Analysis of Cosmic-Ray Fine Structure

Part I. Earlier Missouri Observations

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The present paper gives the results of an analytical study of the directional intensity anomalies measured by Ribner and Cooper in Missouri in 1939–1940. The method used follows directly from the log *vs.* log graph found useful by previous workers. The straight line of Fig. 1 is then represented by a least-squares equation of the form, $j = A \sec^b \zeta$. The anomalous points are shown to fall below the line and to be representable by a second line parallel to the first. The validity of the results is verified by direct comparison with the probable errors of measurement, the best test being Deming's calculation of chi square. The use of this method brings to light the following results. The data of the two observers show an excellent agreement. The least-squares constants of the calculated equations show a real but rather irregular azimuthal variation. Special consideration of the reported time fluctuations of these fine-structure anomalies shows that in practically no case are they distinguishable from statistical fluctuations in counting. The anomaly close to the zenith is shown to be caused by magnetic effects, and not atmospheric absorption. In the case of the other two anomalies, the importance of the secondary formation process is stressed. It must lead to an energy ratio S between the primary and mean secondary energies, the latter being concerned in the absorption. Comparison with the atom-annihilation results for the soft radiation suggests that the absorption anomalies, with $S=2.2$, may show agreement, but the magnetic anomaly, with $S \gg 1$, cannot agree. There is a slight suggestion here of two kinds of primaries, both involved in mesotron production.

SEVERAL observations have been made in recent years of the directional variation of cosmic-ray intensities at stations in Missouri and Mexico City.[1–5] They afford an excellent set of data for use in an analytical representation if such can be found. The key to the form of representation is indeed available in earlier, less detailed measurements of the directional intensity variation. The present paper, Part I, presents the treatment of the data of Cooper[1] and Ribner[2] in Missouri. Part II, on Schremp and Banos's Mexico City measurements,[3] is nearly ready.

The analytical form to be used to represent the fine structure may be inferred as follows. At sea level, the intensity-zenith angle curve has long been represented by the square of the cosine of the zenith angle.[6] Even for the fine structure,[1,2] this represents a helpful first approximation, from which anomalous departures can be calculated. Though this is not even approximately true except at sea level, it does suggest the use of a double logarithmic plot, log intensity *vs.* log cos ζ (ζ is the zenith angle), to represent the findings. A similar indication is found in Johnson's plot[7] of intensity as a function of atmospheric depth. One of the best examples of all is in the work of Wilson,[8] whose logarithmic plot (Fig. 5) is much more readily interpreted than the semilogarithmic Fig. 4 of his paper. Still another excellent example of the use of the logarithmic graph is Greisen's Fig. 2,[8a] in which it is used to interpret results from a considerable range of altitude and zenith angle.

The first step in the present method therefore consists of the plotting of log j as a function of log cos ζ. The result is found to be represented by a straight line except for the fine structure, which naturally represents departures from the smooth

[1] D. M. Cooper, Phys. Rev. **55**, 1272L (1939); **58**, 288 (1940).
[2] H. S. Ribner, Phys. Rev. **55**, 1271L (1939); **56**, 1069 (1939).
[3] E. J. Schremp and A. Banos, Phys. Rev. **58**, 662L (1940); **59**, 614L (1941).
[4] M. L. Yeater, Ph.D. Dissertation, Washington University.
[5] M. L. Yeater, Phys. Rev. **67**, 74 (1945).
[6] T. H. Johnson, Rev. Mod. Phys. **10**, 193 (1938).
[7] Reference 6, Figs. 22, 23. Although the graph represents total intensity, a simple application of the Gross transformation (*ibid.* p. 197) shows that the same power law applies to the vertical intensity.
[8] V. C. Wilson, Phys. Rev. **53**, 337 (1938).
[8a] K. Greisen, Phys. Rev. **61**, 212 (1942).

line. Therefore the second step consists of the determination of the equation of the straight line, together with some means of allowing for the fine structure. In the Missouri data, the fine structure is found to be representable as a lower, parallel straight line, whose equation is also determined. In this way, a semi-analytical representation of the fine structure is obtained.

Considerable care was also taken in testing the fit of the calculated curve. The best fit was shown by Gauss[9] to correspond to the smallest value of Ω, the mean square departure from the assumed "true" curve. In this case, where the accuracy of the individual measurements is known, the use of Ω alone does not permit a close comparison with the expected departures, which are all of different magnitudes. This must be done in a different way, comparing the individual residuals with the corresponding probable errors or standard errors. This is expected to give a normal distribution of V_i/σ_i with a standard deviation $(n-m)^{\frac{1}{2}}/n^{\frac{1}{2}}$, where $n-m$ is the number of degrees of freedom and n is the number of observation points. Though this comparison cannot be made with a high degree of certainty, it does show that the sum of the squares of these relative residuals will have the same distribution as X^2, a fact pointed out by Deming.[10] This was found to be the most sensitive test applicable in the present case, probably because of the large number of degrees of freedom represented. The probability corresponding to the calculated value of X^2 (see appendix) is taken from a table,[9,11] and the result interpreted as the probability that the calculated curve agrees with the measurements within the experimentally determined uncertainties.

APPLICATION TO COOPER'S DATA

The data obtained by Cooper[1] in Columbia, Missouri are particularly complete and suitable for application of the method outlined above. The cycles used were such as to make the observations at each azimuth a complete, reliable unit, and the uncertainties in each measurement are given. The logarithmic graphs of his results in eight chief azimuths are given in the lower part of Fig. 1. The general linearity of the curves, and the suitability of the logarithmic representation are apparent.

In the attempt to fit the different azimuths and fine-structure irregularities with an equation, 11 different least-squares approximations were tried. The important results of these different treatments are summarized in Table I. Where the agreement is poor, the values of Ω are sufficient to determine the adequacy of the treatment; in the closer cases the more complete comparison with the errors of the measurements is necessary. Approximations C, D, I, and K do not include all the points, but were found useful in interpreting the data.

Of these, approximations C and D were used in studying the fine-structure irregularities. Careful scrutiny of the points plotted in Fig. 1 gives the impression that the straight, uniform line passes

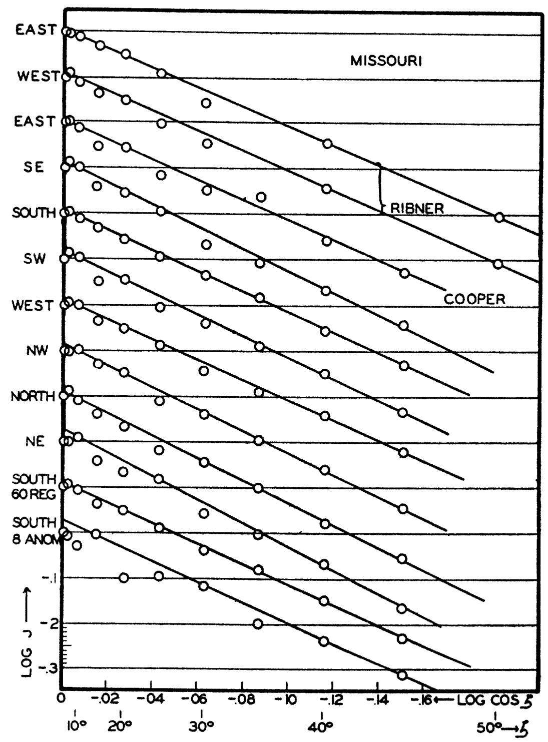

FIG. 1. Logarithmic graph of the cosmic-ray intensities measured by Ribner (top) and Cooper. The ordinates are the logarithms of the relative intensities. The abscissae are the negative logarithms of the cosines of the zenith angles, which differ by a constant term from the logarithms of atmospheric path lengths.

[9] A. G. Worthing and J. Geffner, *Treatment of Experimental Data* (John Wiley & Sons, Inc., New York, 1943), cf. appendix below for notation and formulas.

[10] W. E. Deming, *Statistical Adjustment of Data* (John Wiley & Sons, Inc., New York, 1943).

[11] G. U. Yule and M. G. Kendall, *Introduction to the Theory of Statistics* (Charles Griffin & Company, Ltd., London, 1937), eleventh edition.

TABLE I. Approximation summary—Cooper. Success obtained in fitting Cooper's data with different approximations. The names and constants characterizing the different analytical treatments are given. Ω is the mean-square deviation from the "true" curve of the assumed form, a smaller value indicates closer agreement. $P(>X^2)$ is the probability, based on the known probable errors, of obtaining a poorer agreement with the true curve than that observed. The expected value is 0.5. P(S.D.) is the probability that the deviations contributing to X^2 have the standard deviation theoretically expected from the true curve.

Approx.	Name	Constants	Ω	$P(>X^2)$	P(S.D.)
A	Single line for all α	a, b	135		
B	Single line	$a(\alpha), b(\alpha)$	104		
C	Line high	$a_H(\alpha), b(\alpha)$	41.0		
D	Line low	$a_L(\alpha), b(\alpha)$	69.3		
E	Double line for all α	a_H, a_L, b	102	0.000	
F	Double line Uniform points	$a_H(\alpha), a_L(\alpha), b(\alpha)$	62.9	0.037	0.040
G	Double line Individual points	$a_H(\alpha), a_L(\alpha), b(\alpha)$	29.4	0.999	0.0002
H	Non-‖ lines Uniform points	$a_H(\alpha), b_H(\alpha), a_L(\alpha), b_L(\alpha)$	66.6	0.025	0.014
I	Top line cut short	like F	>60.9		
J	5° Points asymmetrical	like F	52.2	0.37	0.71
K	F, exc. omit 35°E and 25°NE	like F	46.2	0.55	0.75

through the high points of the anomalies. Since Cooper speaks of the high points as the anomalous ones, this will bear closer examination. The method used is the comparison of approximations C (line high, i.e., through the high intensity points and the high ζ points) and D (line low, i.e., through the low intensity points and the high ζ points). The calculation verifies the visual conclusion that the "line high" of approximation C fits onto the high ζ points better than the "line low" of approximation D, since Ω is 40 percent lower. It is for this reason that in Fig. 1 and in all later approximations the high line is extended through all the range. Indeed, it seemed better to draw only the one line, and let the anomalously low points of the fine structure be visible as separate points.

Approximations C and D were also used to test the reality of the fine-structure anomalies, as indicated in Table II. This was carried out by getting the weighted mean departure of the low points from the high line, and the high points from the low line, and comparing with the standard error, σ, of that departure. The probability of obtaining such a departure, if we assume a normal distribution whose true value is zero, is also given. It is seen that all the departures fall below the 0.01 limit of probability except for 30° below the low line. This not only verifies Cooper's own conclusions that the fine structure is real and not the result of statistical fluctuations, but goes much further in showing how it may be expressed. From approximations C and D we may draw the following conclusions: (1) The fine structure is real, and the points cannot all be represented by one line; (2) The points at 5°, 10°, and 20° belong on a high line; (3) The points at 0°, 15°, 25°, and 30° belong on a low line; (4) Better results will be obtained by fitting the points at 35°, 40°, and 45° on the high line. Thus the smooth line is found to be that of the high points, with the anomalous fine structure consisting of the points that fall below it.

Certain other important results can be seen from the comparison of approximations A, B, E, and F. A and B differ from E and F, respectively, by including only one line instead of both a high and low line. As would be expected from the discussion of C and D, the departures, as measured by Ω, are considerably greater in A and B than when the line is double (Table I). Also, A and E are like B and F, respectively, as to nature of

TABLE II. Summary of $\left\{\begin{matrix}\text{high}\\\text{low}\end{matrix}\right.$ lines—Cooper. Test of reality of fine structure. The weighted mean departures ($\bar{V}$) of the measured points from the indicated line are compared with their probable errors (σ). Assuming a normal distribution of departures the probability of obtaining such a large value of $\bar{V}$ if the "true" value is zero is $P(0)$ given in the fourth column. The calculations are made by the formulas:

$$\bar{V}=\Sigma_\alpha\left[\frac{1}{\sigma_\alpha{}^2}V(\alpha)\right]\Big/\Sigma_\alpha\frac{1}{\sigma_\alpha{}^2}\quad \sigma=\left(\Sigma_\alpha\frac{1}{\sigma_\alpha{}^2}\right)^{-\frac{1}{2}}.$$

The value of $P(0)$ was determined from tables of the normal curve.

ζ	$\bar{V}$ $\times 10^3$	σ $\times 10^3$	$P(0)$
	Line high		
0°	−8.1	2.9	0.0039
15°	−13.3	2.8	0.00000
25°	−13.3	3.1	0.00002
30°	−7.7	2.9	0.008
	Line low		
5°	11.3	2.8	0.00008
10°	14.4	3.1	0.00001
20°	8.3	3.1	0.007
30°	−5.2	3.2	0.093

curve, but in the latter the curves are fitted separately in each azimuth. Since this gives again a considerably better fit, we may conclude that the intensities are actually different in the different azimuths, α. This was also born out by a detailed calculation of the standard error of the constants of the equations for one azimuth in approximation F. Since the standard error was about one-fifth of the range of the constants between azimuths, it is pretty certain that the constants of the curves are different in the different azimuths.

Approximations F and H may be compared to determine whether the low points form a line which is actually parallel to the upper, regular line, or whether they constitute a line of a different direction. In this case, the indications are not so certain. The tests of Table I agree in indicating that F is slightly better, but the difference is not very marked. The differences in the calculated constants for F and H are quite small, and there is no indication that the slope of the low line is consistently greater or less than that of the high line, since the means for the eight azimuths in H are identical with those for F, and also with E. It seems quite possible that it is largely accidental that the anomalously low points appear to fall at approximately the same distance below the high, regular line.

In drawing the preliminary lines corresponding to Fig. 1, it was found that the west and southeast azimuths give an appearance of a break at the 35° point, with the line more nearly horizontal at higher zenith angles. To test the reality of this, the partial approximation I was carried out on these azimuths, by stopping the line at 35°. In one case a slight improvement, in the other a slighter worsening was found. It was concluded that the over-all agreement could not be improved in this fashion.

So far, then, approximation F looks the best. It does not appear that any azimuthally uniform change in treatment could give an appreciable improvement. There remains the question of whether it is good enough, i.e., whether the residuals obtained in F agree with the values of the residuals expected from the known probable errors. This question is best treated by considering the probabilities of Table I and Table III. It is seen that the total probabilities fall in the "doubtful," or "significant but not highly significant" range between 0.05 and 0.01, the most commonly used thresholds for determining the reality of an observed effect. From these values alone, we would be tempted to consider approximation F as "probably not good enough," i.e., the probability of obtaining the observed residuals by the mere random operation of the errors of determination of intensities is rather low.

TABLE III. Calculated values—Cooper. The values of the parameters calculated by least squares for the three best approximations are listed in columns 3–6. The last three columns give the values of the quantities used to test the success of the approximations. The notation is explained in the appendix. Note that the values of a_H and a_L are the calculated values at 25°.

Approx.	α	$-a_H$	$-a_L$	a_H-a_L	$-b$	Ω	X^2	$P(>X^2)$
F	E	0.086	0.106	0.020	2.18	129.4	19.33	0.007
	SE	.089	.106	.017	2.40	69.4	8.76	0.27
	S	.089	.093	.004	2.21	5.7	.90	1.00
	SW	.082	.101	.019	2.30	32.9	2.91	0.89
	W	.084	.095	.011	2.23	30.1	2.65	0.91
	NW	.090	.100	.010	2.36	52.3	8.32	0.31
	N	.095	.107	.012	2.43	48.7	9.49	0.16
	NE	.096	.102	.006	2.46	134.3	23.61	0.002
	Total	.711	.810	.099	18.57	502.8	75.97	0.037
	Mean	.089	.101	.012	2.32	62.9		
	s	.0047	.0037	.0054	.10			
G	E	0.088	0.110	0.022	2.14	78.9	8.17	0.32
	SE	.089	.107	.018	2.37	40.6	5.59	0.54
	S	.089	.094	.005	2.21	4.1	.65	1.00
	SW	.082	.101	.019	2.30	32.9	2.91	0.89
	W	.084	.096	.012	2.20	17.3	2.03	0.96
	NW	.087	.103	.016	2.42	19.1	2.50	0.92
	N	.094	.110	.016	2.45	28.7	4.50	0.72
	NE	.085	.109	.024	2.58	13.6	1.37	0.98
	Total	.698	.830	.132	18.67	235.2	27.72	.999
	Mean	.087	.104	.017	2.33	29.4		
	s	.0035	.0059	.0058	.14			
J	E	0.088	0.110	0.022	2.14	78.9	8.17	0.32
	SE	.089	.106	.017	2.40	69.4	8.76	0.27
	S	.089	.093	.004	2.21	5.7	0.90	1.00
	SW	.082	.101	.019	2.30	32.9	2.91	0.89
	W	.084	.095	.011	2.23	30.1	2.65	0.91
	NW	.087	.101	.014	2.40	26.1	3.69	0.81
	N	.095	.107	.012	2.43	48.7	9.49	0.16
	NE	.093	.103	.010	2.49	125.6	22.51	0.002
	Total	.707	.816	.109	18.60	417.4	59.08	0.37
	Mean	.088	.102	.014	2.33	52.2		
	s	.0040	.0055	.0053	0.12			

For this reason we consider approximation G. No uniform treatment of the lines in all azimuths can give improvement over F. But the fact is that neither Fig. 1 nor Cooper's data show the fine structure as uniformly symmetrical. The high and low points do differ in the different azimuths, no two agreeing exactly, and only the southwest showing just the distribution determined from approximations C and D. We may, therefore, try approximation G, in which the form of the equation is as in F, but the distribution of high and low points, instead of being uniformly that determined from approximations C and D ("uniform points"), is arbitrarily made to coincide with the observed distribution in each azimuth ("individual points"). This will necessarily decrease the residuals, without introducing any change in the number of degrees of freedom. This means that Ω must necessarily decrease, and therefore cannot be used as a test of the suitability of the treatment. Instead, any justification for this arbitrary assignment must be sought in the necessity of fitting the observations within the limits expected from the known probable errors.

Since great theoretical interest attaches to just this question of whether the fine structure is actually symmetrical or not, special effort was made in the comparison of approximations F and G. The results for these two approximations are given in Table III. Briefly, they may be summed up in the statement that where approximation F is perhaps not good enough, approximation G is highly significantly too good. For instance, we may consider the over-all probability from the chi-test. This gives us only 0.037 for F, but 0.999 for G. Since the probability given is that of obtaining a value greater than that observed, we can say that we are 99.9 percent certain to get a worse agreement with the "true" calculated equation than is actually observed for approximation G. Again, the standard deviation of the residuals is so small that there is only a 0.0002 probability of obtaining such a close agreement.

It is pointed out below, in the discussion of these anomalies, that there is a strong theoretical reason to believe that they are actually azimuthally exactly symmetrical. At least, this is true of the anomalies at 15° and at 25°–30° (naming them according to the anomalous low points, instead of by the more regular "prominences" used by previous writers), but it is not true of the anomaly at the zenith. Not only is there no theoretical reason to think that this must be azimuthally symmetrical, but in actual fact, the observations of Cooper,[1] Ribner,[2] and Yeater[5] all agree in showing that it is not symmetrical. The difference between the east and west is the most marked, with none of the curves given showing a significant prominence (i.e., above the $\cos^2 \zeta$ curve) at or near 5° in the east, whereas they all do in the west. Except for this exact agreement, the difference might be considered uncertain, and has, indeed, been overlooked by all previous writers. The observations of Yeater[5] show an east-west asymmetry of 3–5 times its standard error (the uncertainty of the multiplying factor makes the exact value indeterminate) at 2° east and west. It seems certain that the asymmetry of this anomaly may be taken as real.

Therefore we consider finally approximation J. Cooper's data show the 5° points low in the northeast and northwest azimuths. Unfortunately there are no other observations in these two directions, but the completeness of the verification of his results in the east and west may give us confidence that they are reliable. These two 5° points are therefore called low in approximation J, though they do not make a great deal of difference in the interpretation, as will be seen from a comparison of approximations F and J in Table III. Approximation J differs from F only in the east (zenith on the high line) and northeast and northwest (5° on the low line). It is seen from Table I that it shows a very good over-all agreement with the observations within the limits of the uncertainties of the individual measurements. However, Table III shows that even approximation J does not fit well in the northeast. Unfortunately, the irregularities (20° low, 25° high) of this curve cannot be checked against any other observations. The single point at 25° shows a 0.0004 probability of agreement in approximation J, so that even out of 80 total points the expectation of such a poor agreement anywhere is low. This presents some evidence of asymmetry in the outer anomalies as well.

The situation, then, is this. We may insist on strict azimuthal symmetry, and select approximation F. In this case, we must explain the low over-all agreement, and the low agreement in east and northeast. Or, secondly, we may abandon insistence on symmetry in the zenith anomaly and take approximation J. In this case, the only difficulty to be explained is in the northeast. Finally, we may abandon symmetry altogether, adopt approximation G, and attempt to explain the fact that the resulting agreement is much too good to be expected from the known statistical fluctuations. These alternatives will be considered in order of increasing probability.

The greatest difficulty is found with approximation G. There is, however, a possible explanation of the low values of X^2 in the time variations of these anomalies. These have been mentioned by all the writers on the fine structure, but it is shown below that there is very good reason to consider them spurious appearances from the random counting fluctuations. If, however, they could be supposed to increase the variance of the individual measurements by 50 percent (an impossibly high value), then the value of X^2 would be reduced to $\frac{2}{3}$ of the expected value, as is seen from the formula for X^2 in the appendix. This is the value found in approximation G. This explanation also permits us to picture the instantaneous fine structure as always symmetrical, though variations in the time fluctuations would make the results of different series average to asymmetrical totals. However, any simultaneous series should show strict symmetry. There is some evidence in Yeater's observations[4] that this is not the case. In any event, the dubious nature of the time fluctuations and the great magnitude required leave approximation G the weakest of the three.

Careful examination of approximation F shows that the difficulty with it is localized in the east and northeast azimuths, and indeed in two bad points on each azimuth. An attempt was therefore made, in approximation K of Table I, to eliminate the difficulty by omitting two bad points. The attempt was very successful, X^2 having practically its most probable value. It seems very dubious, therefore, to rule out approximation F on account of only two bad points out of 80, especially since the over-all probability is not decisively low anyway.

However, these arguments apply still more strongly to approximation J. In this case, the omission of only one point in the northeast would give an even closer fit. And the over-all fit of J is already quite acceptable. There remains only the doubt of the poor fit in the northeast, which is certainly better than is the case with F. It may be ascribed either to fluctuations of the fine structure or to some special disturbance of the 25° point. Altogether, the weight of the evidence seems to favor approximation J considerably over the others.

Table III lists also the values of the constants calculated for the least squares fit to the two parallel lines in approximations F, G, and J. The calculations were all made with uniform weights for the logarithms of the intensities, since trial calculations showed no appreciable improvement in the calculated value of X^2 (theoretically improved by using weights) when the considerably more laborious process was carried out. One complete treatment by the method of Deming[10] was carried out for approximation F in the north azimuth, and the standard errors for the calculated constants were determined from the reciprocal matrix. The results are

$$\sigma_{aH}=0.0025,$$
$$\sigma_{aL}=0.0030,$$
$$\sigma_b=0.046.$$

Since the probable errors and the corresponding weights do not differ greatly from one azimuth to another, these values may be taken as reasonable estimates of the standard errors of the parameters listed in Table III. The values show considerable azimuthal variation. An attempt to account for these variations met with little success. Since the low line includes the origin ($\zeta=0$, $\log j=0$), the value of a_L is largely determined by that of b. The most regular azimuthal variation is that of a_H, whose Fourrier expansion shows a considerable degree of convergence in approximations F and J, with a maximum of the first harmonic in the southwest. The quantities b and a_H-a_L showed poor convergence and not much agreement as to the phases in the different approximations.

APPLICATION TO RIBNER'S DATA

The data of Ribner[2] are far less complete than those of Cooper,[1] and are not so directly suitable for the present form of analysis. The smaller number of cycles, and the necessity of superposing the results of different runs in each azimuth make the measured values subject to rather greater uncertainties than Cooper's. The general agreement, however, is excellent, and Ribner's values furnish very satisfactory corroboration of the deductions from Cooper's data. The general nature of this agreement is shown in Fig. 1 and Table IV. The parallelism is evident, although there are four cases in which the difference is real. Three of these cases are covered by the rather large uncertainties in Ribner's data, as was found by fitting Ribner's measurements to the equations determined from Cooper's data. The exception is the value of a_L in approximation F in the east. Such differences might be expected as a result of small differences in latitude, altitude, or transition and absorption in the counter train, none of whose effects on the fine structure have yet been investigated. There is also a slight indication of azimuthal asymmetry in the 15° and 25°–30° lows when compared with Cooper's approximation G, but these again are covered by the uncertainties in Ribner's data.

There is one very significant additional piece of data given by Ribner and not by Cooper. In addition to the probable errors calculated in the same manner as those quoted by Cooper, Ribner gives the values of the probable errors based on the number of counts alone. The errors by residuals include all variations, such as those caused by instrumental sensitivity or time fluctuations of the fine-structure pattern, as well as the statistical counting irregularities. The striking fact is that the errors by residuals are actually less than those caused by counting, though only by an amount neatly covered by the probable errors of the probable errors. However, there is here no evidence whatsoever of real fluctuations in sensitivity or in fine-structure pattern. This does not prove, of course, that very much higher counts could not reveal the reality of such fluctuations. But it does make it quite certain that such fluctuations cannot amount to 50 percent of Cooper's counting errors and so explain the very high probability found in Cooper's approximation G. It also makes it clear that the fluctuations in pattern reported by Ribner to Cooper cannot be distinguished from statistical fluctuations in counting.

TABLE IV. Summary—Ribner. Comparison between the results of Cooper's and Ribner's data. The columns compare the calculated values with those listed in Table III. The significance of the column headings is the same as in Table III, and is given in full in the appendix.

Approx.	Name	α	$-a_H$	$-a_L$	a_H-a_L	$-b$	Ω	$P(>\chi^2)$
B	Single line							
	Ribner	E	0.090			2.14	111	0.27
	Cooper	E	0.095			2.11	210	
	Ribner	W	0.090			2.12	118	0.13
	Cooper	W	0.089			2.19	59.0	
F	Double line Uniform points							
	Ribner	E	0.085	0.097	0.012	2.18	82.3	0.35
	Cooper	E	0.086	0.106	0.020	2.18	129.4	0.007
	Ribner	W	0.083	0.096	0.013	2.15	27.7	0.70
	Cooper	W	0.084	0.095	0.011	2.23	30.1	0.91
G	Double line Individual points							
	Ribner	E	0.087	0.104	0.017	2.14	53.8	0.51
	Cooper	E	0.088	0.110	0.022	2.14	78.9	0.32
	Ribner	W	0.084	0.099	0.015	2.14	16.8	0.90
	Cooper	W	0.084	0.096	0.012	2.20	17.3	0.96

Let us consider first the matter of time fluctuations of the fine structure. Ribner[2] mentioned such fluctuations in his observations, but it has just been pointed out that they cannot be distinguished in his data from the expected variations inherent in the counting process. Cooper[1] not only mentions the observation, but gives a detailed report of an extreme case occurring in the south azimuth, under the heading of "8 anomalous cycles." For these, he quotes the observations, together with their probable error determined from the number of counts. Two statistical tests of these eight cycles have now been made. The first was the direct measure of the probability of observing such a series of eight cycles, by the chi-test comparison of the departures from the average curve with their errors. The result gave a probability of 0.21 or about $\frac{1}{5}$. Since there were $8\frac{1}{2}$ groups of eight cycles in the south, there is only a 0.14 chance that at least one such group would not show as much departure as this. In other words, the known limits of error prove that departures as great as these would be expected, though that is not to say that the next such irregularity would be again of the nature of a reversal of observed structure.

This group of eight anomalous cycles was also investigated for its effect on the size of the probable errors. This can only be done roughly, with the assumption that the other cycles had total counts the same as those where the effect of the counting is known. But the result was entirely

DISCUSSION

Much of the treatment presented above is clear enough. Qualitatively, it agrees with previous treatments, except for the identification of the low points as anomalies, instead of the prominences. Quantitatively it goes farther and presents a semi-analytical representation of the entire fine structure. There are, however, a number of matters connected with the interpretation of the anomalies that will bear considerable elaboration.

negative. The probable errors listed for the total of 68 cycles are not increased by the presence of this apparently extreme case of fluctuation. Even if this does not prove the non-existence of the time fluctuations, it certainly shows that their effects cannot amount to the 50 percent (in the variance) required to make approximation G agree with expectation.

Again, Yeater,[4, 5] with apparatus and experimental method designed specifically to determine these time fluctuations, was unable to establish any such effect. He further points out that Schremp's earlier apparatus[12] was also inadequate for the purpose. From the comparison of all these lines of evidence, then, one may conclude that most, and probably all, supposed cases of fluctuations can be better explained as caused by statistical counting variations. Certainly the effect cannot be large enough to explain approximation G.

Now we may consider the probable interpretation of the fine structure. From the earliest days of its prediction and consideration[13] the symmetry it shows has been taken as the key to its interpretation. Irregularities caused by atmospheric absorption should depend only on the length of atmospheric path traversed, and therefore be independent of azimuth. The effect of the earth's magnetic field, though imperfectly known[14] at these high latitudes, is not expected to show any such symmetry. It may show an east-west symmetry if there are oppositely charged particles of identical total energy in the primaries, and except to the extreme north and east show little variation from northwest to southwest or northeast to southeast.

The consequence of this reasoning is that these fine-structure irregularities, with their at least approximate azimuthal symmetry, have always been ascribed to the effects of atmospheric absorption. To show such irregularities the absorbed particles must possess certain characteristics. In the first place, their absorption must be of the nature of a range phenomenon, their effects ceasing rather suddenly after a certain amount of absorption. The theoretical work[15] seems to show quite conclusively that this is characteristic of the penetrating, mesotron component of secondaries, but not of the soft electron showers. In the second place, the energies of these absorbed secondaries must fall into definite, well-separated groups, whose suddenly completed absorption is responsible for the sharp drop in intensity. It is often overlooked that this must be the characteristic of the absorbed secondaries, whose energies cannot be expected to be nearly so sharply defined as the primaries from which they are derived. Yeater,[4] for instance, elaborates many possible forms of absorption curves, all based on the assumption that the secondaries have the same energies as the primaries. The strength of the present method is that it allows for the straggling in secondary energies by the empirical selection of a baseline determined by the actual energy distribution.

There is an additional very important characteristic of anomalies caused by absorption. It is clear that any increase in absorbing path, i.e., in zenith angle, can only decrease the intensity, never increase it. This is observed in the case of the present anomalies at 15° and 25°–30°. But it is not true of the anomalies at the zenith. Both Fig. 1 and Yeater's curve[5] show a ring of higher intensity all around the zenith, except that in the east it first falls lower and then rises to the same height. Such an anomaly, whether azimuthally symmetrical or not, could not possibly be caused by atmospheric absorption, and there is therefore no reason to suppose that its approximate symmetry must be exact. In view of the close agreement of all the observations that it is not symmetrical, it seems better to abandon the assumption that it is, as was done in approximation J. It is therefore probably ascribable to magnetic effects. Such magnetic effects are also caused by a discontinuous primary energy spectrum (or possibly by peculiarities of the penumbra, which are not looked for at this latitude.[14])

We are left, then, with two edges of atmospheric absorption and one magnetic anomaly caused by bands of energy in the primary distribution. It is very desirable, therefore, to compare these bands with other work on the energy bands of the

[12] E. J. Schremp, Phys. Rev. **57**, 1061A (1940).
[13] E. J. Schremp and H. S. Ribner, Rev. Mod. Phys. **11**, 149 (1939).
[14] E. J. Schremp, Phys. Rev. **54**, 153, 158 (1938).
[15] Cf. e.g., B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

primaries, together with their theory. The most extensive material of this nature is the atom-annihilation theory of Millikan, Neher, and Pickering.[16] Although this theory has been tested chiefly on the soft, shower component, recent evidence[17] indicates that the mesotron component may show the same primary energy spectrum. The pertinent portion of the primary spectrum is the four lightest atom-annihilation bands, named by Millikan, Neher, and Pickering the helium (1.9 Bev), carbon (5.6 Bev), nitrogen (6.6 Bev), and oxygen (7.5 Bev) bands. Of these, the helium band lies well below the limit (2.5 Bev) for observation at sea level. The other three bands may be compared roughly with the two atmospheric absorption anomalies at 15° and 25°–30° zenith angle. Unfortunately, the shape of the anomalies is not very exactly known from these rather widely-spaced measurements. But it was found by repeated trials that if the significant zenith angle was taken at the succeeding high points, 20° and 35°, a fairly good agreement with the carbon and nitrogen bands could be obtained, together with a prediction of another high point at 44°. On Yeater's more detailed curves, a symmetrical prominence is readily seen at 42°, though it naturally does not show up on the present curves which do not include measurements at this point.

It seems therefore quite possible that the atom-annihilation bands can be used to account for the present atmospheric absorption fine structure. To do so requires the use of a secondary energy factor (cf. appendix) $S=2.16$. This compares favorably with the factor 2.20 deduced from the data of Gill[18] on the assumption that the sea level cosmic-ray shelf edge is determined by the atom-annihilation value 5.5 Bev.

In the case of the magnetic anomaly, the situation is different. The energy determined in this case is the primary energy. At this latitude, magnetic anomalies at the zenith should be caused by primaries with rigidities[6] between the limits 0.215 stoermer (Stoermer cone) and 0.23 stoermer (Lemaitre and Vallarta full light cone[19]). The corresponding energies are slightly less than 2.9 Bev and 3.2 Bev because of the weakness of the earth's magnetic field on this side. Such a band of primaries could evidently only make its presence felt at sea level with a secondary production factor close to one, instead of over two. It appears that the intensity of this anomaly, 3 percent in the vertical, may be so small when averaged over the entire sphere (a rough calculation gives $\frac{1}{8}$ percent) as not to affect Gill's observation[18] of the cosmic-ray shelf.

Such a primary energy band, however, does not fit into the atom-annihilation spectrum. Nor is the situation improved by the assumption that this anomaly is caused, like the others, by absorption, for the energy difference from the next one is then far too small to match the spacing of the atom-annihilation energies. The discrepancy is further emphasized when it is noted that the present observations lie well within the plateau[20] observed between St. George and Pocatello in vertically incident soft radiation.

Similar difficulties beset the attempt to account for the magnetic anomalies observed by Yeater[5] at 20° east and west of the zenith. He ascribes to them a rigidity of 0.20 stoermer, but such particles could not give rise to secondaries capable of penetrating that thickness of air. It seems more probable that the fluctuations observed are in the edge of this same zenith anomaly, either as the fluctuation of the main cone to the west (for positive particles) or the Stoermer cone to the east. The interpretation given by Yeater, of the Stoermer cone in the west, leads to an unobservably low energy. Whether the variations are to the east or west of the zenith (i.e., whether the energy is higher or lower than for the zenith position of the same cone) can probably be determined from the fine structure of the east-west asymmetry, since the inclination of the magnetic meridian plane to the east will cause the edges to lie at different zenith angles, as was actually observed.

The following tentative conclusions emerge. (1) The absorption edges may be obtained from atom-annihilation energy bands, with an S-factor

[16] R. A. Millikan, H. V. Neher, and W. H. Pickering, Phys. Rev. **61**, 397 (1942); **63**, 234 (1943); **66**, 295 (1944).
[17] E.g., M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. **59**, 615L (1941); J. Hamilton, W. Heitler, and H. W. Peng, Phys. Rev. **64**, 78 (1943).
[18] P. S. Gill, Phys. Rev. **55**, 1151 (1939).
[19] G. Lemaitre and M. S. Vallarta, Phys. Rev. **50**, 493 (1936).
[20] Reference 16, Fig. 7 on page 241.

of over two. (2) The magnetic anomaly, at the zenith, and with edges at 20° east and west, does not occur in atom-annihilation bands, but has an S-factor close to one. It seems possible there is here evidence for the dual nature of mesotron primaries, with only one kind able to generate soft electron showers.

APPENDIX

Notation and Formulas

α = azimuth; ζ = zenith angle; Ω = most probable mean square residual from "true" curve; $\Omega = \Sigma V_i^2/(n-m)$, where V_i = residual at point i, n = number of points fitted, and m = number of parameters adjusted from data. $X^2 = \Sigma(V_i/\sigma_i)^2$, where σ_i = standard error of measurement whose residual is V_i. I.e., $\sigma_i = (1/0.675)\times$probable error. When the measurement is a true frequency, this reduces to the more common definition $X^2 = \Sigma V_i^2/f_c$, where f_c = theoretical frequency at point i. $P(>X^2)$ = probability of obtaining a value of X^2 greater than that observed. For the least-squares calculations by which lines were fitted to the logarithmic plot, the following formulas and definitions hold: j = relative intensity from data, $x = \log\cos\zeta_0 - \log\cos\zeta$ ($\zeta_0 = 25°$ in references 1 and 2). The equation fitted has the form $\log j = a + bx$, where a = intercept of calculated line at ζ_0; b = slope of logarithmic line = exponent of generalized hyperbola whose equation is $j = A(\cos\zeta)^{-b}$. When two lines are used to represent the fine structure, a_H is for the higher line; a_L is for the lower; σ_{aH} = (standard error in a_H) = (probable error in a_H)/0.6745; σ_{aL} = standard error in a_L; and σ_b = standard error in b. In Table III, the standard deviations of these quantities are given for the azimuthal variation $s_b = \left\{\frac{1}{n}\Sigma(b-\bar{b})^2\right\}^{\frac{1}{2}}$. When a single constant term is used, $a = \dfrac{\Sigma x^2\Sigma y - \Sigma x\Sigma xy}{N\Sigma x^2 - (\Sigma x)^2}$; $b = \dfrac{N\Sigma xy - \Sigma x\Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$, where $y = \log j$. When two constant terms are used,

$$b = \frac{\frac{1}{n_1}\Sigma_1 x\Sigma_1 y + \frac{1}{n_2}\Sigma_2 x\Sigma_2 y - \Sigma_{1,2}xy}{\frac{1}{n_1}(\Sigma_1 x)^2 + \frac{1}{n_2}(\Sigma_2 x)^2 - \Sigma_{1,2}x^2},$$

$$a_1 = \frac{1}{n_1}\Sigma_1 y - (b/n_1)\Sigma_1 x,$$

$$a_2 = \frac{1}{n_2}\Sigma_2 y - (b/n_2)\Sigma_2 x.$$

In these equations, the subscripts 1 and 2 distinguish the points assigned to the high and low lines, respectively. $S = E_{\text{pri}}/E_{\text{sec}}$ = ratio of primary energy to most probable energy of the secondaries it generates.

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Radiation Losses in the Induction Electron Accelerator

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This paper discusses the possibility that radiation losses because of the high radial accelerations experienced by the electrons in an induction electron accelerator may introduce limitations in the design of accelerators for energies above 100 million electron volts. The effects of radiation losses on the electron orbits are calculated, and it is shown that not only should the orbit shift pulse necessary to bring electrons to a target inside the equilibrium orbit fall below the value expected in the absence of radiation, but also electrons should eventually arrive at the target with no orbit shift pulse whatever, at a phase of the field wave predictable from the theory. Both effects have been observed in the General Electric 100-Mev unit in a manner consistent with the predictions of the theory. The radiation itself has not yet been detected.

1. INTRODUCTION*

In the induction electron accelerator, the electrons are subjected continually to radial accelerations of the order of 10^{17} meters per second per second. It has been pointed out by

* *Symbols:*—Unrationalized m.k.s. units will be used throughout: The following symbols will be employed:

A = peak value of applied magnetic flux density at the equilibrium orbit (webers per sq. m)
A' = peak value of magnetic flux in orbit shrinking pulse at the equilibrium orbit (webers per sq. m)
B_0 = applied magnetic flux density at the equilibrium orbit (webers per sq. m)
B_r and B_z are components of magnetic flux density (webers per sq. m)
c = velocity of light = 3.00×10^8 m per sec.
e = charge on the electron = 1.602×10^{-19} Coulomb
E_r and E_z are components of electric field (volts per m)
f_n and f_t are normal and tangential components of the acceleration vector **f** (m per sec. per sec.)
$F(\omega t) = (\omega t/\sin\omega t) - \cos\omega t - (2/3)\sin^2\omega t\cos\omega t$
h = Planck's constant = 6.624×10^{-34} joule sec.
H_r and H_z are components of magnetic field
I = beam current (amperes)
m_0 = rest mass of the electron = 9.107×10^{-31} kg