On Isotopic Spin

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SUPPOSE we have a dynamical system containing N heavy nuclear particles. A complete set of commuting observables for a state of system will be r_α, σ_α, τ_α where σ_α is spin and τ_α is isotopic spin. Thus the state of a dynamical nuclear system is determined by the wave functions $\psi(r_1\cdots r_N;\ \sigma_1\cdots\sigma_N;\ \tau_1\cdots\tau_N)$. For the two-valued spin variables σ_α and τ_γ, we have $q\cdot 2^{2N}$ functions

$$\psi^{\xi}\sigma_1\cdots\sigma_N;\ \tau_1\cdots\tau_N{}^{(r_1\cdots r_N)},\tag{1}$$

where q is the number of the independent stationary states determined by the Cartesian forces.

The properties of the stationary states of the dynamical system with the N particles can be expressed by the *irreducible* representations of the symmetrical group of N variables.

The $2^{2N}\cdot q$ dimensional representation D^A with the basis (1) splits under the transformations of the symmetrical group into irreducibly invariant subspaces D_α,

$$D^A = D_\sigma \times D_t \times D_\omega = \Sigma_\alpha D_\alpha,\tag{2}$$

each defined by *Partitio Numerorum*

$$N = K_1 + K_2 + \cdots + K_\alpha,\quad K_1 \leq K_2 \leq \cdots \leq K_\alpha,\tag{3}$$

where K_s are the whole positive numbers.

The reduction of the D_σ (and D_τ) is determined by the following theorem:

$$D^\sigma = \sum_{s,m} D^{s,m},\quad D^\tau = \sum_{\tau,r} D^{\tau,r},\tag{4}$$

where $D^{s,m}$, $D^{\tau,r}$ are the irreducible representations and

$$-s \leq m \leq +s,\quad s = (N/2),\ (N/2)-1,\ \cdots\tfrac{1}{2}\text{ or }0,$$
$$-T \leq r \leq +T,\quad T = (N/2),\ (N/2)-1,\ \cdots\tfrac{1}{2}\text{ or }0,\tag{5}$$

the number of the rows and columns of the matrices $(D^{s,m})_{\alpha\beta}$, $(D^{\tau,r})_{\rho\sigma}$; i.e., the number of independent states of the basis of the irreducible representation is:

$$\lambda_T,\ \lambda_s = \left[\binom{N}{(N/2)-s} - \binom{N}{(N/2)-s-1}\right].\tag{6}$$

The *Partitio Numerorum* $(D^{s,m})$ and $(D^{\tau,r})$ are to be[1]

$$N,\ (N-1)+1,\ (N-2)+2,\ \cdots((N/2)+|s|)-((N/2)-|s|).\tag{7}$$

The reduction of the D^A is thus determined by the properties of the irreducible representation of the symmetrical group D^ω (i.e., by the representation of the Cartesian co-ordinates) and by the Pauli principle.

The formulation of this problem for the ρ-valued spin and ν-valued isotopic spin, i.e., for the many-valued electric charges, is very interesting. The physical properties of the dynamical quantum system are determined by the characters X_v of the irreducible representations D_α of the symmetrical group.

The study of the possible types of the nuclear interaction is made especially easy by consideration of the possible irreducible representation of the symmetric group. The zero-approximation is determined by Hamiltonian H_0 and the representation D_ω. The interaction may be considered as introducing a splitting of the stationary states of the zero-approximation (the interchange effects for a spin and isotopic spin-variables). The energetic relations can be determined by the general Heitler-Dirac[2] formula by means of the characters X_v.

[1] J. v. Neumann and E. Wigner, Zeits. f. Physik **47**, 203; **49**, 73 (1928).
[2] W. Heitler, Zeits. f. Physik **51**, 805 (1928); Dirac, Proc. Roy. Soc. **123**, 714 (1929); H. Weyl, *Gnippentheorie and Quantenmechanik* (S. Hirzel, Leipzig, 1931), Vol. 2, Chap. V.