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THE INTERFERENCES OF THE DIRECT SPECTRA OF TWO GRATINGS.

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1. *Introduction. Method.*—The chief purpose of the present paper is the search for phenomena similar to those recently discussed,[2] but in which the two spectra brought to interference are not inverted relatively to each other. Incidentally the strong interferences may have a value on their own account. It has been shown that the totality of the phenomena with spectra reversed on a transverse or a longitudinal axis are quite complicated and a series of companion researches in which similar results are aimed at, in the absence of inversion, is thus very desirable.

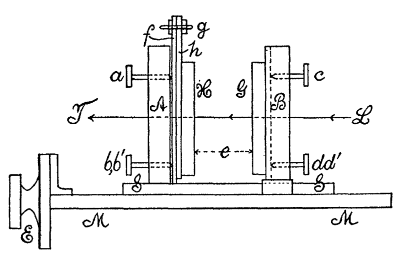

Fig. 1.

The apparatus (Fig. 1) is the same (nearly) as that shown in my last paper,[3] MM being the base of the Fraunhofer micrometer, SS the slide, E the micrometer screw. The brass capsules A and B are securely mounted on the slide S, free from the base M, and on the base M free from the slide S, respectively. Each capsule is provided with three adjustment screws relative to horizontal and vertical axes, a, b, b', and c, d, d', together with strong rearward acting springs, by which the gratings G and H at a distance e apart, may each be rotated slightly around a vertical or a horizontal axis (plane-dot-slot mechanism). The two gratings G and H must be identical, or very nearly so, as to the number of lines per inch and with their ruled faces toward each other.

[1] Abridged from a forthcoming Report to the Carnegie Institution of Washingotn, D. C.

[2] C. Barus, Am. Journ. Science, XL, pp. 486–498, 1915.

[3] C. Barus; PHYSICAL REV., VII, pp. 79–86, 1916.

These faces, as well as the ruled lines, are to be in parallel. To secure the latter adjustment a bolt, *g*, normal to the face of the grating *H*, serves as an axis and an available tangent screw and spring (not shown) is at hand for fine adjustment. This device is of great importance in bringing the longitudinal axes of the two spectra due to *G* and *H*, into coincidence and a fine wire must be drawn across the slit of the collimator to serve as a guiding line through the spectrum. Any lack of parallelism in slit and rulings rotates the fringes.

The beam of light, *L*, either white or homogeneous as the experiment may require, is furnished by a collimator (not shown), which with the telescope at *T* (in plan, in Fig. 1*a*, at *T* or *D*) are the usual parts of a spectroscope. The collimator with slit is always necessary for adjustment. It may then be removed if the phenomenon is to be studied in the absence of the slit. The telescope is frequently replaced to advantage by a lens.

The adjustments in case of white light are simple and the interferences usually very pronounced, large and striking. Brilliant spectra, chanelled with vertical narrow black lines, are easily obtained when the longitudinal axes are placed accurately in coincidence by rotating the plate *h* carrying the grating *H*, on the plate *f*, around the axis *g*. If the gratings are quite identical the sodium lines will also be in coincidence. Otherwise the two doublets D_1D_2 and $D_1'D_2'$ of the two spectra (nearly identical in all their parts and in the same direction) are placed in coincidence by rotating either grating around a vertical axis. Thereupon the strong fringes will usually appear for all distances, *e*, less than 2 cm. These fringes are nearly equidistant, vertical and they intersect the whole spectrum transversely. They are not complicated with other fringes, as in the experiments of my last paper (l. c.). They increase in size till a single shadow fills the field of view, in proportion as the distance *e* is made smaller and smaller to the limit of complete contact. During manipulations great care must be taken to keep the angle of incidence, *i*, rigorously constant; *i. e.*, to avoid rotating both gratings together, or the apparatus as a whole, as this displaces the sodium doublets relative to each other and seriously modifies the equations.

2. *White Light. Colored fringes.*—The two sodium doublets seen in the arc spectrum are usually equally brilliant and but one set of strong fringes is present in the field of the telescope.

If both gratings are rotated, changing the angle of incidence from 0^0 to i^0, the fringes disappear from the principal focal plane, but reappear strongly in another focal plane (ocular forward or rearward). In such a case the *D* lines are no longer superposed. To be specific let *i* and *i'*,

θ and θ', be the angles of incidence and diffraction at the two gratings in question, the angle between their ruled faces being $i - i'$. Let D and D' be the two grating constants, and nearly equal. Then for a given color, λ, in relation to the individual normals of the two gratings it may be shown that

$$\frac{i - i'}{D - D'} = \frac{\lambda}{D^2(1 - \cos\theta)}, \text{ nearly.} \tag{1}$$

In case of the Wallace grating below $D = 1.75 \times 10^{-4}$, $\lambda = 58.93 \times 10^{-6}$, $i - i' = 10^4 \times 3.29\ (D - D')$. Thus if the enclosed angle $i - i'$ between the plates is one degree, or .0175 radian, $D - D' = 5.3 \times 10^{-7}$, about 3/10 per cent. of D, and equivalent to about 43 lines to the inch. With adequate facilities for measuring i this method may be useful for comparing gratings, not too different, in terms of a normal or standard, practically, since the finite equations may also be expanded. In a similar way the slight adjustments of the longitudinal axes of the two spectra may be made by rotating one grating around a horizontal axis, but this correction is less easily specified. Finally one should bear in mind that with film gratings there is liable to be an angle $i - i'$ between the adjusted plates.

The range of displacement of grating within which the fringes may be used with an ordinary small telescope, extends from contact of the two gratings, to a distance of $e = 2$ to 3 cm. beyond.

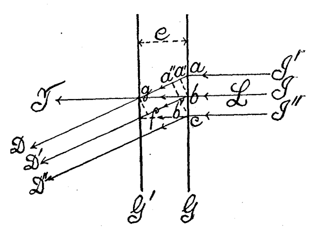

Fig. 1a.

In Fig. 1a, which is a plan of the essential planes of the apparatus, G, G' being the ruled faces of the gratings in parallel, I, I', I'', three impinging rays of white light diffracted into D, D', the points a, b, c, a', a'', b are in the same phase, so that the path difference of the rays from b at g and f is easily computed. If the single ray I is diffracted into D and D', or I and I' into D, I and I'' into D', etc., the micrometer value of a fringe for a color λ under normal incidence, is

$$\delta e = \frac{\lambda}{1 - \cos\theta}. \tag{2}$$

For two colors λ and λ'

$$n\lambda = e(1 - \cos\theta) = eM;\ (n + n')\lambda' = e(1 - \cos\theta') = eM'$$

if n is the number of fringes between λ and λ'. Thus

$$n' = e\,\frac{M'\lambda - M\lambda'}{\lambda\lambda'} \tag{3}$$

or the number of fringes increases as e is greater.

Equation (2) does not, as a rule, reproduce the phenomenon very well. Since the grating space D of the two gratings is rarely quite the same, the air plate enclosed, in case of apparent coincidence, of the sodium lines, is slightly wedge shaped, as in Fig. 2. Hence the two diffractions take place at incidences 0^0 and α^0, respectively, and the corresponding angles of diffraction will be θ and θ'. If we consider the two corresponding rays I and I'', diffracted at the first and second face, respectively, and coinciding at c in the latter, the points ab and a' (ba' normal to ac) are

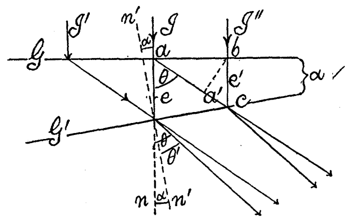

Fig. 2.

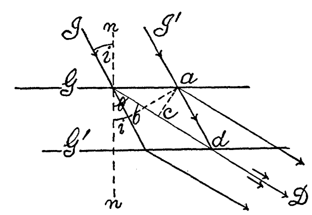

Fig. 2a.

in the same phase and we may compute the phase difference at the coincident points at c. The distance bc is

$$e' = e\frac{\cos\alpha\cos\theta}{\cos(\theta-\alpha)}$$

whence eventually

$$\delta e = \frac{\lambda\cos(\theta-\alpha)}{\cos\theta\cos\alpha\cdot(1-\cos\theta)}, \tag{4}$$

which changes into equation (3) when $\alpha = 0$ and $n = 1$. Fortunately this correction is, as a rule, small. In case of the Wallace gratings ($D = 1.75 \times 10^{-4}$ cm. for instance), if $\lambda = 58.93 \times 10^{-6}$, $\theta = 19^0\ 40^1$ or $\delta e = 1.01 \times 10^{-3}$; whereas if $\alpha = 5^0$, $\delta e = 1.04 \times 10^{-3}$; etc.

If the incidence is at an angle i and the plates are parallel, Fig. 2a, the inquiry leads in the same way to an equation of more serious import. If the gratings G and G' are at a distance e apart and the incident rays are I and I', the points a, b, c, are in the same phase. Hence the two rays leaving d and diffracted along D correspond to

$$\delta e = \frac{\lambda\cos i}{1-\cos(\theta-i)}. \tag{5}$$

Fig. 2c shows the variation of fringes with the angle of incidence i (equation (5)).

Hence if the angle of incidence is changed from -5^0 to $+5^0$, δe increases to nearly 3 times its first value. This therefore accounts for the large discrepancies of δe found in the successive data below. To secure increased sensitiveness and to make the apparatus less sensitive to slight changes of i, this angle should be about 25^0, in which case δe is about three wave-lengths per fringe. But normal incidence is frequently more convenient.

Finally in Fig. 2*b*, if the angle of incidence is i and the two faces G and G' make an angle α with each other and are initially at a distance

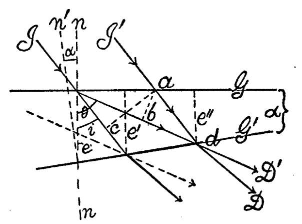

Fig. 2*b*.

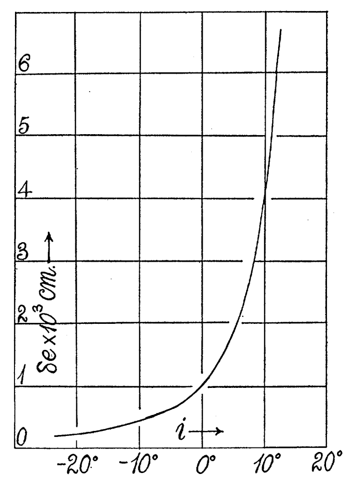

Fig. 2*c*.

e apart, changing successively to e' and e'', the points a, b, c, being in the same phase, the two rays D and D' leaving at d, at an angle $\theta - i$, will conform to

$$\delta e = \frac{\lambda \cos i \cos (\theta - \alpha)}{\cos \alpha \cos \theta \, (1 - \cos (\theta - i))}. \qquad (6)$$

This equation reproduces the preceding equation (5) if $\alpha = 0$ and the original equation (2) if $\alpha = i = 0$. It shows that a discrepancy or angle between the plates is of minor importance. Hence the change of this angle may be used to bring the sodium lines in coincidence when the gratings differ slightly in their grating constants D. On the other hand, changes of incidence i are of extreme importance.

Experiments made with the film gratings showed that equation (2) not only fits very badly, but that δe per fringe appears as a fluctuating quantity. Fringes were distinctly seen however within 3 cm. of displacement, by an ordinary telescope. I omit the data.

The reason for the lack of accord is given in equation (5) and (6) and Fig. 2*c*. To show that the irregularity of the above results is to be sought in the accidental variations of the angle of incidence i at both gratings, the rough experiments in Table I. suffice.

TABLE I.

Effect of Changing i.

Ives's grating,[1] i negative, ocular inward,[2] $i = \pm 0°$; i positive, ocular outward. $10^3 \delta e = .77, .78, .74; 1.18, 1.12; 1.69, 1.66$ cm.

Thus, as equation (6) implies, small variations of i produce relatively large variations of δe and if i passes continuously through zero, from negative to positive incidence, δe increases continually and may easily be more than doubled. If the phenomenon is in focal planes in front of the principal plane (ocular in), δe is small and vice versa. Moreover this enormous discrepancy is quite as marked for thin glass (2 millimeters) as for thick glass plates (8 millimeters). Again the rather stiff screw of the micrometer, which twisted the whole apparatus slightly, was sufficient to introduce irregularity.

An additional result was obtained by placing a plate of glass between the two gratings G and G'. The effect was an unexpected *enlargement* of fringes, increasing with the thickness of the glass plate (.6 cm. or more). The reason for this is given by equation (3) in paragraph 2, for the number of fringes n' between two colors λ and λ',

$$n' = \frac{e(M'\lambda - M\lambda')}{\lambda\lambda'},$$

where $M = 1 - \cos\theta$, $M' = 1 - \cos\theta'$. Since n' is a number, the glass plate can only be effective in changing θ and θ'. As both are diminished by refraction, the cosines are increased and $1 - \cos\theta$, $1 - \cos\theta'$ are both decreased. Hence n' is decreased or the number of fringes is decreased and their distance apart is thus larger.

It is obvious that when the sodium lines are not superposed, the fringes cannot lie at infinity, but are found in a special focal plane depending on the character of coincidence; *i. e.*, whether the rays are convergent or divergent. Finally a slight rotation of the slit around the axis of the collimator rotates the fringes in the opposite direction to the sodium lines and it is rather surprising, at first, that as much rotation of slit (10^0 or 20^0) is permissible, without fatally blurring the image. The slightest rotation of one grating relatively to the other destroys the fringes.

[1] About 15,000 lines to inch. Normal $\delta e \times 10^3 = .94$ cm.

[2] Focal plane changing.

3. *Homogeneous Light. Wide Slit. Transverse Axes Coincident.*— If there is no coloı difference, fringes of the same kind will nevertheless be seen in the telescope, on widening the slit indefinitely. Path difference is here due to differences of obliquity in the interfering rays. As in the preceding case, accurate adjustments of the longitudinal and transverse axes (in case of sodium, D_1 and D_1' or D_2 and D_2' coincide, horizontally and vertically) of the homogeneous color field are essential, if strong fringes are to appear in the principal focus. These fringes are as a rule well marked and widening the slit merely increases the width of the channelled, homogeneous field of view. If, owing to slight differences of grating space, the sodium lines are not quite superposed automatically, they may be corrected by rotating either grating, or else the apparatus as a whole, until the fringes are strongest. The fringes may be made to vanish under inverse conditions. The experiments (to be omitted here) showed their close relation to the preceding colored set, so far as motion of the micrometer is concerned. The fringes decrease in size as e increases and exhibit the same irregularity of δe values, due to the same causes (equation 6). Moreover δe was here usually found to be below the normally computed value, supposing the angle i to be negligible. In fact, Fig. 3 shows the optical center of the collimator C, so that Ca and Ca' are the axes of parallel pencils diffracted by the gratings G and G' at the angles θ for Ca and θ' for Ca'. The rays are subsequently condensed at F the focus of the telescope, L being the principal plane of the objective. The general path difference is thus for negative i, given by equation (5) which distributes the fringes from right to left with variation of i.

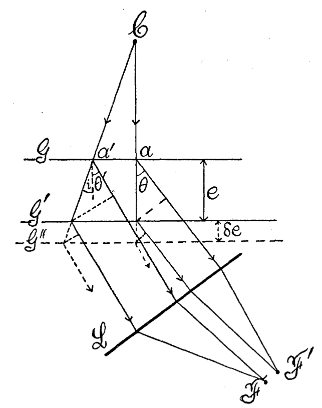

Fig. 3.

If the grating G' is displaced, δe, parallel to itself, the path difference will again be increased by λ whence

$$\delta e = \frac{\lambda \cos i}{1 - \cos(\theta' + i)}.$$

Since i is small, this equation will not differ appreciably from equation (2), with which it coincides for the central fringes.

If the sodium lines are not superposed, these fringes may still be seen; but they are not in the principal focal plane and the new focal plane changes continually as the fringes grow in size. Experiments similar to the above bearing on the reason for the discrepancies (equation (6)) were tried with the thin Wallace gratings. As before the effect of i passing from negative (through zero) to positive values, is enormous, δe increasing nearly threefold for a change of i estimated as within 20^0.

4. *Homogeneous Light. Fine Slit. Transverse Axes Not Coincident.*— To obtain this group of interferences, the two sodium lines from a very fine slit are thrown slightly out of coincidence; *i. e.*, by not more than the D_1D_2 distance. In the principal focal plane, therefore, these doublets are seen sharply; while if the ocular is drawn sufficiently forward or rearward, an interesting class of fringes soon appears which resemble Fresnel's fringes for two virtual slits. These fringes may be seen, however, on both sides of the focal plane and increase in size with the distance of the plane of observation (focus or ocular) in front or behind the principal focal plane. In Fig. 4 the two gratings G and G' are struck by parallel pencils from the collimator at different angles of incidence (0^0 and i^0). The two diffracted pencils of parallel rays are caught by the objective L of the telescope and condensed at the principal foci F and F', appearing as two bright yellow lines. In front and behind the plane FF', therefore, are two regions of interference I and I', throughout which the Fresnellian phenomenon may be seen in any plane parallel to FF', observed by the ocular. When the electric arc is used with a very fine slit, these sodium fringes often appear at the same time as the colored fringes, and though they are usually of different sizes, their lateral displacement with a change of distance apart of the gratings, δe, is the same. The fringes in question appear alone when the sodium burner is used. They may (at times) be observed with the naked eye, with or without a lens and they then fail to appear in the telescope, unless the objective is strengthened by an additional lens. They are always vertical, but finer in proportion as the D_1D_2 and $D_1'D_2'$ doublets are moved further apart. They become infinite in size, but still strong, when the doublets all but coincide, showing a tendency to become sinuous or possibly horizontal. Rotation of either grating G around an axis normal to itself and relative to the other, produces greatly enhanced rotation of the fringes, as in all the above cases, but they soon become blurred.

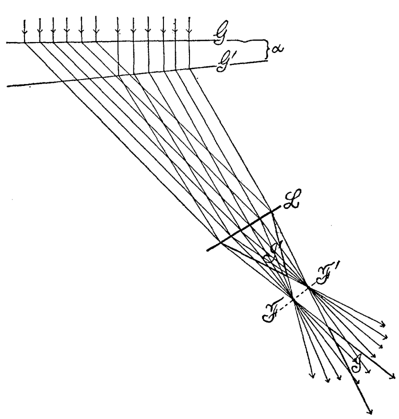

Fig. 4.

Only in the case when the horizontal axes of the field coincide (parallel rulings, etc.) do they appear strong. When the angle of incidence (or non-coincidence) is increased for both gratings, the size of the fringes increases; but when the e distance is increased by the micrometer, the fringes are apparently constant as to size. However, after displacement of 4 millimeters they are liable to become irregular and stringy, though still moving. A fine slit is not essential, particularly when e is small. They vanish gradually when the slit is too wide. If a telescope with a strong objective is used, these fringes may be seen, retaining their constant size long after those of the next paragraph vanish. Examples of data are superfluous here. With the Wallace gratings, these fringes were best produced by the aid of the sodium lines in the ordinary electric arc, simultaneously with the colored fringes and for the case of a very fine slit. They were apparent both with an ocular drawn out or drawn in. In the former case several successive groups were observed. Beginning with the sharp sodium lines in principal focus (D_2 and D_2' coincident), a slight displacement of the ocular outward showed the first group, this being a grid of very fine striations. Further displacement outward produced a second set, equally clear but larger. A third displacement of the ocular outward showed the third set and these now coincided with and moved at the same rate as the colored fringes in the same field. Other groups could not be found. No doubt for these four successive steps the interference grids of D_1 and D_1', D_2 and D_2' are coincident and superposed, until they finally find their place in the colored phenomenon.

5. *Homogeneous Light. Slit and Collimator Removed.*—Fringes similar to those seen with the wide slit above, may be observed to better advantage by removing the slit altogether. The sodium flame is then visible as a whole; and if the adjustments are perfected, it intersected with strong vertical black lines, visible to the naked eye or through a lens or a suitably strengthened telescope. They decrease rapidly with increase in e, but vanish to the eye before the preceding set (§ 4). The sodium lines need not be in adjustment, but the longitudinal axes of the field must be, as usual. If the collimator only partly fills the field of view, these diffuse light fringes and the preceding set may occur together. Both rotate markedly for slight rotation of either grating in its own plane. There seems to be a double periodicity in the yellow field, but it is too vague to be discerned. When magnified with a lens, they admit of a play of e within about .6 cm. from contact. If the sodium lines are not coincident, the focal plane continually changes with e. Otherwise it remains fixed. Again if either of the gratings is displaced and if they

are parallel, the focal plane will not change; but if G and G' are not parallel, the focal plane differs from the principal plane and now moves with the grating.

6. *Inferences.*—The above experiments showed that the equation underlying all the interferences observed is the same. These may result from different causes; but their variation in consequence of the motion of the grating, δe, is due to one and the same cause. This is best seen by producing them simultaneously in pairs. As a means of finding an accurate comparison of the number of lines per cm. on any grating, in comparison with those on the given grating, the method used in § 2 deserves consideration.

If the fringes are to be used for practical purposes, great care must be taken to keep the angle of incidence of the impinging light constant. This was not done in the present experiments, where the purpose is merely an identification of the phenomena. Moreover a micrometer with the screw running easily is essential, as otherwise the frame is liable to show appreciable twist (change of incidence) during displacement of the fringes.

The fringes are not of the sensitive type; but on the other hand they admit of a large range of displacement and are therefore adapted to special purposes.

With regard to their bearing on the behavior of reversed spectra, for the interpretation of which the present experiments were undertaken, it is obvious that the interferences with homogeneous light and a wide slit (§ 3), or in the absence of a slit (§ 5), are of analogous origin in both cases. It makes no difference, therefore, whether one of the spectra is reversed or not, except perhaps that in the former case (inversion), the coincidence of longitudinal and transverse axes is a more insistent condition. The colored fringes of § 2 obviously cannot be produced with reversed spectra. There remain the fringes with the fine slit and homogeneous light (§ 4). In other words the occurrence of a sort of generalized Fresnellian interference within the telescope, modified by causes which lie outside of it. Thus D_1 and D_1' or D_2 and D_2' may be placed sufficiently close together, to produce a region of interference before and behind the principal plane in which the sodium lines are in focus. If the D_1D_1' lines are x = .01 cm. apart and the fringes seen likewise at e = .01 cm. apart, their position measured from the principal plane will be at

$$R = \frac{xc}{\lambda} = \frac{5}{3}\,\text{cm.},$$

or less than 2 cm. The ocular would then have to be displaced forward or rearward by this amount. But there are two sodium doublets, each pair of which is to interfere. Suppose that D_2 and D_1' are in coincidence so that the scheme is $D_1 : D_2D_1' : D_2'$ as in Fig. 5, where o is the principal plane of the objective and D_1 to D_2' the principal focal plane. We should then have the separate regions of interference I and I' and the combined regions I'' and I'''. When the breadth of the latter is the whole number of fringes, the two patterns clearly merge into a single pattern. The experiments show several of these stages, terminating outermost in the focal plane of the colored fringes under the given conditions. Since

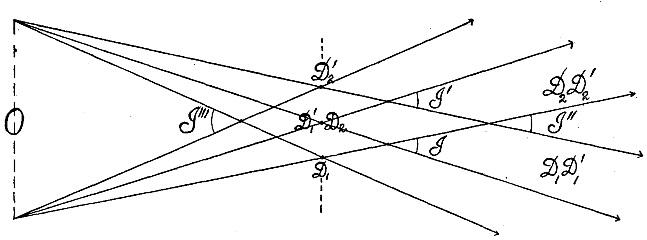

Fig. 5.

the fringes lie on hyperbolic loci, the problem itself is beyond the present purposes; but it appears that the colored fringes will not occur until the correspondence of D_1D_2 lines is shared by the whole of the two continuous spectra.

The final question at issue is the bearing of the present Fresnel phenomenon on the reversed spectra. If in Fig. 6 S and S' represent the traces of two reversed spectra in the principal focal plane, superposed

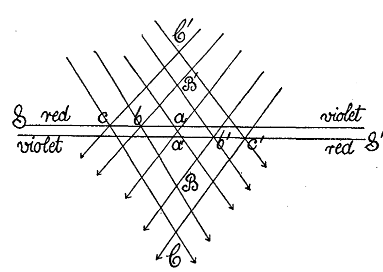

Fig. 6.

throughout their extent (*i. e.*, in longitudinal coincidence), the rays a, a', through the line of symmetry, are at once in a condition to interfere with a given difference of phase; but so are all the symmetrically placed pairs of colors c, c', b, b' of the two spectra (the distances c, c', b, b', being arbitrary), provided the corresponding rays meet. As they do not meet in the principal focal plane, they can only interfere outside of it, b and b' at B, c and c' at C, etc. Similar conditions must hold at B' and C', inside the principal focal plane. The linear interference is thus successively transferred to different pairs of wave-lengths. The phenomena of this paper can not therefore be detected in case of reversed spectra, because in the principal focus different wave-lengths or colors are everywhere superposed, except in the narrow strip of spectrum at aa', which experiment shows to be about 1/3 the width of the sodium doublet in apparent size. The latter, however, is a question of diffraction. Beyond the principal focus, the corresponding conditions, in turn, hold for the rays at B, C, etc., B', C', etc. Hence they can not be any Fresnellian interferences (§ 4), for there are not two but only one virtual slit, as it were, and interferences are laid off in depths, along the normal $C'C$. The phenomenon may, in fact, be detected along this line for 2 or 3 meters.

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