

It follows⁵ that the infinite hypergeometric series oscillates finitely. Abel's theorem⁶ then shows that the hypergeometric function in (12) remains bounded as $z \rightarrow 1$ or $\alpha \rightarrow \infty$.

This establishes the existence of a continuous eigenvalue spectrum of all positive λ or

$$E > \hbar^2/2\mu R^2. \quad (13)$$

In order to obtain an indication of how a relativistic treatment might affect this problem, the authors repeated the calculations by use of the Gordon-Klein equation. The general character of the solution was found to be unchanged.

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¹ E. Schrödinger, Proc. Roy. Irish Acad. **A46**, 9 (1940). Also, L. Infeld, Phys. Rev. **59**, 737 (1941) and A. F. Stevenson, Phys. Rev. **59**, 842 (1941).

² C. Gilbert, Quart. J. Math. **9**, 187, Eq. (9) (1938), and A. G. Walker, M. N. R. A. S. **95**, 263 (1935).

³ E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge, 1940), Secs. 10.7, 10.71, 10.72 and Chaps. 14.

⁴ T. J. I. Bromwich, *An Introduction to the Theory of Infinite Series* (The Macmillan Company, New York, 1926), Sec. 79, p. 241, example.

⁵ Reference 4, Sec. 79, p. 241, rule (ii).

⁶ Reference 4, Sec. 51, p. 149.

Classical Theory of the Point Electron

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THE field produced by a point charge can be split in two parts:

(1) The attached field, $F_{at}^{\mu\nu}$;

(2) The radiated field, $F_{rad}^{\mu\nu}$.

We assume the total field to be the sum of the attached, the radiated and the external fields and that the sum of the attached and radiated fields gives the usual retarded field $F_{ret}^{\mu\nu}$ of the particle.

The radiated field behaves as an external field and reacts on the emitting charge. The attached field does not act on the generating charge but acts on other charges. Therefore, each charge is under the action of the retarded fields of the others, but only under the radiated part of its own field.

The particle of charge e and mass m has two kinds of momentum and energy: (1) the kinetic four vector

$$m \frac{dx^\mu}{ds} = G_{kin}^\mu,$$

and (2) the four vector of the acceleration energy and momentum

$$G_{ac}^\mu = -\frac{2}{3}e^2 \frac{d^2x^\mu}{ds^2}.$$

The four vector G_{ac}^μ arises from the interaction of the charge and its radiated field. Since the particle is under the action of the external field and its own radiated field,

the equations of motion ought to be:

$$m \frac{d^2x^\mu}{ds^2} = \frac{e}{c} \frac{dx^\nu}{ds} (F_{ext,\nu}^\mu + F_{rad,\nu}^\mu). \quad (1)$$

If we apply the laws of conservation of energy and momentum we get:

$$\frac{d}{ds} (G_{kin}^\mu + G_{ac}^\mu) - \frac{2}{3}e^2 \frac{dx^\mu}{ds} \frac{d^2x^\rho}{ds^2} \frac{d^2x_\rho}{ds^2} = \frac{e}{c} F_{ext,\nu}^\mu \frac{dx^\nu}{ds}, \quad (2)$$

because the rate of loss of energy and momentum is

$$\frac{2}{3}e^2 \frac{dx^\mu}{ds} \frac{d^2x^\rho}{ds^2} \frac{d^2x_\rho}{ds^2}$$

according to the well-known Larmor formulae. Comparing Eqs. (1) and (2) we see that:

$$\frac{e}{c} F_{rad,\nu}^\mu \frac{dx^\nu}{ds} = -\frac{dG_{ac}^\mu}{ds} + \frac{2}{3}e^2 \frac{dx^\mu}{ds} \frac{d^2x^\rho}{ds^2} \frac{d^2x_\rho}{ds^2}. \quad (3)$$

Equation (3) is satisfied by taking:

$$F_{rad}^{\mu\nu} = \frac{1}{2} (F_{ret}^{\mu\nu} - F_{adv}^{\mu\nu}). \quad (4)$$

From Eq. (4) it results that:

$$F_{at}^{\mu\nu} = \frac{1}{2} (F_{ret}^{\mu\nu} + F_{adv}^{\mu\nu}). \quad (5)$$

We define the stress-tensor $T^{\mu\nu}$ of the field in which there are n point charges in the following way:

$$4\pi T_{\nu}^{\mu} = F^{\mu\rho} F_{\rho\nu} + \frac{1}{4} \delta_{\nu}^{\mu} (F^{\rho\sigma} F_{\rho\sigma})$$

$$- \sum_i F_{i,at}^{\mu\rho} F_{i,at;\rho\nu} - \frac{1}{4} \delta_{\nu}^{\mu} (\sum_i F_{i,at}^{\rho\sigma} F_{i,at;\rho\sigma}). \quad (6)$$

Equation (6) ensures the conservation of the energy and momentum of the system of the field and particles, and leads to a finite field energy which is not always positive. The energy and momentum of the field derived from the stress tensor $T^{\mu\nu}$ are the components of a four vector. A detailed exposition of the theory will be given elsewhere.

Our theory leads to the Lorentz-Dirac classical equations of motion and to a finite field energy without introducing any energies and momenta of non-electromagnetic nature besides the kinetic ones. There are no subtractions of infinite quantities.

The Radiation Field of a Point Electron

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THE radiation field of the electron has been defined in different ways. The most usual is to take it as the part of the retarded field of the charge that depends on the retarded acceleration and varies inversely with the retarded distance to the charge. Though this definition presents many obvious inconveniences, it has the advantage of giving the right amount of the radiation emission. Dirac¹ has proposed the following definition of the radiation field of a point charge:

$$F_{rad,D}^{\mu\nu} = F_{ret}^{\mu\nu} - F_{adv}^{\mu\nu}, \quad (1)$$