eclipses, radio fade-outs, sporadic E, $F2$ scatter, and abnormal stratifications. Projection makes possible quick scanning of an enormous wealth of data, selection of portions for critical study, and visualization of dynamic events of short duration.

Lowering of Electrical Breakdown Field Strength at Microwave Frequencies Due to Externally-Applied Magnetic Field

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THE following effects have been observed in connection with 3-cm microwave breakdown studies[1] conducted on air gaps in wave guide:

(1) An approach of a permanent magnet to a gap which is on the verge of microwave sparking can make the gap spark over. In one favorable experiment 35 spark-overs took place out of 36 approaches with a magnet.

(2) A magnet of pole-face cross section smaller than the patch of wave guide over which it is poised may lower the breakdown field strength by 20 percent or more, depending on certain conditions, such as strength of the magnetic field and height of the breakdown gap.

(3) When the magnet is swept along on the outside of the vertically-narrowed wave guide (which encloses the breakdown gap) the gap may spark at a field strength lower by as much as a factor of 2 or more than when the magnet is absent. The sweeping magnet must remain upright, for the effect mentioned.

(4) No sparking effect can be reliably reported when the magnetic field is applied at right angles to the microwave electric vector, either along the direction of propagation, or crosswise to it.

(5) When broadface permanent magnets are used, the poised (stationary) effect and the sweep effect are much diminished.

The following possible explanations have been considered: (a) Vertical magnetostriction of the wave guide, including the possibility of interaction between the applied magnetic field and the currents in the wave guide surface. (b) Magnetic spiral-focusing in the gap (oscillating helices) leading to more intense concentrated ionization. (c) Larmor precession resonance with accompanying increase of gas conductivity.[2] For various reasons, and the fact that the lowering of breakdown peak power by a factor of 4 (Poynting vector proportional to field squared) in the sweep cases is equivalent in violence to the shrinking of the height of the gap[3] by the same factor 4, none of the mentioned possible explanations seems at the moment to be adequate. However, further investigation of both the effect and the possible causes is being continued.

[1] D. Q. Posin, Bull. Am. Phys. Soc. **21**, No. 2, April, abstract L6.
[2] Appleton and Childs in experiments cited by K. K. Darrow, Bell Sys. Tech. J. **11**, No. 4, 605.
[3] D. Q. Posin, Ina Mansur and H. Clarke, Radiation Laboratory Report 731.

The Two-Body Problem in Einstein's and Birkhoff's Theories

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C. GRAEF[1] has obtained the solution of the two-body problem according to Birkhoff's theory.[2] After the substitution $x_1\rightarrow\eta_1$, $x_2\rightarrow\zeta_1$, $y_1\rightarrow\eta_2$, $y_2\rightarrow\zeta_2$, and after the correction of some obvious typographical errors his equations may be written as follows

$$\begin{aligned}\ddot{\eta}_j/m_2=+(\eta_j-\zeta_j)/r^3[3/2r^2\{(\eta_k-\zeta_k)\dot{\zeta}_k\}^2\\ -2\dot{\zeta}_k\dot{\zeta}_k-\dot{\eta}_k\dot{\eta}_k+4\dot{\eta}_k\dot{\zeta}_k-1]\\ +(\eta_k\zeta_k)(\dot{\zeta}_k-2\dot{\eta}_k)(\dot{\zeta}_j-\dot{\eta}_j)/r^3,\end{aligned}\tag{1}$$

where

$$r^2=(\eta_k-\zeta_k)(\eta_k-\zeta_k).$$

The repetition of an index implies summation from one to two. Another set, for $\ddot{\zeta}_j$ is obtained by replacing η with ζ and m_2 with m_1. Let $\alpha_j=M^{-1}(m_1\eta_j+m_2\zeta_j)$ be the coordinates of the center of gravity where $M=m_1+m_2$. Then

$$\begin{aligned}\ddot{\alpha}_j=m_1m_2(\eta_j-\zeta_j)/Mr^3[3/2r^2(\{(\eta_k-\zeta_k)\dot{\zeta}_k\}^2\\ -\{(\zeta_k-\eta_k)\dot{\eta}_k\}^2)-\dot{\zeta}_k\dot{\zeta}_k+\dot{\eta}_k\dot{\eta}_k]\\ +m_1m_2(\eta_k-\zeta_k)(\dot{\eta}_k-\dot{\zeta}_k)(\dot{\eta}_j-\dot{\zeta}_j)/Mr^3.\end{aligned}\tag{2}$$

Einstein, Infeld, and Hoffman[3] have obtained the equations for the two-body problem on the basis of the general theory of relativity. The equations that correspond to (2) are

$$\begin{aligned}\ddot{\alpha}_j=m_1m_2(\eta_j-\zeta_j)/Mr^3[3/2r^2(\{(\eta_k-\zeta_k)\zeta_k\}^2\\ -\{(\zeta_k-\eta_k)\dot{\eta}_k\}^2)-\dot{\zeta}_k\dot{\zeta}_k+\dot{\eta}_k\dot{\eta}_k+(m_1-m_2)/r]\\ +m_1m_2(\eta_k-\zeta_k)(\dot{\eta}_k-\dot{\zeta}_k)(\dot{\eta}_j-\dot{\zeta}_j)/Mr^3.\end{aligned}\tag{3}$$

Thus, as far as motion of the center of gravity is concerned, the only difference between (2) and (3) is the term

$$(Mr^4)^{-1}m_1m_2(\eta_j-\zeta_j)(m_1-m_2)\text{ in (3).}\tag{4}$$

Robertson[4] has shown that the average value of $\ddot{\alpha}_j$, $\langle\ddot{\alpha}\rangle_{\mathrm{Av}}=\frac{1}{T}\int_0^T\ddot{\alpha}_jdt$ over a period of the classical motion, is zero for Eq. (3); i.e., there is no secular change in the velocity of the center of gravity. The average value of (4) however, is not zero, and for Eq. (2) this leads to the result that $\langle\ddot{\alpha}_1\rangle_{\mathrm{Av}}=m_1m_2(m_2-m_1)\epsilon/2Ma^{3/2}p^{3/2}$ and $\langle\ddot{\alpha}_2\rangle_{\mathrm{Av}}=0$ where $r^{-1}=p^{-1}[1+\epsilon\cos w]$. This acceleration is in the direction of the periastron of the minor component of a double star. Levi-Civita[5] has given an example of a binary star for which such an acceleration might be detected.

Berenda[6] has suggested that the difference in the results of the two theories, may be caused by different methods of approximation. This remark would not apply to the result deduced above, since the extra term in (3) is an interaction term which arises on account of the nonlinear character of the field equations.[7]

I wish to thank Dr. Infeld for suggesting this problem to me.

[1] C. Graef, Boletin Sociedad Mat. Mexicana **1**, Nos. 4 and 5, 25.
[2] G. Birkhoff, Boletin Sociedad Mat. Mexicana **1**, Nos. 4 and 5, 1.
[3] Einstein, Infeld, and Hoffman, Ann. Math. [1] **39**, 65.
[4] H. P. Robertson, Am. Math. [1], **39**, 101.
[5] T. Levi-Civita, Am. J. Math. **59**, 225. Oddly enough the magnitude of the acceleration is the same as that obtained by Levi-Civita from relativity theory through a mistake in the calculation.
[6] C. W. Berenda, Phys. Rev. **67**, 56 (1945).
[7] H. P. Robertson, reference 4, p. 103 footnote.