

CATHODE MEASUREMENTS WITH GLOW CURRENT IN HYDROGEN.

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PROFESSOR SKINNER¹ has recently developed a theory of the cathode fall in gases which suggests that with a plane cathode the "normal" current density should be proportional to the square of the gas pressure provided the number of mean free paths of the electrons from the cathode to the point of minimum gradient in the negative glow is the same for all pressures. In measurements made with aluminum and steel cathodes in hydrogen both of these conditions were found to exist.

He has also found that, provided the current density used is the same multiple of the "normal" current density, potential measurements in the region between the cathode and the negative glow, taken at various gas pressures and plotted against the number of mean free paths from the cathode, all lie on the same curve. For example, the potential difference obtained at "normal" current density at various gas pressures gave one curve, at twice "normal" current density a second curve, and so on.

The purpose of this research was to repeat some of Skinner's measurements with aluminum and steel and to extend the investigation to various other metals.

I. APPARATUS.

This consisted of a discharge tube (Fig. 1) about three centimeters in diameter, having a movable disc cathode *K*, carried by the micrometer screw *B*. The graduated drum *M*, on the ground joint *J*, served to give the axial position of the cathode. The current was confined to the front face of the cathode by the glass hood *H*. A probe wire *P* (Al, .24 mm. in diam.) sheathed to very near its end with a fine glass tube *S*, served to give the poten-

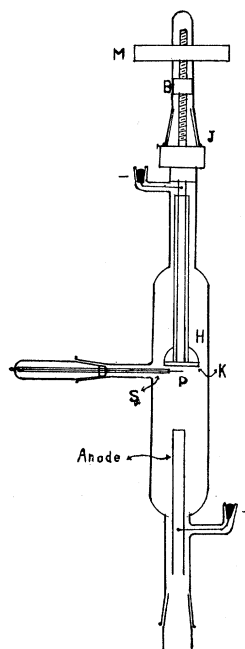


Fig. 1.

¹ C. A. Skinner, *PHYS. REV.*, June and August, 1915.

tial in the gas. This sheath was renewed from time to time to prevent its becoming conductive as a result of metal sprayed off from the cathode.

Potential measurements were made by a quadrant electrometer in which a definite P.D. (6 volts) was placed between the quadrants, the potential sought being placed between one pair of quadrants and the needle. The electrometer was, from time to time, compared on a battery circuit with a Weston voltmeter. Customary precautions were taken against the possible influence of static charges and the more important leakage effects avoided by using sealing wax insulators throughout. This leakage was tested by placing a charge on the electrometer, disconnecting, and noting the rate at which the deflection changed. The change was not appreciable over a considerable length of time. In order to work rapidly—quite necessary in view of the deterioration of the gas—the deflections of the electrometer (read by telescope and scale) were taken only in one direction.

The current was furnished by a battery of small storage cells, giving in all about 1,100 volts; measured by means of a mil-ammeter or, for smaller currents, a d'Arsonval galvanometer; and regulated by means of a liquid rheostat consisting of a solution of cadmium iodide in amyl alcohol.

A description of the modified Dolazalek electrometer used may not be out of place. The regular needle of the electrometer was replaced by one of aluminum and the ordinary suspension by a ribbon of phosphor-bronze. A damping arrangement consisting of a fork of platinum, dipping into concentrated H_2SO_4 , was used.

Another factor demanding care was the cathode surface. To insure uniform results, each day before beginning a series of observations (and often during the series) the cathode was freshly polished by scouring with Tripoli composite, by rubbing on a pad of felt, and finally by wiping carefully on a new piece of cheese cloth.

The hydrogen was obtained by the action of KOH on aluminum. Great care was exercised to rid the gas of air by initial pumping and flushing with hydrogen; also to thoroughly dry it by passing it, first through a tube containing P_2O_5 , and then allowing it to stand for about a half hour in a chamber containing the same material. The hydrogen was then stored in a bulb (freed from occluded gases) connected with the apparatus, from which it could be drawn off as desired.

II. EXPERIMENTAL RESULTS.

1. *Current Density and Number of Mean Free Paths to the Negative Glow with 'Normal' Cathode Fall.*—Cathodes of aluminum, steel, nickel,

zinc, and platinum were used. The gas pressures (measured by a McLeod gauge) were necessarily confined to the range of about one to four millimeters. With pressures below one millimeter the negative glow became crowded so closely to the cathode as to make distance measurements too uncertain. With each pressure the "normal" current density was obtained by increasing the current until the glow just covered the cathode and ceased to curl away from it, or (measuring with the electrometer) when the cathode fall began to rise. The former method was used in most cases, though at the very low pressures the latter was often resorted to. The point of minimum gradient was found, by moving the cathode toward the probe wire, and observing where the P.D. began to drop appreciably below the normal value given by the probe in the main part of the negative glow.

Before each set of observations, the cathode was cleaned by passing a heavy current through the tube; the gas then pumped out; and fresh gas introduced. With each gas filling the "normal" current, the "normal" cathode fall, and the distances to the point of minimum gradient were measured.

Representative results are given in Tables I.-V. The first column gives the pressures—each representing a fresh filling of gas. Column 2 gives the "normal" fall in volts; Column 3, the current density in milliamperes. Column 5 gives the mean free path (in mm.) of the electrons

TABLE I.

Aluminum Cathode in Hydrogen—Area = 3.0 sq. cm.

Gas Pressure (mm.).	Normal Fall (Volts.)	j_n (m.a.)	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.)	No. of m. f. p. to Neg. Glow.	$j_n \bar{\lambda}^2$
1.00	206	.067	10.80	.720	15.0	.0358
1.69	194	.193	6.60	.426	15.4	.0353
2.00	196	.287	5.33	.360	14.8	.0374
2.08	199	.307	5.36	.348	15.4	.0369
2.30	197	.400	4.82	.313	15.4	.0384
2.70	199	.550	3.99	.266	15.0	.0389
2.80	194	.607	3.98	.257	15.5	.0405
2.85	197	.613	3.98	.253	15.6	.0395
2.96	197	.683	3.72	.243	15.3	.0400
3.00	196	.653	3.75	.240	15.0	.0379
3.05	196	.713	3.66	.238	15.5	.0400
3.72	194	1.067	2.91	.194	15.0	.0400
3.90	196	1.167	2.78	.185	15.0	.0400
4.00	194	1.267	2.78	.180	15.4	.0410
Mean	197				15.3	.0387
Max. dev.	4.5%				3%	9%
Mean dev.	1%				1.5%	4%

TABLE II.

Steel Cathode in Hydrogen Area = 3.14 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts.).	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. m.f.p. to Neg. Glow.	$j_n \bar{\lambda}^2$
1.94	315	.191	7.70	.371	20.7	.0267
2.20	299	.270	6.70	.327	20.5	.0291
2.30	311	.245	6.42	.313	20.5	.0241
2.45	299	.314	5.88	.294	20.0	.0272
2.50	294	.350	5.76	.288	20.0	.0291
2.53	299	.350	5.70	.285	20.0	.0285
2.64	304	.334	5.70	.273	20.9	.0250
3.07	293	.540	4.70	.234	20.0	.0294
3.40	303	.620	4.24	.212	20.0	.0281
3.55	294	.700	4.15	.203	20.4	.0291
3.90	299	.780	3.85	.184	20.9	.0265
Mean	301				20.3	.0275
Max. dev.	4.5%				3%	12%
Mean dev.	2%				1.5%	5.5%

in hydrogen, at the corresponding gas pressure, as calculated from that of the molecule by using the ratio given by Franck and Hertz.¹ Column 6 gives the number of mean free paths to the minimum gradient, obtained by dividing the magnitudes in 4 by the corresponding ones in 5. Column 7, giving the product $j_n \bar{\lambda}^2$, is simply another way of expressing the relation

TABLE III.

Nickel Cathode in Hydrogen Area = 3.14 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts.).	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. of m.f.p. to Neg. Glow.	$j_n \bar{\lambda}^2$
1.72	325	.159	8.47	.419	20.2	.0281
2.50	328	.382	5.85	.288	20.3	.0317
2.81	319	.430	5.37	.256	21.0	.0283
2.90	322	.493	4.97	.248	20.0	.0307
3.06	323	.535	4.85	.235	20.6	.0296
3.54	316	.669	4.06	.203	20.0	.0278
3.60	325	.700	4.00	.200	20.0	.0281
3.73	320	.795	3.86	.193	20.0	.0296
3.80	324	.764	3.80	.190	20.0	.0275
4.03	320	.891	3.70	.178	20.8	.0286
Mean	322				20.3	.0290
Max. dev.	2%				3.5%	9%
Mean dev.	1%				1.5%	4%

¹ J. Franck and G. Hertz (Deutsch. Phys. Gesell., No. 9, p. 373, 1913) found the mean free path of the electron to be $4\sqrt{2}$ times the mean free path of the molecule. For 1 mm. pressure then the m.f.p. of the electron in hydrogen is about .72 mm.

TABLE IV.

Platinum Cathode in Hydrogen Area = 1.76 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts.).	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. of m. f. p. to Neg. Glow.	$j_n \bar{\lambda}^2$
1.61	310	.114	8.50	.446	19.0	.0231
1.95	306	.176	7.01	.369	19.0	.0239
2.21	302	.233	6.30	.326	19.3	.0249
2.35	300	.256	5.82	.306	19.0	.0240
3.05	303	.466	4.70	.236	19.9	.0260
3.10	303	.482	4.61	.232	19.9	.0260
3.35	300	.545	4.30	.215	20.0	.0252
3.60	299	.636	4.10	.200	20.5	.0254
3.86	297	.739	3.70	.186	19.9	.0257
4.00	304	.767	3.20	.180	20.0	.0249
4.34	302	.938	3.30	.166	19.9	.0260
Mean	302.5				19.7	.0250
Max. dev.	2.5%				3.5%	7%
Mean dev.	1%				2%	3%

j_n/p^2 , since $\bar{\lambda}$ varies inversely as the pressure, p . Columns 2, 6, and 7 show for all metals merely irregular fluctuations about a certain mean value, thereby proving the constancy of the relations already stated.

Comparing the different metals, aluminum gives a much higher current density than the others, the lowest cathode fall, and the least number of

TABLE V.

Zinc Cathode in Hydrogen Area = 3.14 sq. cm.

Gas Pressure (mm. Hg.).	Normal Fall (Volts.).	j_n (m.a.).	Dist. to Neg. Glow (mm.).	$\bar{\lambda}$ (mm.).	No. of m.f.p. to Neg. Glow.	$j_n \bar{\lambda}^2$
1.40	284	.108	9.25	.515	18.0	.0285
1.50	278	.130	9.15	.480	19.1	.0301
1.85	273	.203	7.35	.389	18.9	.0309
2.33	280	.302	5.85	.309	18.9	.0290
2.45	277	.346	5.30	.294	18.0	.0301
2.45	276	.344	5.30	.294	18.0	.0290
2.80	269	.460	4.85	.257	18.9	.0306
2.97	275	.509	4.36	.242	18.0	.0301
3.02	270	.557	4.40	.238	18.5	.0316
3.25	272	.620	3.99	.222	18.0	.0306
3.33	272	.685	4.00	.217	18.4	.0322
3.48	265	.764	3.81	.207	18.4	.0327
4.00	267	.923	3.24	.180	18.0	.0301
4.18	271	1.08	3.22	.172	18.7	.0322
Mean	273.5				18.4	.0306
Max. dev.	4%				4%	7%
Mean dev.	1.5%				2%	3%

mean free paths to the negative glow. The mean values of the cathode fall given here for aluminum and steel agree very closely with the values given by both Warburg¹ and Skinner.² The other results from these metals are also in excellent agreement with Skinner's as shown by the following table.

	Cheney.		Skinner.	
	Al.	Steel.	Al.	Steel.
Cathode fall (volts)	197	301	197	303.5
No. of m.f.p. from Cathode to minimum gradient	15.3	20.3	15.3	20.5
$j_n \lambda^2$	.0387	.0275	.039	.0273

These comparisons, along with that of individual values given in the tables I–V., indicate that one is but slightly less certain of reproducing both $j_n \lambda^2$ and the distance of the minimum gradient from the cathode than one is of reproducing the cathode fall itself.

Zinc ranges next to aluminum in the series of metals, having the next lowest cathode fall, the next highest current density, and the next smallest number of m.f.p.'s to the negative glow. Both zinc and nickel were rather difficult to work with, owing to tarnish. This necessitated frequent polishing of the cathode and many observations. The values given for these (tables III. and V.) are only those results which were obtained immediately after polishing and before any tarnish became apparent.

It is interesting to note that in the case of nickel, both the current density and the cathode fall are higher than in the case of steel, while the number of mean free paths from the cathode to the minimum gradient are the same.

With platinum much trouble arose from the distintegration of the cathode. In a relatively short time the metal would spray off so badly that it necessitated renewing the sheath and cleaning both the probe wire and the walls of the tube. With this metal, the cathode fall (302.5 volts) runs slightly higher than the value given by Warburg¹ (300 volts) and Capstick² (298 volts). The "normal" current density runs a trifle lower than for steel or nickel.

2. *Potential Curves in the Gas with 'Normal' Current Density.*—Before each series of observations the cathode was cleaned as usual by a heavy

¹ E. Warburg, Wied. Annalen, 31, p. 542.

² C. A. Skinner, PHYS. REV., *l. c.*

³ E. Warburg, Wied. Annalen, 31, p. 542.

⁴ Capstick, Roy. Soc. Proc., 63, p. 356, 1898.

current, the tube evacuated, and fresh gas introduced. Starting at the negative glow, the P.D. between the cathode and points at various mean free paths from it (measured to the center of the probe) were obtained as rapidly as possible—the current being maintained at its normal value during the series of observations. Check values were always taken after each series to indicate if disturbing changes had arisen during the operation. Only those values are given where the change in the cathode fall during a series was less than 2.5 per cent.

The observations are incorporated in Tables VI. and VII. Table VI. gives the results in detail obtained from platinum. It shows that the magnitudes are practically the same at all gas pressures. In Table VII. are given therefore only the mean values obtained from Al 1, Al 2, Ni, steel, and Pt. These mean values were obtained in the same way as the values in Table VI.—Al 1 from 8 different gas pressures ranging from 1.7 to 5.0 mm.—Al 2 from 4 different pressures, from 1.1 up to 2.0 mm. only—steel from 11 different pressures, from 2.2 to 3.9 mm.—Ni from 8 different pressures from 1.7 to 4.0 mm.—and also Pt from those of Table VI.

TABLE VI.

Potential Curve—Platinum—"Normal" Current Density ($j_n \lambda^2 = .0387$).

No. of m. f. p.	Gas Pressure in Mm.										Mean P. D.
	1.41	1.75	1.95	2.25	2.65	3.05	3.10	3.35	3.60	3.86	
3	175	178	175	170	173	174	176	—	—	173	174.5
4	—	190	190	182	185	189	191	183	183	187	187
5	200	202	204	193	196	200	203	198	197	201	199.5
6	—	—	215	204	207	214	215	210	208	213	211
7	222	224	228	215	218	227	228	222	219	224	223
9	245	244	248	237	238	246	249	241	237	244	243
13	274	278	281	265	274	276	269	270	265	269	272
17	293	293	301	290	286	294	295	289	290	288	292
20	302	302	306	299	294	303	303	300	299	297	301

Al 1 was the same cathode used in previous measurements. Comparing these results with those obtained by Skinner, the P.D. between the cathode and two mean free paths from it differed by about 7 per cent. To ascertain if this were due to a difference in the samples of the metal his cathode (Al 2) was also tried. The results are practically the same as given by Al 1 at the same gas pressure. The discrepancy in our results is somewhat larger than the conditions would ordinarily allow.

Skinner calls attention to the divergence of the potential curves at low gas pressures from the constant relation illustrated in Table VI. the difference shown here between Al 1 and Al 2 arises from this fact—the

observations for Al 1 being simply those obtained below 2 mm., while for Al 2 they range from 1.7 up to 5 mm.

Observations were also made with zinc but the results were very inconsistent, indicating conductivity of the probe sheath.

TABLE VII.

Potential Curves "Normal" Current Density.

Number m.f.p. from Cathode.	Mean P.D.'s				
	Al 1.	Al 2.	Steel.	Nickel.	Platinum.
2	107	100	171	—	—
3	119.5	124	187	213	174.5
4	131	137	198	225	187
5	142	152	208	237.5	199.5
6	154	164	218	248	211
7	163	172	227	257	223
9	179	186	243	274	243
11	190	194	—	—	—
12	—	—	267	291	—
13	193	—	—	—	272
15	196	197	—	—	—
16	—	—	285.5	309	—
17	—	—	—	—	292
20	—	—	—	320	301
20.5	—	—	297	—	—

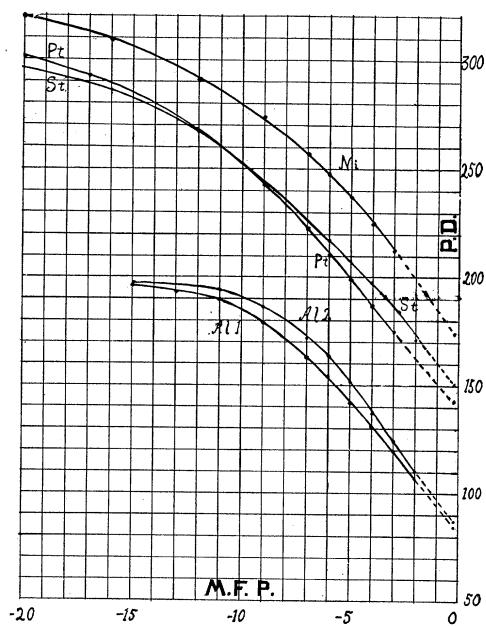


Fig. 2.

The results of the observations in Table VII. are plotted as curves in Fig. 2 with mean free paths as abscissæ and P.D.'s as ordinates. The broken line parts of these curves (practically straight) are their theoretical extension up to one m.f.p. of the positive ion from the cathode—over which the fall of potential between the cathode and the gas is conceived to occur. The method of obtaining these theoretical values follows later. A comparison of these values with the total cathode fall in the gas is given in the following table (VIII.):

TABLE VIII.

Cathode.	Total Cathode Fall (Volts).	Fall in Gas.	Fall at Cathode.
Al 1.....	196	111.5	84.5
Al 2.....	197	110	87
Steel.....	297	146	151
Nickel.....	320	145.5	174.5
Platinum.....	301	158	143

It is interesting to note that with Al and Pt the larger part of the fall is in the gas while with the other metals the larger part is between the cathode and gas.

III. SOME APPLICATIONS OF THE RESULTS.

1. *Mobility of the Positive Ions.*—This is defined as their velocity in cm. per sec. in unit field. From the potential measurements given in Table VII., the upper and lower limits of the mobility of the positive ions can be calculated from expressions given by Skinner (*l. c.*). These are

$$\begin{aligned} V_0 - V_1 &< \left\{ \left(\frac{16\pi j}{9\mu^+} \right)^{\frac{1}{3}} (x_1 - x_0) \right\}^{\frac{3}{2}} \\ &> \left\{ \left(\frac{16\pi j}{9\mu^+} \right)^{\frac{1}{3}} (x_1 - x_0) - \epsilon^{\frac{1}{3}} \right\}^{\frac{3}{2}} + \epsilon \end{aligned} \quad (1)$$

in which $V_0 - V_1$ is the difference of potential (in electrostatic units) between some point x_1 near the cathode and x_0 the point of minimum gradient; j is the current density in electrostatic units; μ^+ is the mobility of the positive ions also in electrostatic units; and ϵ is the ionizing potential of the electron¹ (11 volts or .037 electrostatic unit).

From these expressions it follows that

$$\begin{aligned} \mu^+ &< \frac{16\pi}{9} j \frac{(x_1 - x_0)^3}{(V_0 - V_1)^2} \\ &> \frac{16\pi}{9} \frac{j(x_1 - x_0)^3}{\{(V_0 - V_1 - \epsilon)^{\frac{3}{2}} + \epsilon^{\frac{3}{2}}\}^2} \end{aligned} \quad (2)$$

¹ J. Franck and G. Hertz, Deutsch. Phys. Gesell. Verh., 15, 2, p. 34, 1913.

To get the mobility in a field of one volt per cm. the values of μ^+ given by (2) must be divided by 300.

Substituting from Table VII.— x_1 being taken as the point where measurements were made nearest the cathode and $(x_1 - x_0)$ obtained by multiplying the distance in m.f.p. by .072 cm. (the m.f.p. of the electron at 1 mm. pressure)—the limits of μ^+ for a gas pressure of 1 mm. were calculated. These are given in Table IX. The values found here for the two samples of aluminum (2.3×10^4 and 3.7×10^4 cm./volt) agree closely with Skinner's values (2.2×10^4 and 3.6×10^4). There is also good agreement for Al, steel and Ni, while Pt gives a distinctly lower mobility—probably the effect of the presence of the metallic vapor arising from the disintegration of the cathode, the vapor hindering the progress of the positive ions toward the cathode.

TABLE IX.

Mobility of Positive Ions at One mm. Pressure.

Metal.	Lower Limit of μ .		Upper Limit of μ .	
	Electrostatic.	cm./volt.	Electrostatic.	cm./volt.
Al 1.....	$7.0 \times 10^6 \frac{\text{cm.}}{\text{sec.}}$	$2.3 \times 10^4 \frac{\text{cm.}}{\text{sec.}}$	$11.0 \times 10^6 \frac{\text{cm.}}{\text{sec.}}$	$3.7 \times 10^4 \frac{\text{cm.}}{\text{sec.}}$
Al 2.....	7.0×10^6	2.3×10^4	11.0×10^6	3.7×10^4
Steel.....	8.0×10^6	2.7×10^4	11.8×10^6	3.9×10^4
Nickel.....	7.0×10^6	2.3×10^4	10.5×10^6	3.5×10^4
Platinum.....	5.6×10^6	1.9×10^4	8.4×10^6	2.8×10^4

2. *Fall at the Cathode.*—By using the above values of μ^+ in their respective expressions in (1) the potential curves may be extended to the cathode, or better to one mean free path of the positive ion from the cathode—this being considered as the region within which the positive ions causing the immediate fall between the gas and the cathode accumulate.

These calculations give the following values for the fall between the cathode and the adjacent gas. It appears surprising that the upper and lower limits of μ^+ give so nearly the same values.

TABLE X.

Metal.	Fall at One m. f. p. of Positive Ion from the Cathode.		
	Upper Limit.	Lower Limit.	Mean.
Al 1.....	85	84	84.5
Al 2.....	88	86	87
Pt.....	143	143	143
Steel.....	152	150	151
Nickel.....	177	172	174.5

3. *The Magnitude of the Electron Current from the Cathode.*—Assuming that each electron leaving the cathode results in two electrons at one m.f.p., four at two m.f.p., and so on; and, that in the region of lower gradient the number of electrons doubles in moving through the ionizing potential, Skinner calculates that the ratio of the electron current density ($\bar{j}_k$) from the cathode to the total current (j) is

$$\frac{\bar{j}_k}{j} = \frac{\bar{\mu}}{\bar{\mu} + \mu^+} \cdot 2^{-\left\{ \frac{x_k - x_a}{\lambda} + \frac{V_0 - V_a}{\epsilon} \right\}} \quad (3)$$

where $[(x_k - x_a)/\lambda]$ is the number of mean free paths between the cathode and the point where ionization ceases to take place at every mean free path, $V_0 - V_a$ the fall of potential between this point and the negative glow, ϵ the ionizing potential, and $\bar{\mu}$ and μ^+ the mobilities in the low gradient region of the negative glow. Assuming that $\bar{\mu}$ and μ^+ are practically equal the following values for $\bar{j}_k/j$ were obtained.

TABLE XIV.

Metal.	$\frac{x_k - x_a}{\lambda}$	$\frac{V_0 - V_a}{\epsilon}$	$\frac{\bar{j}_k}{j}$
Al 1.....	6	3.8	57×10^{-5}
Al 2.....	6	3.0	98×10^{-5}
Ni.....	6	6.5	8.4×10^{-5}
Steel.....	6	7.2	5.3×10^{-5}
Pt.....	7	7.1	2.8×10^{-5}

Al 2 gives nearly twice the electron current of Al 1 (which is somewhat larger than the value obtained by Skinner). The effect of this is shown in the potential curve (Fig. 2)—Al 2 reaching a constant value at a somewhat shorter distance from the cathode. The difference does not arise from a difference in the samples of the metal but merely shows the effect of lowering the gas pressure, as already noted. The ratio $\bar{j}_k/j$, although very small for all metals, is about 7 times as large with Al 1 as with Ni, 10 times as large as with steel, and about 19 times as large as with Pt.

It is also interesting to note that with all metals, the point at which ionization at every collision takes place extends to about 6 m.f.p. from the cathode.

SUMMARY.

1. The "normal" current density at the cathode has been measured in hydrogen with Al, steel, Ni, Zn, and Pt as cathodes, and found in all cases to be proportional to the square of the gas pressure.

2. The number of mean free paths of the electron from the cathode to the point of minimum gradient in the negative glow has been measured for these metals under the same conditions and found to be independent of the gas pressure.

3. Potential curves in the gas between the cathode and the negative glow have been obtained at "normal" current density with these metals as cathodes. The potentials plotted against the number of mean free paths of the electron from the cathode lie, for all pressures, practically on the same curve.

4. From these potential measurements, the mobility of the positive ions has been calculated and consistent values obtained for all metals except platinum where the mobility is somewhat low, probably arising from the presence of metal vapor.

5. The potential curves have been theoretically extended to the cathode, and the fall between the cathode and the gas obtained therefrom.

6. The ratio of the electron current from the cathode to the total current has been calculated for the various metals.

I wish to express my sincere thanks to Professor C. A. Skinner for suggesting this problem and directing its course, also to Dr. A. Q. Tool for many helpful suggestions.

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