Letters to the Editor

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The Shape of Betatron Pole Faces

James H. Bartlett
Department of Physics, University of Illinois, Urbana, Illinois
August 25, 1943

FOR there to be focusing of the electron beam in the betatron, it is necessary that the magnetic field near the beam fulfill certain requirements.[1] In practice, these have been met by selecting a certain shape of pole face, determining the field by approximate methods such as flux plotting, and then modifying the original shape if it was not a suitable one. It is the purpose of this note to point out that the pole face shape may be determined in a more direct manner.

As in the article of Kerst and Serber, we use cylindrical coordinates, and $z=0$ describes the median plane. In empty space, curl $H=0$ and div $H=0$, so that $H=-\text{grad }\varphi_m$, where φ_m is the magnetostatic potential, and $\nabla^2\varphi_m=0$. We shall assume that $H_z(r)$ at $z=0$ is a known function, which is such that it satisfies the focusing conditions. A surface of constant magnetostatic potential will be, to a high degree of approximation, an acceptable boundary for the magnet. The problem thus reduces to the calculation of φ_m subject to the condition at $z=0$. In what follows, we drop the subscript m.

We require the solution of:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{\partial^2\varphi}{\partial z^2}=0.$$

Expand φ as a power series in z, so that:

$$\varphi=\sum_{n=0}^{\infty}\varphi_n(r)z^n. \tag{1}$$

Substituting in the differential equation, one obtains the recursion formula:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi_n}{\partial r}\right)+(n+1)(n+2)\varphi_{n+2}=0. \tag{2}$$

Now:

$$H_z=-(\partial\varphi/\partial z)=-\sum n\,\varphi_n z^{n-1}.$$

Therefore, $(H_z)_{z=0}=-\varphi_1$. The potential φ_0 is arbitrary and may therefore be set equal to zero. The solution (1) of the problem is thus obtained by differentiating expressions involving φ_1 [as prescribed by (2)], and will be an odd function of z.

If the series (1) should prove to be valid in too narrow a range, one could extend the range by analytic continuation, i.e., a power series in $z-z_0$, where z_0 is such a value that φ may be readily calculated by (1).

[1] Stated by Kerst and Serber, Phys. Rev. **60**, 58 (1941).