

The Electron Optics of Mass Spectrographs and Velocity Focusing Devices

R. G. E. HUTTER*

Division of Electron Optics, Stanford University, California

(Received January 29, 1945)

The well-known results of the theory of mass spectrometers and velocity focusing devices are derived again by a different method which is simpler than the ones previously used and which brings out more clearly the electron optical nature of the deflecting and focusing properties of the fields employed in these types of instruments.

THE deflecting and focusing properties of electrostatic and magnetic fields used in mass spectrometers and velocity focusing devices have been treated theoretically by R. Herzog¹ and W. Henneberg.² These authors derived expressions for the optical quantities of such systems. The methods employed are, however, not adequate to the electron optical nature of the problem. Because Newton's force equations were used as the basic equations, the time had to be eliminated by special manipulations so that path equations could be obtained. These derivations are therefore rather involved. It might be useful to give a derivation of the results in a manner which has become the typical method for the treatment of electron optical phenomena. This method has been used by O. Scherzer³ for electron lenses of rotational symmetry about an axis, by J. Picht, J. Himpan,⁴ and G. Wendt⁵ for the deflecting fields in television tubes and cathode ray tubes. A paper by G. Wendt⁶ deals with the dioptrics of electron optical devices with arbitrarily curved axes. The general expressions obtained there could be applied to yield the formulae derived in the present work as far as the first-order properties

are concerned. Since we are going to investigate also second-order aberrations and chromatic aberration, it seems advisable to give a complete theory of the problems involved.

(1) THE GENERAL PATH EQUATION

The index of refraction of an electromagnetic field in cartesian coordinates x, y, z is given by:

$$n = [(2e/m)\varphi(xyz)]^{\frac{1}{2}} - (e/m)(\mathbf{A} \cdot \mathbf{s}), \quad (1)$$

where $\varphi(xyz)$ is the electrostatic potential, $\mathbf{A}$ is the magnetic vector potential, $\mathbf{s}$ is a unit vector in the direction of the path, e and m are charge and mass of the particle respectively, and $(\mathbf{A} \cdot \mathbf{s})$ is the scalar product of the two vectors $\mathbf{A}$ and $\mathbf{s}$.

Fermat's principle can then be written in the form

$$\delta \int_{P_1}^{P_2} [(2e/m)\varphi(xyz)]^{\frac{1}{2}} - (e/m)(\mathbf{A} \cdot \mathbf{s}) ds = 0. \quad (2)$$

Transforming this expression to cylindrical coordinates r, θ, z , we get:

$$\delta \int_{P_1}^{P_2} [(2e/m)\varphi(r, \theta, z)]^{\frac{1}{2}} [r^2 + (r')^2 + (z')^2]^{\frac{1}{2}} - (e/m)(A_r r' + A_\theta r + A_z z') d\theta = 0, \quad (3)$$

since

$$ds = [r^2 + (r')^2 + (z')^2]^{\frac{1}{2}} d\theta. \quad (4)$$

The prime signifies derivation with respect to θ .

The necessary conditions for Eq. (3) to be true are the following differential equations, called the Euler-Lagrange equations:

* The author is now a research physicist with Sylvania Electric Products, Inc., New York. The paper was completed while he was Research Associate in the Division of Electron Optics at Stanford University, California.

¹ R. Herzog, *Zeits. f. Physik* 89, 447 (1934).

² W. Henneberg, *Ann. d. Physik* 19, 335 (1934); *Ann. d. Physik* 20, 1 (1934); *Ann. d. Physik* 21, 390 (1934).

³ O. Scherzer, *Zeits. f. Physik* 101, 593 (1936).

⁴ J. Picht and J. Himpan, *Ann. d. Physik* 39, 409 (1941); *Ann. d. Physik* 39, 436 (1941); *Ann. d. Physik* 39, 478 (1941).

⁵ G. Wendt, *Die Telefunkenroehre* 15, 100 (1939).

⁶ G. Wendt, *Zeits. f. Physik* 120, 720 (1943).

$$\begin{aligned}
& \left[\frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \frac{d}{d\theta} \left\{ r' \left[\frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \right\} \\
&= \frac{\partial \varphi(r\theta z)}{\partial r} + \frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} r \\
&\quad - \left(\frac{e}{m} \right)^{\frac{1}{2}} [rH_z - z'H_\theta] \cdot \left[\frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}}, \quad (5) \\
& \left[\frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \frac{d}{d\theta} \left\{ z' \left[\frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \right\} \\
&= \frac{\partial \varphi(r\theta z)}{\partial z} - \left(\frac{e}{m} \right)^{\frac{1}{2}} [r'H_\theta - rH_r] \\
&\quad \cdot \left[\frac{2\varphi(r\theta z)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}}.
\end{aligned}$$

Use has been made of the definition of $\mathbf{A}$ as:

$$\mathbf{H} = \text{curl } \mathbf{A},$$

or in components

$$\begin{aligned}
H_r &= -\frac{\partial A_\theta}{\partial z} + \frac{1}{r} \frac{\partial A_z}{\partial \theta}, \quad H_\theta = -\frac{\partial A_z}{\partial r} + \frac{\partial A_r}{\partial z}, \\
H_z &= \frac{1}{r} \left(\frac{\partial}{\partial r} (rA_\theta) - \frac{\partial A_r}{\partial \theta} \right). \quad (6)
\end{aligned}$$

We now consider the following field combination given by:

$$H_\theta = H_r = 0, \quad H_z = H \neq 0, \quad \varphi = \varphi(r). \quad (7)$$

Equations (5) become:

$$\begin{aligned}
& \left[\frac{2\varphi(r)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \frac{d}{d\theta} \left\{ r' \left[\frac{2\varphi(r)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \right\} \\
&= \frac{2\varphi(r)}{r^2 + (r')^2 + (z')^2} - \left(\frac{e}{m} \right)^{\frac{1}{2}} \left[\frac{2\varphi(r)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} r \\
&\quad \cdot H + \frac{\partial \varphi(r)}{\partial r}, \quad (8a)
\end{aligned}$$

$$\left[\frac{2\varphi(r)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \frac{d}{d\theta} \left\{ z' \left[\frac{2\varphi(r)}{r^2 + (r')^2 + (z')^2} \right]^{\frac{1}{2}} \right\} = 0. \quad (8b)$$

From Eq. (8b) we can see that z' is identically equal to zero if it was equal to zero for $\theta=0$, i.e., electron paths remain in the plane $z=\text{const.}$ if their initial slope z' was zero. If we therefore

exclude all rays whose initial slope z' is different from zero, we need consider only Eq. (8a) which can be reduced to:

$$\begin{aligned}
& \frac{d}{d\theta} \left\{ r' \left[\frac{2\varphi(r)}{r^2 + (r')^2} \right]^{\frac{1}{2}} \right\} = \left[\frac{2\varphi(r)}{r^2 + (r')^2} \right]^{\frac{1}{2}} r \\
&\quad - \left(\frac{e}{m} \right)^{\frac{1}{2}} r \cdot H + \left[\frac{r^2 + (r')^2}{2\varphi(r)} \right]^{\frac{1}{2}} \frac{d\varphi(r)}{dr}. \quad (9)
\end{aligned}$$

The electrostatic field $\varphi(r)$ shall now be that of a cylinder condenser which we can write in the form

$$\varphi(r) = \phi_0 + [(V_2 - V_1)/(\ln R_2/R_1)] \ln(r/r_0) \quad (10)$$

where $V_2 - V_1$ is the potential difference between the two coaxial cylinders of radii R_1 and R_2 . ϕ_0 is the superimposed constant potential. We therefore have:

$$\varphi(r_0) = \phi_0.$$

The condition for a path to be circular is:

$$r' \equiv 0.$$

Hence the radius of the circular path is given by:

$$r_0 = \frac{(V_2 - V_1)/(\ln R_2/R_1) + 2\phi_0}{(2e\phi_0/m)^{\frac{1}{2}}} \cdot \frac{1}{H}, \quad (11)$$

with

$$v_0 = (2e\phi_0/m)^{\frac{1}{2}}, \text{ initial velocity of the particle,}$$

and

$$\omega_0 = v_0/r_0, \text{ angular velocity of the particle,}$$

we get:

$$ev_0 \cdot H = e \frac{V_2 - V_1}{\ln R_2/R_1} \cdot \frac{1}{r_0} + m \frac{v_0^2}{r_0} = e \frac{d\varphi}{dr} + m \frac{v_0^2}{r_0}.$$

This is Newton's force equation for a particle moving on a circular path through the combined electrostatic and magnetic field.

It has been shown by W. Henneberg that the final formulae are simple if expressed in terms of a quantity y defined by:

$$y = -(V_2 - V_1)/(2\phi_0 \ln R_2/R_1), \quad (12)$$

or

$$y = -[e(V_2 - V_1)(1/r_0)/(\ln R_2/R_1)]/m(v_0^2/r_0).$$

y is therefore that fraction of the centrifugal force on the circular path which is compensated

by the electrostatic force. We have $y=0$ for a purely magnetic field and $y=1$ for a purely electrostatic field.

Equation (9) can only be solved by methods of approximation. In order to achieve this, we need a number of expansions of expressions occurring in Eq. (9). With

$$r=r_0+x, \text{ or } r/r_0=1+x/r_0=1+z, \quad (13)$$

and Eqs. (11) and (12) we get:

$$\begin{aligned} (2\varphi)^{\frac{1}{2}} &= (2\phi_0)^{\frac{1}{2}}(1-yz+\frac{1}{2}y(1-y)z^2 \\ &\quad -\frac{1}{6}y(2-3y+3y^2)z^3+\dots), \\ 2\varphi^{-\frac{1}{2}} &= (2\phi_0)^{-\frac{1}{2}}(1+yz-\frac{1}{2}y(1-3y)z^2 \\ &\quad +\frac{1}{6}y(2-9y+15y^2)z^3+\dots), \\ (r^2+(r')^2)^{\frac{1}{2}} &= r_0(1+z+\frac{1}{2}(z')^2-\frac{1}{2}z(z')^2+\dots), \\ (r^2+(r')^2)^{-\frac{1}{2}} &= (1/r_0)(1-z+z^2-\frac{1}{2}(z')^2 \\ &\quad +\frac{3}{2}z(z')^2-z^3+\dots), \\ d\varphi/dr &= -(1/r_0)2\phi_0y(1-z+z^2-z^3)+\dots, \\ H \cdot r_0 &= (2\phi_0m/e)^{\frac{1}{2}}(1-y). \end{aligned} \quad (14)$$

If we exclude all third and higher order terms in z and z' Eq. (9) becomes:

$$\begin{aligned} [d/d\theta][z'-(1-y)zz'] &= -[z+y^2z \\ &\quad +\frac{1}{2}y(1-3y^2)z^2+\frac{1}{2}(1-y)(z')^2]. \end{aligned} \quad (15)$$

The method which is typical to electron optical investigations proceeds now as follows. The first step towards a solution of Eq. (15) will lead to a path function fundamental to the Gaussian dioptrics of the system by taking into account only first order terms in Eq. (15). After this has been achieved, the effect of second-order terms will be investigated.

(2) THE GAUSSIAN DIOPTICS

We assume that:

$$z \ll 1, \quad z' \ll 1, \quad (16)$$

so that $z^2, zz', (z')^2$ can be neglected compared with z and z' . We then get the following so-called paraxial equation:

$$[d/d\theta][z'_\theta] = -[1+y^2]z_\theta. \quad (17)$$

The index g shall indicate that the solution is

correct only if the conditions (16) are satisfied, i.e., within the limits of the Gaussian dioptrics.

The solution of Eq. (17) is well known; it is

$$z_\theta = a \sin(1+y^2)^{\frac{1}{2}}\theta + b \cos(1+y^2)^{\frac{1}{2}}\theta, \quad (18)$$

where a and b are two constants of integration. If $r=r_0$ for $\theta=0$, we have with (13):

$$r = r_0 + ar_0 \sin(1+y^2)^{\frac{1}{2}}\theta. \quad (19)$$

We therefore see that all (paraxial) electron paths leaving the point $\theta=0, r=r_0$ pass again through the (circular) axis ($r=r_0$) at:

$$\theta = \pi/(1+y^2)^{\frac{1}{2}}. \quad (20)$$

For a purely magnetic field we have $y=0$ and hence

$$\theta = \pi = 180^\circ.$$

For a purely electrostatic field we have $y=1$ and hence

$$\theta = \pi/\sqrt{2} = 127^\circ 17'.$$

In order to derive the expressions for the optical quantities of the system, we make use of the general results of the theory developed by W. Glaser.⁷

If the field has a finite extent along the optical axis and if the paraxial ray equation is a linear differential equation of the second order, the optical quantities can be expressed in terms of two linearly independent solutions of the ray equation. The shape of the field is shown in Fig. 1. Let $z_{\theta 1}$ and $z_{\theta 2}$ be two linearly independent solutions of Eq. (17) which satisfy the following conditions:

$$\begin{aligned} z_{\theta 1}(\theta_a) &= 1, \quad z'_{\theta 1}(\theta_a) = 0, \\ z_{\theta 2}(\theta_a) &= 0, \quad z'_{\theta 2}(\theta_a) = 1. \end{aligned} \quad (21)$$

Such a pair of solutions is:

$$\begin{aligned} z_{\theta 1} &= \cos(1+y^2)^{\frac{1}{2}}(\theta - \theta_a), \\ z_{\theta 2} &= (1+y^2)^{-\frac{1}{2}} \sin(1+y^2)^{\frac{1}{2}}(\theta - \theta_a). \end{aligned} \quad (22)$$

We then obtain the expressions for the optical quantities by substituting the functions (22) into Glaser's formulae.

⁷ W. Glaser, Zeits. f. Physik **80**, 451 (1933); Zeits. f. Physik **81**, 649 (1933); Zeits. f. Physik **83**, 104 (1933); Zeits. f. Physik **97**, 177 (1935).

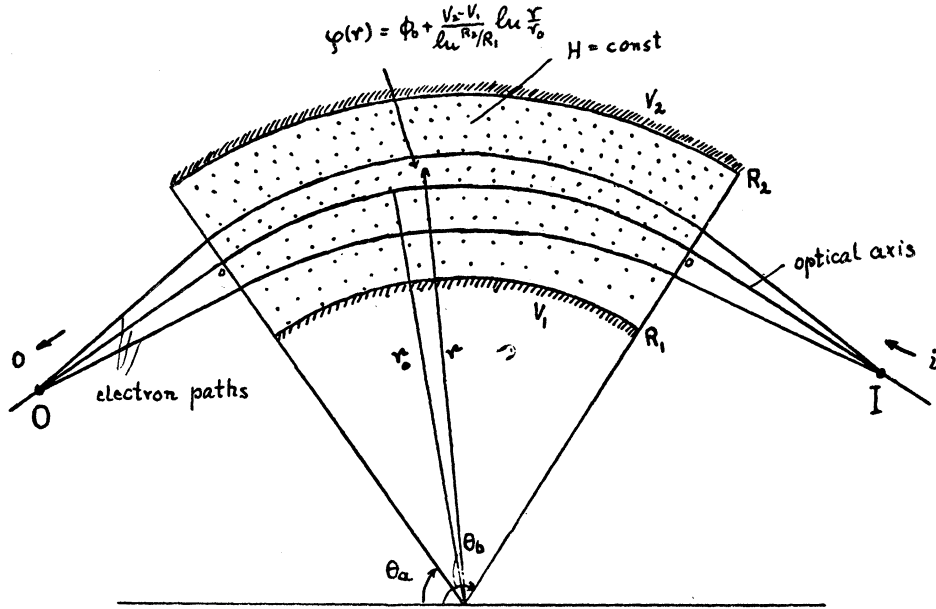


FIG. 1. The geometry of the fields.

Focal lengths:

$$f_0 = \frac{z'_{o1}(\theta_b)z_{o2}(\theta_b) - z_{o1}(\theta_b)z'_{o2}(\theta_b)}{z'_{o1}(\theta_b)} r_0$$

$$= \frac{r_0}{(1+y^2)^{\frac{1}{2}} \sin (1+y^2)^{\frac{1}{2}}(\theta_b - \theta_a)}, \quad (23)$$

$$f_i = -\frac{1}{z'_{o1}(\theta_b)} r_0$$

$$= \frac{r_0}{(1+y^2)^{\frac{1}{2}} \sin (1+y^2)^{\frac{1}{2}}(\theta_b - \theta_a)}.$$

Focal points:

$$F_0 = -\frac{z'_{o2}(\theta_b)}{z'_{o1}(\theta_b)} r_0 = -\frac{r_0}{(1+y^2)^{\frac{1}{2}}} \cot (1+y^2)^{\frac{1}{2}}(\theta_b - \theta_a), \quad (24)$$

$$F_i = -\frac{z_{o1}(\theta_b)}{z'_{o1}(\theta_b)} r_0 = -\frac{r_0}{(1+y^2)^{\frac{1}{2}}} \cot (1+y^2)^{\frac{1}{2}}(\theta_b - \theta_a).$$

Principal points:

$$H_0 = -f_0 + F_0 = -\frac{r_0}{(1+y^2)^{\frac{1}{2}}} \tan \frac{1}{2}(1+y^2)^{\frac{1}{2}}(\theta_b - \theta_a), \quad (25)$$

$$H_i = -f_i + F_i = -\frac{r_0}{(1+y^2)^{\frac{1}{2}}} \tan \frac{1}{2}(1+y^2)^{\frac{1}{2}}(\theta_b - \theta_a).$$

Object-image relation:

$$(i-H) \cdot (0-H) = f^2. \quad (26)$$

The system of coordinates is chosen so that the origins are the boundaries of the field θ_a and θ_b . In both regions the direction from the left to the right is defined as the positive direction.

(3) SECOND-ORDER ABERRATION

Taking now into account the second-order terms in Eq. (15), we can determine the line width which is the actual image of a mathematical line of the object space. To obtain this line width, we proceed in a manner typical to electron optics. We replace in Eq. (15) z and z' in the second-order terms by z_o and z'_o since these terms will differ very little from the actual ones. We therefore have:

$$[d/d\theta][z'] + [1+y^2]z$$

$$= (1-y)(d/d\theta)(z_o z'_o) - \frac{1}{2}y(1-3y^2)z_o^2$$

$$+ \frac{1}{2}(1-y)(z'_o)^2. \quad (27)$$

The general solution of this equation is the sum of the general solution of Eq. (17) and a particular solution of Eq. (27). With the expression (22) Eq. (27) becomes:

$$z'' + A \cdot z = C_1 + C_2 \cos m\theta + C_3 \sin m\theta. \quad (28)$$

A particular solution is given by:

$$z_p = (C_1/A) + [C_2/(A-m^2)] \cos m\theta + [C_3/(A-m^2)] \sin m\theta, \quad (29)$$

where

$$\begin{aligned} A &= 1+y^2, \\ C_1 &= -\frac{1}{4}(a^2+b^2)(1+y^2-4y^3), \\ C_2 &= \frac{1}{4}(a^2-b^2)(3-2y+3y^2-6y^3), \\ C_3 &= \frac{1}{2}ab(3-2y+3y^2-6y^3), \\ m &= 2(1+y^2)^{\frac{1}{2}}. \end{aligned} \quad (30)$$

The general solution of Eq. (27) is therefore:

$$\begin{aligned} z &= a \sin (1+y^2)^{\frac{1}{2}}\theta + b \cos (1+y^2)^{\frac{1}{2}}\theta \\ &\quad - \frac{1}{4}(a^2+b^2) \frac{1+y^2-4y^3}{(1+y^2)} \\ &\quad - \frac{1}{12}(a^2-b^2) \frac{3-2y+3y^2-6y^3}{(1+y^2)} \cos 2(1+y^2)^{\frac{1}{2}}\theta \\ &\quad - \frac{1}{6}ab \frac{3-2y+3y^2-6y^3}{(1+y^2)} \sin 2(1+y^2)^{\frac{1}{2}}\theta. \end{aligned} \quad (31)$$

The initial conditions for z shall now be:

$$\begin{aligned} z(0) &= b - \frac{1}{4}(a^2+b^2) \frac{1+y^2-4y^3}{1+y^2} \\ &\quad - \frac{1}{12}(a^2-b^2) \frac{3-2y+3y^2-6y^3}{1+y^2} = 0, \\ z'(0) &= a \left(1 - \frac{1}{2}b \frac{3-2y+3y^2-6y^3}{1+y^2} \right) (1+y^2)^{\frac{1}{2}} = \alpha. \end{aligned} \quad (32)$$

At $\theta = \pi/(1+y^2)^{\frac{1}{2}}$ we then have:

$$\begin{aligned} z \left[\frac{\pi}{(1+y^2)^{\frac{1}{2}}} \right] &= -b - \frac{1}{4}(a^2+b^2) \frac{1+y^2-4y^3}{1+y^2} \\ &\quad - \frac{1}{12}(a^2+b^2) \frac{3-2y+3y^2-6y^3}{1+y^2}, \end{aligned} \quad (33)$$

or with Eq. (32)

$$z[\pi/(1+y^2)^{\frac{1}{2}}] = -2b. \quad (34)$$

Since b will be proportional with α^2 we have:

$$b = C\alpha^2. \quad (35)$$

If we neglect now all terms higher than α^2 in (31) we get:

$$C = 1 + \frac{1}{3}y + y^2 + 3y^3/(1+y^2)^2, \quad (36)$$

hence

$$z \left[\frac{\pi}{(1+y^2)^{\frac{1}{2}}} \right] = -\alpha^2 \frac{1 + \frac{1}{3}y + y^2 + 3y^3}{(1+y^2)^2}, \quad (37)$$

the line width will then be

$$L = -\alpha^2 \frac{1 + \frac{1}{3}y + y^2 + 3y^3}{(1+y^2)^2} r_0. \quad (38)$$

In case of a purely magnetic field ($y=0$)

$$L_m = -\alpha^2 r_0.$$

For an electrostatic field ($y=1$)

$$L_e = -(4/3)\alpha^2 r_0.$$

(4) CHROMATIC ABERRATION

If the wedge of electrons entering the field is not homogeneous with respect to the velocity, the image line will again be of a finite width. In order to obtain an expression for this width we let:

$$\varphi(r) = \phi_0 + \Delta\phi_0 + [(V_2 - V_1)/(\ln R_2/R_1)]/\ln r/r_0, \quad (39)$$

where

$$|\Delta\phi_0/\phi_0| \ll 1. \quad (40)$$

The solution of Eq. (9) will be assumed to be of the form:

$$r = r_0 + x + \delta$$

or

$$\frac{r}{r_0} = 1 + \frac{x+\delta}{r_0} = 1 + z + \sigma = 1 + z_1. \quad (41)$$

We expand again the expressions in Eq. (9) in power series of z_1 and z_1' similarly to those of Eq. (14). The differential equation for z_1 then becomes:

$$\begin{aligned} \frac{d}{d\theta} \left[z_1' \left(1 + \frac{1}{2} \frac{\Delta\phi_0}{\phi_0} \right) \right] &= \frac{1}{2} \frac{\Delta\phi_0}{\phi_0} (1+y) \\ &\quad - z_1(1+y^2) - \frac{1}{2} \frac{\Delta\phi_0}{\phi_0} y z_1(1+3y). \end{aligned} \quad (42)$$

If the particle travels initially perpendicularly to the field lines and would describe the circular path of radius r_0 ($\Delta\phi_0=0$) we have $z=0$. If

$\Delta\phi_0 \neq 0$ the deviation from the circular path is therefore given by the following differential equation:

$$\sigma'' + \sigma(1+y^2) = \frac{1}{2}(1+y)\Delta\phi_0/\phi_0. \quad (43)$$

The general solution of this equation is given by:

$$\sigma = a \sin(1+y^2)^{1/2}\theta + b \cos(1+y^2)^{1/2}\theta + \frac{1}{2}(1+y)\Delta\phi_0/\phi_0. \quad (44)$$

With the initial conditions:

$$\sigma = \sigma' = 0 \text{ for } \theta = 0, \quad (45)$$

we get:

$$a = 0, \quad b = -\frac{1}{2} \frac{(1+y)\Delta\phi_0}{(1+y^2)^{1/2}\phi_0}. \quad (46)$$

At $\theta = \pi/(1+y^2)^{1/2}$ we therefore have:

$$\sigma \left[\frac{\pi}{(1+y^2)^{1/2}} \right] = \frac{1+y}{1+y^2} \frac{\Delta\phi_0}{\phi_0}. \quad (47)$$

Since $(2e\phi_0/m)^{1/2} = v_0$ we have:

$$\frac{d\phi_0}{\phi_0} = 2 \frac{dv_0}{v_0}, \quad (48)$$

and since $\delta = \sigma \cdot r_0$, we get for the line width D :

$$D = 2 \frac{1+y}{(1+y^2)} r_0 \frac{dv_0}{v_0}. \quad (49)$$

Since the velocity resolution is only noticeable if it causes the line width to be greater than that due to second-order aberrations, a quantity τ has been introduced called the reduced resolution. It is defined by:

$$\tau = |D/L| \alpha^2. \quad (50)$$

We get for τ :

$$\tau = 2 \left| \frac{1+y+y^2+y^3}{1+\frac{1}{2}y+y^2+3y^3} \right|. \quad (51)$$

On Permanent Charges in Solid Dielectrics

I. Dielectric Absorption and Temperature Effects in Carnauba Wax

B. GROSS AND L. FERREIRA DENARD

National Institute of Technology, Rio de Janeiro, Brazil

(Received February 6, 1945)

The influence of the temperature upon dielectric absorption is studied for carnauba wax. Isothermic and non-isothermic current-time curves are measured. It is shown that a considerable part of the absorbed charge can be "frozen in," if the temperature is reduced to a value sufficiently inferior to that prevailing during the charging period before the system is short-circuited. The "frozen" charge dissipates extremely slowly, if the temperature is kept low, but it is liberated rapidly if the temperature is raised again. The effect is explained by the increase of the charging and discharging rates with increasing temperature. It is closely related to the permanent moment of the electret.

I. INTRODUCTION

AN electret is a dielectric body that retains an electric moment after the externally applied electric field has been reduced to 0. It was shown by Eguchi¹ that permanent electrets are obtained when certain waxes and resinous materials, mainly carnauba wax, previously melted, are allowed to solidify while exposed to strong electric fields. Eguchi's results

were considered very startling ones because for a long time it was the general opinion that any self-maintained electric field in solid bodies would soon be neutralized in consequence of the movement of the free electric charges existing in all substances. But these results have been confirmed and completed by various authors.² While the behavior of the electret is now fairly well known, no generally accepted explanation of its

¹ M. Eguchi, *Phil. Mag.* **49**, 178 (1925).

² A. Gemant, *Phil. Mag.* **20**, 929 (1935); W. M. Good and J. D. Stranathan, *Phys. Rev.* **56**, 810 (1939).