Letters to the Editor

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The Electron's Self-Energy

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IN a preceding letter, we have shown the possibility of building a satisfactory classical theory of a point-electron based on two principles:

(a) The total field created by a point charge results from the superposition of an attached field and a radiated one.

(b) Only the radiated field reacts on the emitting charge.

For the radiated field we had:

$$F^{\mu\nu}_{\text{rad}} = \tfrac{1}{2}(F^{\mu\nu}_{\text{ret}} - F^{\mu\nu}_{\text{adv}}). \tag{1}$$

Now we are going to indicate how the assumptions (a) and (b), together with Eq. (1), afford the basis for a satisfactory quantum theory without any self-energies or zero-point energy. Let us expand the potentials $\mathbf{A}_{\text{ret}}$ and ϕ_{ret} in Fourier series, using notations similar to those of Heitler's *Quantum Theory of Radiation* (Oxford, 1936):

$$\left.\begin{aligned} \mathbf{A}_{\text{ret}} &= \sum_{\sigma}(q_\sigma \mathbf{A}_\sigma + q_\sigma^* \mathbf{A}_\sigma^*) + \sum_{\lambda}(q_\lambda \mathbf{A}_\lambda + q_\lambda^* \mathbf{A}_\lambda^*), \\ \phi_{\text{ret}} &= \sum_{\sigma}(a_\sigma \phi_\sigma + a_\sigma^* \phi_\sigma^*). \end{aligned}\right\} \tag{2}$$

For the advanced potentials we have the expansions:

$$\left.\begin{aligned} A_{\text{adv}} &= i\sum_{\sigma}(\bar{q}_\sigma \mathbf{A}_\sigma - \bar{q}_\sigma^* \mathbf{A}_\sigma^*) + i\sum_{\lambda}(\bar{q}_\lambda \mathbf{A}_\lambda - \bar{q}_\lambda^* \mathbf{A}_\lambda^*), \\ \phi_{\text{ret}} &= i\sum_{\sigma}(\bar{a}_\sigma \phi_\sigma - \bar{a}_\sigma^* \phi_\sigma^*). \end{aligned}\right\} \tag{3}$$

The $(q_\sigma, q_\sigma^*; q_\lambda, q_\lambda^*; a_\sigma, a_\sigma^*)$ and $(\bar{q}_\sigma, \bar{q}_\sigma^*; \bar{q}_\lambda, \bar{q}_\lambda^*; \bar{a}_\sigma, \bar{a}_\sigma^*)$ are our field variables. We must find an hamiltonian $H = H_{\text{el}} + H_{\text{field}}$ such that the resulting Hamilton equations be equivalent to the Dirac wave equation for the electron and the inhomogeneous d'Alembert equations for the potentials. It is of capital importance that both the retarded and advanced potentials have the *same equations of motion*. From this it results that H_{field} is the sum of the energies of the various waves arising from $\mathbf{A}_{\text{ret}}$ and ϕ_{ret} *minus the energies of the waves* arising from $\mathbf{A}_{\text{adv}}$, and ϕ_{adv}, because the advanced and retarded potentials appear *with opposite signs* in H_{el}:

$$H_{\text{el}} = c\left(\mathbf{p} - \frac{e}{c}\mathbf{A}_{\text{rad}}\right)\cdot \boldsymbol{\alpha} + \beta mc^2 + e\phi_{\text{rad}}. \tag{4}$$

The contributions of the advanced field behave as negative energy photons would do and the respective values of the frequencies in the matrix elements are negative, because of the factor i in expansions (3).

The longitudinal terms in H disappear when we apply Fermi's method of replacing them by Coulomb energies. The transverse self-energy results equal zero and the zero-point energy vanishes, because there are field oscillators with both positive and negative energies.

Similar considerations can be developed both in the meson and the positron theories, allowing one to eliminate difficulties, as will be shown elsewhere.

Part of the results of this letter were discussed at the meeting of the Brazilian Academy of Sciences, December 12, 1944.

High Energy Neutron-Proton Scattering and the Saturation Problem

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THE well-known property of heavy atomic nuclei called saturation—i.e., the fact that nuclear volume and binding energy are roughly proportional to the number of particles present, the mass number—is one of the most important empiric foundations of the theory of nuclear forces. Ever since the primary work of Heisenberg in 1932,[1] it has been regarded almost as an axiom that the saturation is to be explained as a pure exchange phenomenon analogous to the homopolar chemical binding. From a theoretical point of view this is a simple and attractive explanation, but it is very difficult to reconcile with the recent experiments on the angular distribution of fast neutrons scattered by protons.[2,3]

To illustrate this difficulty, let us consider the theory of Møller and Rosenfeld,[4] where the above-mentioned exchange character is particularly prominent. Neglecting the non-static effects which are of secondary importance in this connection, we find the interaction between two nucleons '1' and '2' can, according to the MR-theory, be written (cf. Ref. 4, p. 38):

$$V_{\text{MR}}(r) = (\tau^{(1)}\tau^{(2)})(g_1^2 + g_2^2(\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}))\frac{e^{-xr}}{4\pi r}.$$

From the Pauli principle (and the fact that the force is attractive in a ^1S-state) follows that this force is *repulsive* in all states of *odd* parity, the average of $V_{\text{MR}}(r)$ over the different possible spin and charge states being O. It is well-known that just this property of the force leads to saturation.

But this characteristic feature, that the force is attractive in S, D··· states but repulsive in P, F··· states also leads to a preferentially *backward* scattering of neutrons by protons, i.e., the intensity of the scattered neutrons (referred to the mass-center system of neutron and proton) has a maximum in a direction making 180° with the direction of incidence. It has already been pointed out that this theoretical prediction is inconsistent with the recent experiments.[6]

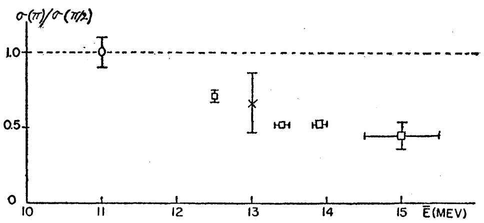

FIG. 1. Intensity ratio $\sigma(\pi)/\sigma(\pi/2)$ (=int. (180°)/int. (90°) in the mass-center system) for various high energies ($\bar{E}$=average energy of the incident neutrons in the laboratory system), according to different measurements; 0—Tatel, ×—Champion and Powell, □—Amaldi, Bocciarelli, Ferretti, and Trabacchi.

It may be of interest to note that the discrepancy becomes less striking if we turn to a pure pseudoscalar symmetrical theory. The interaction of dipole type ('tensor force'), which is of particular importance in this theory, has an angular effect opposite to that of the central force, and therefore the angular distribution is more uniform than in the MR-theory where the terms of dipole interaction type are entirely eliminated.[6] Thus, comparing different symmetrical theories, the pseudoscalar one seems to yield the least pronounced backward scattering. But the calculations give no ground for a hope that this theory might lead to an angular distribution with $\sigma(\pi)/\sigma(\pi/2)<1$—although the dipole interaction is so dominant as to make it doubtful, according to Volkoff,[7] if such a theory leads to saturation at all.

Recent experiments on the scattering of high energy neutrons by protons indicate a fairly marked forward scattering, as appears from the diagram where the intensity ratio $\sigma(\pi)/\sigma(\pi/2)$ is plotted against the energy of the incident neutrons. (Tatel[8] measured the ratio for 148° and 90°, but the difference is immaterial.) The errors of $\sigma(\pi)/\sigma(\pi/2)$ are those assigned by the several authors themselves. For low energies theory and experiment both give a practically uniform distribution.

It should also be pointed out that these results seem to be qualitatively supported by the experiments of Barschall and Ladenburg on the scattering of 2.5 Mev neutrons by some heavier nuclei.[9] The manifest forward elastic scattering found in this case would be hard to understand theoretically, if the neutron-proton scattering were not of the same type.

Provided the experimental results concerning the neutron-proton scattering are correct, it seems necessary to re-examine the problem of the causes of the saturation and the charge-dependence of the nuclear forces. In this connection it should be mentioned that the non-symmetrical scalar-pseudoscalar theory[10] affords a rather simple possibility for reconciling the saturation properties of heavy nuclei with a neutron-proton scattering of the type indicated by the experiments.[11] At the same time the theory is able to account for the approximate equality of the neutron-proton and proton-proton forces in a ^1S-state. In this case the saturation is explained as a consequence of the repulsive forces with very short range which appear in the higher approximations of the perturbation theory.

In any case it is evident that the experiments in question are of fundamental importance for the further development of the theory, as has already been emphasized by Rarita and Schwinger.[6]

[1] W. Heisenberg, Zeits. f. Physik **77**, 1 (1932).
[2] E. Amaldi, D. Bocciarelli, B. Ferretti, G. C. Trabacchi, Naturwiss. **30**, 582 (1942); Ricerca Sci. **13**, 502 (1942).
[3] F. C. Champion, C. F. Powell, Proc. Roy. Soc. **A183**, 64 (1944).
[4] C. Møller, L. Rosenfeld, Danske Vidensk. Selsk. math.-fys. Medd. **17**, No. 8 (1940).
[5] L. Hulthén, Phys. Rev. **63**, 383 (1943). Formula (1) in this note should read:

$$I(\theta)=1-0.18\cos\theta+0.45\cos^2\theta-0.22\cos^3\theta+0.12\cos^4\theta;$$

thus $I(\pi)/I(\pi/2)=1.97$. A later calculation (Arkiv f. mat. astr. fysik **29A**, No. 33 (1943)) has shown that even F-states must be taken into account: for 14.5 Mev neutrons we then have $I(\pi)/I(\pi/2)\cong 2.2$.
[6] Arkiv **31A**, No. 15 (1944). Cf. also W. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941).
[7] G. M. Volkoff, Phys. Rev. **62**, 134 (1942). It is, however, possible that this difficulty will disappear if the interactions of higher order in the perturbation theory are taken into account; cf. a remark on the non-symmetrical theory above.
[8] H. Tatel, Phys. Rev. **61**, 450 (1942).
[9] H. H. Barschall, R. Ladenburg, Phys. Rev. **61**, 129 (1942).
[10] L. Hulthén, K. Fysiograf. Sällsk. Lund Förhandl. **14**, Nr 2 (1944).
[11] Arkiv **31A**, No. 15 (1944).

Range of Light Variation in Cepheid Variables

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THE beautiful investigation of Bethe[1] has shown that the energy output of the main sequence stars is satisfactorily accounted for in terms of the C-N cycle of nuclear reactions, whereby hydrogen is transformed into helium, the carbon-nitrogen content remaining unchanged. In the case of the giant stars, however, the observed luminosity is far greater than the computed value on the basis of the C-N cycle, and to explain the observed values, recourse must be had to protonic reactions with H^2, Li^6, Li^7, Be^9, B^{10}, B^{11}—unlike the C-N cycle in these reactions, the nuclei are consumed—which can take place rapidly enough at the comparatively low temperatures prevailing in the interiors of these stars. Gamow and Teller,[2] and also Greenfield,[3] have shown that the above reactions account broadly for the luminosities of the giants. It seems that the short-period group of cluster variables "live" probably on the B^{10} reaction; the Cepheid variables "live" on Li, Be, B^{11}; and the long-period variables, on H^2.

The present note is concerned with the range of light variation and its relation with the amplitude of pulsation. As the star pulsates, the temperature and density inside it undergo a periodic variation, and the rate of energy evolution, due to nuclear reactions at any stage of the pulsation, can be readily computed. If L_1 and L_2 denote the maximum and minimum rates of energy generation for a star of mass M and radius R, then, we find for the reaction $Li^7+H^1=2He^4$, using Eddington's solution[4] for the equation of pulsation,

$$\Delta m\equiv 2.5\log_{10}L_1/L_2\sim\frac{\Delta R}{R}\Big[0.25\{15-10(r'-1)\}+1.19\times 10^9(r'-1)\Big(\frac{R}{M\beta\mu}\Big)^{\frac{1}{3}}\Big], \quad (1)$$

where the radius R oscillates between the limits $R+\Delta R$ and $R-\Delta R$; i.e., ΔR is the amplitude of pulsation; μ is