A Thermodynamic Criterion for the Fracture of Metals

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THE theoretical evaluation of the tensile stress necessary to bring about the rupture of a metal in a purely brittle manner has been calculated on the basis of interatomic forces[1,2] and on the basis of the energy required to form new surfaces.[3] All of these calculations have led to the values of tensile strength of metals both single crystal and polycrystalline, which are a hundred to a thousand times higher than the observed values. On this account, it has been thought necessary by some investigators, to postulate the existence of a micro-crack structure or distribution of flaws throughout the specimen such that the resulting stress concentrations provide the factor necessary to bring about agreement between calculation and experiment.

TABLE I. (Multiply stresses by 10^6 for lb./sq. in.).

Metal	Fracture stress from (1) and (2)	Theoretical value, Seitz[a]
Fe	2.5	1.7
Cu	1.65	1.26
Al	1.40	0.53
Zn	0.70	0.82
Pb	0.32	0.37
Sn	0.69	0.51

[a] Seitz, *Physics of Metals* (McGraw-Hill Book Company, Inc., New York, 1943).

Another attempt to resolve this seeming discrepancy was made by Fürth[4] who advanced a thermodynamic theory of the tensile strength of metals based on the work of Born relating to the melting of crystals. Fürth compares fracture stresses computed from his equation with the fracture stresses of metals as given in the *Landolt-Börnstein Tables* (1927) and in the *Handbuch der Physik* (1928), extrapolated to absolute zero. The principal objections to Fürth's theory are that his treatment cannot account for variations in fracture strength with alloying elements, heat treatment, cold work, etc.; and that extrapolation of fracture stresses to absolute zero is meaningless.

A theory of fracture proposed by the author, also related to the melting phenomenon, is based on the following assumptions:

1. All of the strain energy is available for the abolition of cohesive strength.
2. The heat of fusion is uniformly partitioned throughout the volume occupied by the substance.
3. The quantity of energy required for the abolition of cohesive strength is that fractional part of the energy of fusion which is associated with the change in volume on passing from the solid to the liquid state.

On the basis of these assumptions, the criterion for fracture may be expressed in the form

$$U = JQ\Delta V/V, \tag{1}$$

where U is the strain energy per unit volume, Q is the latent heat of melting in kg-cal. per mole, V is the volume occupied by a mole of the substance, ΔV is the change in volume of a mole of the substance on passing from the solid to the liquid state, and J is the mechanical equivalent of heat.

The general expression for U is

$$U = \int (\sigma_1 d\delta_1 + \sigma_2 d\delta_2 + \sigma_3 d\delta_3), \tag{2}$$

where σ_1, σ_2, σ_3, and δ_1, δ_2, δ_3, are the principal stresses and strains, respectively. In the case of a purely elastic one-dimensional state of stress, Eq. (2) reduces to

$$U = \sigma^2/2E, \tag{3}$$

where E is the modulus of elasticity.

With data for ΔV from Masing,[5] and that for Q and E from any of the standard tables, the fracture strength of metals in simple tension for the purely brittle case may be calculated. Several examples are shown in Table I. In the case where plastic flow has preceded fracture and little error is involved in neglecting the elastic part of the strain energy, Eq. (2) may be conveniently expressed in the form

$$U = \int \bar{\sigma} d\bar{\delta}, \tag{4}$$

where

$$\bar{\sigma} = \{[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]/2\}^{\frac{1}{2}},$$

and

$$\bar{\delta} = [(\delta_1^2+\delta_2^2+\delta_3^2)\tfrac{2}{3}]^{\frac{1}{2}}.$$

Under the assumptions that the hydrostatic component of the stress system has no effect on the flow curve and that in the plastic region no changes in volume occur, $\bar{\sigma}$ and $\bar{\delta}$ are invariants. Since in simple tension $\bar{\sigma} = \sigma_1$ and $\bar{\delta} = \delta_1$, the general relationship between $\bar{\sigma}$ and $\bar{\delta}$ may be established from a single tension test. This relationship may be approximated by $\bar{\sigma} = \kappa\bar{\delta}^n$ and U may be evaluated from Eq. (4). This has been done, and in Table II typical comparisons for several metals tested in tension are given.

TABLE II.

Metal	Fracture stress calculated	Observed
SAE 1112 at 70°F	132,000 p.s.i.	121,500 p.s.i.
SAE 1112 at −119°F	139,000	135,000
SAE 2345 Room temp.	406,000	396,500
Pure copper Room temp.	73,000	70,000
Al 24 S-O Room temp.	49,600	48,000
Al 24 S-T Room temp.	101,000	99,600

It is thus seen that the proposed criterion for fracture leads to the theoretical calculations in the case of purely brittle fracture and gives good results in the case where plastic flow precedes fracture. It is doubtful whether purely brittle fractures have been observed in the laboratory even at very low temperatures of testing or at very high rates of strain. Under such conditions plastic flow is probably confined to small regions but sufficient plastic flow in these regions may occur to bring the fracture stresses down to the observed values.

[1] Zwicky, Physik. Zeits., **24**, 1923, 131.
[2] Born and Fürth, Proc. Camb. Phil. Soc. **36**, 1940, 454.
[3] Polyani, Zeits. f. Physik **7**, 1921, 323.
[4] Fürth, Proc. Roy Soc. London **177A**, 1940–41, 217–227.
[5] Masing, *Handbuch der Metallphysik* (Leipzig, 1935), Vol. I, p. 231.