq. (3) (putting $t = \infty$) is seen to be Mi/q^2; consequently,

$$R_1 = 1 - \rho^{-\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}} \cdot \frac{2 \cdot \frac{\Lambda}{\pi}}{\sqrt{1 + \frac{\Lambda^2}{\pi^2}}}. \tag{10}$$

Comparing equations (9) and (5), and noting that $\log_\epsilon \rho = \Lambda$, it is seen that

$$R_2 = R_1 \frac{\rho}{1 + \rho}. \tag{11}$$

Inasmuch as the actual motion of the coil is the result of the superposition of the motions defined by (3) and (7), the exact equations of the two possible kinds of motion are obtained, respectively, first, by adding the two equations, and second, by subtracting the one from the other. The observed maximum then occurs not at the instant given by eq. (8), but a trifle sooner or later than this, depending upon the relative directions of the current and impulsive discharge. By equating to zero the angular velocities obtained by differentiation of each of the combined equations and solving for t, we may write the result for both cases:

$$t = \frac{1}{b}\tan^{-1}\frac{b}{a \pm \frac{i}{bQ}}. \tag{12}$$

The upper or lower sign is taken according as the current and discharge are in the same or in opposite directions, respectively.

It would be a rare occurrence in an ordinary measurement to encounter an extraneous thermoelectric current which is appreciable as compared with bQ, considered in its effect upon the time required for the throw. Should it occur, however, the procedure would be to find the correction by several approximations, using (12) instead of (8) in eq. (3). To substitute (12) directly in the equations of motion obtained by the addition or subtraction of (3) and (7), and then to form the ratios to be used for correcting the observed throws, would lead to expressions which are too complex for practical application.

EXPERIMENTAL.

Procedure in Correcting Observed Throws.—Consideration of equations (10) and (11) shows that for a determination of either R_1 or R_2 for any particular case a single experimental determination suffices, namely of the quantity ρ. The factors $\rho^{1/\pi \tan^{-1} \pi/\Lambda}$ and $\sqrt{1 + (\Lambda^2/\pi^2)}$ occurring in the equations are obtainable from tables.[1] This makes the formulas readily applicable. To use the correction given by eq. (10), the observed steady deflection due to the thermoelectric current flowing in the closed circuit is multiplied by R_1 and the result is added to or subtracted from the observed throw, depending upon the relative directions of current and discharge. To apply eq. (11), the first maximum amplitude due to the thermoelectric current only is observed, and the correction

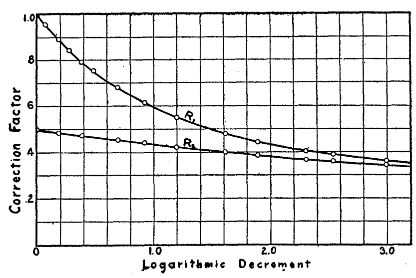

Fig. 1.

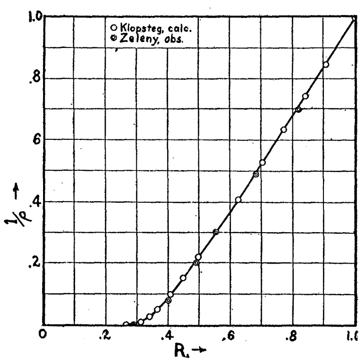

Fig. 2.

obtained by multiplying this value by R_2. Having once obtained R_2, its application is more economical of time than the use of R_1, since the first maximum elongation may be observed $T/2$ seconds after closing the circuit. This obviates the necessity of waiting for the deflection to attain its steady value, or of using a short-circuiting key for bringing the coil to rest.

The curves of Fig. 1 show the variation of each of the ratios with logarithmic decrement. In these curves the abscissas have been carried to $\Lambda = 3.2$ only, since, in practice, Λ seldom exceeds 1 when the galvanometer is used ballistically. If the logarithmic decrement is large, it is most convenient to render the coil just aperiodic by diminishing the resistance in the circuit. In the critically damped case $R_1 = R_2 = 26.4$ per cent. This is shown in Fig. 2.

Test of the Equations.—The experimental test of the method here outlined was taken from the work of Zeleny.[2] From eq. (10) a set of

[1] Kohlrausch, Lehrb. d. prakt. Phys., 12th ed., p. 723.

[2] Loc. cit.

corresponding values was obtained for R_1 and $1/\rho$, the abscissas and ordinates, respectively, of the experimental curve to which reference has been made. Fig. 2 shows the curve obtained by plotting these values, together with representative points taken from Zeleny's curve. There is good agreement between the theoretical and experimental curves.

Summary.

From a consideration of the equations of periodic motion of a galvanometer coil under the conditions, first, that a steady electromotive force exists in the circuit and second, that the electromotive force is zero, two expressions are deduced. Each expression gives the correction for thermoelectric current to be applied to the ballistic throw on closed circuit, the discharge having taken place on open circuit. The equations involve simply the determination of the ratio of one amplitude of the coil to the one next following, together with factors which are obtainable from existing tables. The calculated and observed corrections are found to be in good agreement.

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