(a) Most of the energy in (1) lies in very high harmonics.
(b) The coherence between the high harmonics from different electrons tends to become very small if the group has an appreciable angular spread.
(c) The low harmonics are partially coherent, and give an energy loss per electron per turn (L') depending on N, but not on E if $E>>E_r$.
(d) Because of fluctuations from a uniform distribution, each electron also radiates the same amount L that it would if alone in the orbit. The total radiation per electron is thus $L+L'$.

Values of L' have been computed numerically from Schwinger's formula for the case of N electrons covering uniformly an arc with an angular extent which is $1/m$ of a circle. This was done for $m=2$, 4, and 6; also the asymptotic form for large m was obtained. These values can all be fitted within a few percent by the formula:

$$L'\sim 400\pi(e/R)\times 2.4(m^{4/3}-1)N. \tag{2}$$

Applying (1) and (2) to the case where $R=100$ cm, $E/E_r=600$, $N=10^{11}$ (1/60 microcoulomb, giving 1 microampere at a 60 cycle repetition rate), and $m=6$, we get:

$$L=780 \text{ volts}, \quad L'=1400 \text{ volts}.$$

Thus the radiation loss will not seriously affect the operation of the synchrotron. Furthermore, L. I. Schiff has shown that the coherent part L', which is mostly in the very low harmonics, can be strongly reduced by shielding.

A Note on the Separation of Gases by Diffusion into a Fast-Streaming Vapor

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WHILE it is well known that gas mixtures may be separated by diffusion into a fast-streaming vapor, the potential speed of the process is not generally appreciated. The simple calculation given below may serve to correct this situation. The calculation is meant to be illustrative and not quantitative in the sense that it would agree with experimental data.

In Fig. 1, A and B are two channels connected by a crossduct, C, of very small cross-sectional area. A pure gas is assumed to be traveling in A at a high velocity in the downward direction. Similarly, a vapor is flowing in B. The pressure differential across C is assumed to be such that the fluid velocity in the x direction is constant. To express the rate of transfer of the gas the equation:

$$\mathbf{i}=\rho\mathbf{v}-D \operatorname{grad} \rho \tag{1}$$

may be used.[1] In this equation $\mathbf{i}$ is the rate of gas transfer per unit area; ρ, the density of the gas; $\mathbf{v}$, the fluid velocity; and D, the mutual coefficient of diffusion of gas and vapor. For the steady state $\operatorname{div} \mathbf{i}=0$, so that

$$\operatorname{div}(\rho\mathbf{v})-D \operatorname{div} \operatorname{grad} \rho=0. \tag{2}$$

TABLE I. Rate of diffusion, i, of H_2 into water vapor. (i is expressed in $cm^3/cm^2\times$sec. at 150°C and 1 atmos.)

v (cm/sec.)	$b=0.01$ cm	$b=0.1$ cm	$b=1.0$ cm
−1000	2.34	000	000
−100	120	0.234	000
−10	156	12.0	0.023
−1	164	15.6	1.20
0	165	16.5	1.65
+1	166	17.6	2.20
+10	177	22.0	10.0
+100	220	100	100
+1000	1002	1000	1000

TABLE II. Separation currents and compositions. (i is expressed in $cm^3/cm^2\times$sec. at 150°C and 1 atmos.)

v	$b=0.01$ cm		$b=0.1$ cm		$b=1.0$ cm	
cm/sec.	i	f_{H_2}	i	f_{H_2}	i	f_{H_2}
−1000	1.17	1.00	000	—	000	—
−100	62.3	0.964	0.117	1.00	000	—
−10	91.5	0.852	6.23	0.964	0.012	1.00
−1	97.8	0.839	9.15	0.852	0.623	0.963
0	98.5	0.837	9.85	0.837	0.985	0.837
+1	99.3	0.836	107	0.826	1.63	0.678
+10	107	0.827	16.3	0.678	10.0	0.500
+100	162	0.678	100	0.500	100	0.500
+1000	1001	0.500	1000	0.500	1000	0.500

If ρ is considered a function of x only, and $v_x=v$ is considered constant, then

$$\frac{\partial^2\rho}{\partial x^2}-\frac{v}{D}\frac{\partial\rho}{\partial x}=0. \tag{3}$$

If it is further assumed that the velocities in A and B are so large that the concentration of gas in B and of the vapor in A are substantially zero, (3) must be solved subject to the boundary conditions:

$$\rho(x)=\rho_0 \quad \text{at} \quad x=0,$$

and

$$\rho(x)=0 \quad \text{at} \quad x=b.$$

The result is

$$\rho=\rho_0[e^{vx/D}-e^{vb/D}]/[1-e^{vb/D}]. \tag{4}$$

For the current (1) gives

$$i=\rho_0 v/[1-e^{-vb/D}]. \tag{5}$$

The limit of i as v approaches zero is

$$i_0=\rho_0 D/b. \tag{6}$$

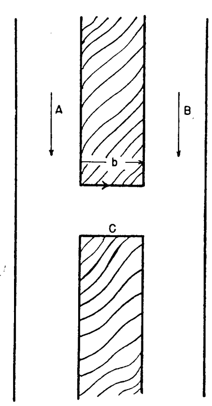

FIG. 1. Gas and vapor streaming by a connecting duct.

In Table I the calculated values of i for the diffusion of hydrogen into water vapor are presented for $D=1.65$ cm²/sec. at 150°C.

The value of i for $b=0.01$ cm and $v=0$ corresponds to 19,500 cu. ft. of hydrogen per square foot per hour.

If now it is assumed that two gases will diffuse independently of one another, the ratio of the currents will be

$$\frac{i_2}{i_1}=\frac{\rho_{0,2}[1-\exp(-vb/D_1)]}{\rho_{0,1}[1-\exp(-vb/D_2)]} \tag{7}$$

and the fraction, f_1, of the first gas in the separated portion will be

$$f_1=\left\{1+\frac{\rho_{0,2}[1-\exp(-vb/D_1)]}{\rho_{0,1}[1-\exp(-vb/D_2)]}\right\}^{-1}.$$

In Table II the values of i and $f_1=f_{H_2}$ are given for a gas mixture originally containing 50 percent H_2 and 50 percent CO_2. $D=0.32$ cm²/sec. was used for the mutual coefficient of diffusion of CO_2 and water vapor.

Table II shows that, at a fixed value of v, an increase in H_2 concentration may be effected merely by increasing the length, b, of the cross-duct, but only at the cost of a severe reduction in the gas current.

The calculations suggest that a thin, perforated metal sheet would serve as the nucleus of an effective gas separator.[2]

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[1] G. Hertz, Physik. Zeits. **23**, 433 (1922); Proc. Acad. Sci. Amsterdam **25**, 434 (1922).

[2] C. G. Maier, Bureau of Mines Bulletin 431.