TABLE I. Cross section and mean life.

T	x_0	y_0	Cross section for photo-effect	Mean life in sec.
6×10^{9}	1	20	10^{-26}	30
1.2×10^{10}	0.5	15	10^{-26}	0.1
2.4×10^{10}	0.25	7.5	10^{-26}	3×10^{-5}
6×10^{9}	1	20	10^{-27}	300
2.4×10^{10}	0.25	7.5	10^{-27}	3×10^{-4}
3×10^{9}	2	40	10^{-26}	6×10^{11}

nucleus before a transformation by a photo-effect are indicated in the Table I. The assumed values of the cross sections correspond to the approximated ones for photons near the threshold.

The purpose of this remark is to emphasize the role of the photo-effect and of the neutrino emission in the general problem of statistical equilibrium of nuclei, electrons, and photons at $T\sim 10^{10}$ degrees. Similar considerations could be applied to other processes, such as production of excited states of nuclei in non-elastic collisions. G. Gamow pointed out to me that also the evaporation of neutrons and induced fission become important at temperatures considered above. A simple calculation shows that, at temperatures $\sim 10^{11}(kT\sim 10^{7}$ ev) and nuclear densities $\rho\sim 10^{6}$, the neutrino emission per cm^3 and per sec. becomes larger than the total density of energy of radiation and of electron pairs, and that thermodynamical equilibrium becomes impossible, if light particles and nuclei are present. If the average energy per particle is $>10^{8}$ ev, then the production of groups of mesons (in collision processes involving nuclei and nucleons) changes even more profoundly the statistical problem. The damping effect of such collisions is caused by the subdivision of primary energy between created particles and to the neutrino emission. The microscopic reversibility of such processes is questionable. The relationship between the postulate of irreversibility of some of these processes and the existence of a finite number of distinguishable states of elementary particles, postulated in the principle of supplementary indeterminacy,[6] will be discussed elsewhere.

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On the Method of Second Quantization

R. BECKER AND G. LEIBFRIED
Göttingen, Germany

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IN the quantum theory of wave fields one usually starts with a Hamiltonian function $\mathfrak{H}=\int\psi(x)^{+}H_x\psi(x)dx$ from which one derives in the usual manner the equation of motion $(-i/\hbar)\dot{\psi}=H_x\psi$ as a classical wave equation in the three-dimensional space, if the quantities $\psi(x)$ and $i\hbar\psi(x)^{+}$ are considered as a continuous set of conjugate variables. x is an abbreviation for x, y, z and eventually the spin variable.

The transition to quantum mechanics is obtained by regarding $\mathfrak{H}$ as an operator. The quantities $\psi(x)^{+}$ and $\psi(x)$ are now operators which in the case of Bose statistics satisfy the commutation rules.

$$\begin{aligned}\psi(x)\psi(x')^{+}-\psi(x')^{+}\psi(x)&=\delta(x-x'),\\ \psi(x)\psi(x')-\psi(x')\psi(x)&=0,\\ \psi(x)^{+}\psi(x')^{+}-\psi(x')^{+}\psi(x)^{+}&=0.\end{aligned}\tag{1}$$

We are going to exhibit the elements in Hilbert space on which these operators operate.[1] For this purpose we assume that the operator $A=\int\psi(x)^{+}\psi(x)dx$ exists. (It follows then by (1) that the operator $\int\psi(x)\psi(x)^{+}dx$ does not exist.) We prove first:[2] there exists an element φ_0 with the property that $\psi(x)\varphi_0=0$ for all x. Let φ_λ be a normalized eigenvector of A belonging to the eigenvalue λ, that is $A\varphi_\lambda=\lambda\varphi_\lambda$, then it follows from (1), that $\psi(x)^{+}\varphi_\lambda$ belongs to the eigenvalue $\lambda+1$ and $\psi(x)\varphi_\lambda$ belongs to the eigenvalue $\lambda-1$. In addition to that it follows from $A\varphi_\lambda=\lambda\varphi_\lambda$ that the norm: $\int dx(\psi(x)\varphi_\lambda,\ \psi(x)\varphi_\lambda)=\lambda\geq 0$.

In contradiction with that we would obtain a negative eigenvalue if the above mentioned φ_0 would not exist. Starting with φ_0 we construct an element

$$\psi(x_1)^{+}\cdots\psi(x_n)^{+}\varphi_0=(n!)^{\frac{1}{2}}\varphi x_1\ldots x_n,\tag{2}$$

then $\varphi x_1\ldots x_n$ is normalized to unity if $(\varphi_0,\varphi_0)=1$ in the sense that:

$$\begin{aligned}(\varphi_x,\ \varphi_y)&=\delta(x-y),\\ (\varphi' x_1x_2,\ \varphi y_1y_2)&=\tfrac{1}{2}[\delta(x_1-y_1)\delta(x_2-y_2)+\delta(x_1-y_2)\delta(x_2-y_1)],\\ (\varphi x_1\ldots x_n,\ \varphi y_1\ldots y_n)&=(n!)^{-1}\Sigma\delta(x_1-y_\alpha)\delta(x_2-y_\beta)\cdots\delta(x_n-y_\zeta),\end{aligned}\tag{3}$$

where $\alpha\beta\cdots\zeta$ is a permutation of the numbers 1, 2, $\cdots$, n and the summation extends over all permutations of these numbers. For these vectors, we have then:

$$\begin{aligned}\psi(x_s)^{+}\varphi x_1\ldots x_n&=(n+1)^{\frac{1}{2}}\varphi x_sx_1\ldots x_n.\\ \psi(x_s)\varphi x_1\ldots x_n&=n^{-\frac{1}{2}}\{\delta(x_s-x_1)\varphi x_2\ldots x_n+\cdots+\delta(x_s-x_n)\varphi x_1\ldots x_{n-1}\}.\end{aligned}\tag{4}$$

The most general vector in this Hilbert space is then:

$$\mathfrak{F}=f_0\varphi_0+\sum_{n=1}^{\infty}\int\cdots\int f_n(x_1,\ \cdots,\ x_n)\varphi x_1\ldots x_n dx_1\cdots dx_n.\tag{5}$$

On account of (1) these $\varphi x_1\ldots x_n$ are symmetrical in the $x_1\cdots x_n$. The same holds true for the coefficients $f_n(x_1,\ \cdots,\ x_n)$. The norm of $\mathfrak{F}$ is

$$(\mathfrak{F},\mathfrak{F})=|f_0|^2+\sum_{n=1}^{\infty}\int\cdots\int|f_n(x_1,\ \cdots,x_n)|^2dx_1\cdots dx_n.\tag{6}$$

Now the operation $\mathfrak{H}$, applied on $\mathfrak{F}$ leads directly to the Schrödinger equations for $n=1$, 2, 3, $\cdots$ particles which obey the Bose statistics.

One can treat the case of Fermi statistics in an equally simple way by replacing on the left-hand side of Eq. (1) the $-$ sign by a $+$ sign. The Eqs. (2), (5), and (6) remain unchanged. However $\varphi x_1\ldots x_n$ and $f(x_1,\ \cdots,\ x_n)$ are now antisymmetrical in the coefficients.

[1] We obtain in this directly a representation which was obtained by Fock, Zeits. f. Physik **75**, 622 (1932).
[2] This theorem was communicated to us by F. Rellich.