The Doppler shift of the light emitted from a distant point may be attributed in a conventional manner to the alteration in the emission time of a signal with the coordinate system, since

$$\Delta t' = (1-\alpha_{14}t)\Delta t,$$

where $t=r/c$ is now the travel time of the signal from the source to the observer.

The structure of the conformal group shows that this type of transformation is introduced as soon as one expands the Lorentz group by adjoining to it the transformations to accelerated axes, owing to the non-commutativity of the velocity and acceleration transformations. This procedure in turn rests on the hypothesis that the invariance of the Maxwell field equations is a fundamental physical requirement.

While a more complete analysis of the physical interpretation of the full group of transformations will be required before all of its implications can be appreciated, the simple treatment given here does seem to strengthen Milne's idea of an approach to the red-shift phenomenon without an explicit appeal to the relativistic theory of gravitation.

[1] E. L. Hill, Phys. Rev. 67, 360 (1945).

The Pair Production of Light Mesons by Electrons

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In a recent series of seminars given at the Massachusetts Institute of Technology, M. Schein discussed certain experiments in cosmic rays which seemed to indicate the existence of mesons of low mass of about 20 electron masses. The development of the betatron has made available a source of electrons of sufficiently great energy to make the laboratory production of these mesons feasible.

The mesons may originate either in the meson field or in the electric field associated with a nucleus. In this note we shall compute the second effect only, inasmuch as the only unknown constant involved is the meson mass whereas the first effect would also involve the unknown interaction between nucleons and light mesons. The method of virtual quanta, as given for example by E. J. Williams,[1] is used. In this procedure an electron of energy E may be replaced by a photon field such that the number of photons with an energy between ϵ and $\epsilon+d\epsilon$ is:

$$N(\epsilon)d\epsilon = \frac{2\alpha}{\pi}\left(\ln\frac{E}{\epsilon}\right)\frac{d\epsilon}{\epsilon}, \quad (1)$$

$$\alpha = e^2/\hbar c.$$

These photons will under the influence of the Coulomb field of a nucleus produce meson pairs. The cross section for this process for "scalar" particles[2] of spin 0 and for particles[3] of spin $\hbar/2$ is

$$Q(\epsilon) = Z^2\alpha\left(\frac{e^2}{\mu c^2}\right)^2\left[A\ \ln\frac{2\epsilon}{\mu c^2} - B\right], \quad (2)$$

where Z is the charge of the nucleus, μ = meson mass, $A=16/9$, $B=104/27$ for spin 0, and $A=28/9$, $B=218/27$ for spin $\hbar/2$. The cross section for the total process, the pair production of mesons by electrons becomes:

$$\sigma = \int^{E} N(\epsilon)Q(\epsilon)d\epsilon \sim \frac{A}{3\pi}(\alpha Z)^2\left(\frac{e^2}{\mu c^2}\right)^2\left(\ln\frac{E}{\mu c^2}\right)^3. \quad (3)$$

A similar formula has been derived[4] for the case of the production of an electron-positron pair by an electron. It differs from (3) with $A=28/9$ only in the replacement of the meson mass by the electron mass. Formula (3) is valid only when both the energy of the incident electrons and the emitted mesons is high compared to their rest mass. Moreover the energy lost by the electron should be small compared to its initial energy.

When the incident electron energy is 100 Mev, the meson mass is 20 electron masses, and the nucleus $Pb(Z=82)$ then for scalar mesons, $\sigma \sim 2\times10^{-28}$ cm^2 while for spin $\hbar/2$ mesons $\sigma\sim 3\times10^{-28}$ cm^2.

It should be emphasized that formula (3) underestimates the total cross section since only one of the possible processes in which mesons can be produced is calculated. In addition, if the mesons were of the vector type, i.e., of spin $\hbar$, one would judge by analogy with the known meson bremstrahlung formulae[5] that the cross section for the pair production of vector mesons is greater than that given by (3).

[1] E. J. Williams, Kgl. Danske Vid. Sels. Math.-Fys. Medd. 13, 4 (1935).
[2] W. Pauli and V. Weisskopf, Helv. Phys. Acta. 7, 709 (1934).
[3] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, Oxford, 1936), p. 200.
[4] See reference 3, p. 203.
[5] R. F. Christy and S. Kusaka, Phys. Rev. 59, 405 (1941).

Method of Increasing Betatron Energy

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Conventional betatrons used to date have followed the conservative designs similar to those described previously.[1,2] In these betatrons the flux density at the orbit of the electron is limited to a value less than half of the flux density which can be used in the center of the orbit. These betatrons have customarily been operated with approximately 4000 gauss at the orbit and 11,000 or 12,000 gauss in the iron within the orbit. This distribution of flux was necessary to have an equilibrium orbit at radius, r, since the flux, ϕ, within the orbit must satisfy $\phi-\phi_0=2\pi r^2H$. ϕ_0 is the flux linking the orbit when the field at the orbit is zero. With the conventional machine, ϕ_0 is zero.

The purpose of this letter is to point out that the flux, ϕ_0, need not be zero when the electrons are injected; it can be biased reversely so that the flux through the center of the orbit has a large negative value initially and changes to a large positive value during the acceleration process. Thus, since the maximum flux density in the center of the orbit can be made to change by twice the conventional amount, the field at the orbit and hence the final energy