The experimental data do not permit evaluation of the individual λ's and the τ's, but nevertheless, η_r and η_t can be calculated from the c's, which are obtainable directly from the experimental data. We have from (1), (2), (3), and (4) above,

$$c_3/c_2 = (\lambda_1\lambda_2)/(\tau_1\tau_2), \quad (7)$$

$$c_1/c_4 = (\lambda_1^2+\lambda_2^2)/(\tau_1^2+\tau_2^2), \quad (8)$$

$$2c_3 = (\lambda_1/\tau_1)+(\lambda_2/\tau_2), \quad (9)$$

and

$$2c_2 = (\tau_2/\lambda_2)+(\tau_1/\lambda_1). \quad (10)$$

Combining (9) and (10), one obtains

$$(\lambda_2/\tau_2) = c_3[1-(1-[1/c_2c_3])^{\frac{1}{2}}] \quad (11)$$

and

$$(\lambda_1/\tau_1) = c_3[1+(1-[1/c_2c_3])^{\frac{1}{2}}]. \quad (12)$$

Substituting (11) and (12) into (8), we can obtain the ratios τ_1/τ_2 and λ_1/λ_2 and these ratios suffice in determining η_r and η_t.

Using data given for gold foils in Tables II and IV, we can evaluate the c's and from these, then,

$$\eta_t = 1.11 \quad \text{and} \quad \eta_r = 1.02.$$

JANUARY 1 AND 15, 1943 PHYSICAL REVIEW VOLUME 63

The Effect of Oblique Incidence on the Conditions for Single Scattering of Electrons by Thin Foils

GERALD GOERTZEL AND R. T. COX
New York University, University Heights, New York
(Received October 13, 1942)

In setting up and applying criteria for the single scattering of electrons by thin foils, it has been usual to assume that the principal deviations from single scattering are caused by the combination of small deflections with one large one. It is here shown that with electrons obliquely incident on the foil much more serious deviations may be caused by the combination of two deflections of the same order of magnitude. It is concluded that the difference observed by Chase and Cox between the scattering at 90° to the two sides of a foil on which electrons are incident at 45° may be ascribed to this cause. The bearing of this factor on the negative result of Dymond's experiment on the polarization of electrons is also considered.

IN the theory of scattering of charged particles by thin foils it has been usual to assume normal incidence. In this case most of the deviations from single scattering are caused by the combination with one large deflection of one or several much smaller ones. Scattering through a large angle by two deflections of the same order of magnitude is an occurrence of lower probability.

The theory so derived has often been used in the discussion of experiments at oblique incidence. Thus when theoretical criteria of single scattering, such as Wentzel's, have been applied to experiments, it has been usual to assume that no more allowance for obliquity had to be made than to use the oblique thickness as the effective thickness of the foil. Bartlett[1] has called attention to the need for a more careful consideration of the effects of obliquity, but he published no calculations on the subject. Lately Chase and Cox,[2] studying the scattering of electrons incident at 45° on thin aluminum foils, observed that the scattering at 90° was greater toward the side from which the electrons were incident than toward the other side. This was in spite of the fact that their foil was much thinner than was necessary at this angle to satisfy Wentzel's criterion. Another, more empirical criterion, which they developed for the purpose, was also much more than satisfied. However, in developing this criterion they assumed that the probability of scattering at a large angle by the combination of two deflections of the same order of magnitude could be ignored.

This assumption is investigated in the present calculation. The calculation is made for incidence at 45° with the normal and scattering in the plane of incidence, also at 45° with the normal and thus at right angles to the direction of incidence. In Fig. 1, Oz is the normal. The electrons come from the direction of A, and hence the direction of their velocity at incidence is that of OA', the

[1] J. H. Bartlett, Jr., Phys. Rev. **57**, 843 (1940).
[2] C. T. Chase and R. T. Cox, Phys. Rev. **58**, 243 (1940).

extension of AO. The collecting chamber for the scattered electrons is in the direction of B, OB being perpendicular to OA' and in the plane of incidence.

The electrons most likely to undergo plural scattering will be those first deflected in a direction giving them the possibility of a rather long path in the foil, so that their chance of a second large deflection is relatively great. Consider an electron deflected in a direction OC parallel to the plane of the foil. Let θ denote the angle of this deflection, which is the angle between OA' and OC. Let Oy be the trace of the plane of incidence in the plane of the foil, and let φ be the angle between OC and Oy. Then φ is the projection of θ on the plane of the foil. Since the angle between OA' and Oy is 45°, it follows that

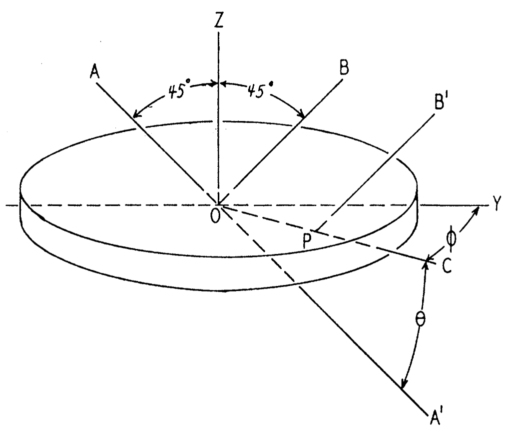

FIG. 1. Directions of incident and scattered electrons.

$$\cos\varphi = \sqrt{2}\cos\theta. \tag{1}$$

At a point P along OC let the electron undergo a second deflection to the direction PB', parallel to OB. The angle of this deflection is that between PC and PB', equal to the angle between OC and OB. But, since OB and OA' are symmetrical with respect to the plane of the foil, they must make equal angles with OC, which is in that plane. Hence the angle of the second deflection is also θ.

The problem now is to find the probability that two deflections through approximately the same angle θ will result in a combined deflection to the direction of the collecting chamber. This probability is to be compared with that of a single deflection through 90° to the collector. The estimate of the effect of plural scattering obtained by the comparison will be a minimum estimate, since only that plural scattering is included in which the first deflection gives the electron a path roughly parallel to the plane of the foil. It turns out that it is unnecessary to define a "roughly parallel" path except to say that its length in the foil, if the electron undergoes no second deflection, is taken as several times greater than the thickness of the foil. The minimum length of the path to be considered will be denoted by kh, where h is the thickness of the foil and k is then a number several times unity.

For convenience, the first deflection will be assumed to occur midway between the surfaces of the foil, at O in Fig. 2. The electrons considered are those deflected at any azimuth φ to directions between the lines OD and OD', the length of each of these lines being kh. The angle between these lines is approximately h/kh or $1/k$. If also the direction of the deflected ray is in the element of azimuth $d\varphi$, it must lie within the solid angle $d\varphi/k$.

Let the probability P of a single deflection to a small solid angle Ω in a direction making an angle α with the incident ray be given by

$$P = ntf(\alpha)\Omega,$$

where n is the number of scattering centers per unit volume of the foil, t is the length of path in the foil of the undeflected ray, and $f(\alpha)$ is the function given by the theory of single scattering. For the electron incident at 45°, $t=\sqrt{2}h$. The solid angle Ω has just been shown to be $d\varphi/k$, and α is approximately the angle θ, related to φ by Eq. (1). The probability of this deflection is thus $n\sqrt{2}hf(\theta)d\varphi/k$.

The probability to be found next is that of a second deflection which will bring the electron to the collecting chamber. For this deflection α is again approximately equal to θ, as shown earlier, and Ω is now the solid angle subtended by the collector at the foil. The length of the undeflected path may be taken as kh, since this is the minimum length for the electrons considered, and since the problem is that of finding the minimum effect of plural scattering. The use of the equation of single scattering for a path as long as kh might be questioned, but it may be justified by the fact that the results are to be compared with observations in which the criteria of single scattering were more than satisfied except for the effects of obliquity. The obliquity is being allowed for here by considering the probability of two deflections. The minimum probability of the second deflection is, by the foregoing discussion, $nkhf(\theta)\Omega$. The minimum probability that both deflections shall occur is the product of the two probabilities found, or $n^2\sqrt{2}h^2f^2(\theta)d\varphi\Omega$. It is to be noted that k has disappeared. This product must be integrated over all values of φ to find the minimum probability that two roughly equal deflections shall bring the electron to the collecting chamber. If this probability be denoted by $P(2)$, then

$$P(2)=2\sqrt{2}n^2h^2\Omega\int_0^\pi f^2(\theta)d\varphi.$$

The factor 2 on the right takes account of the integration clockwise and counter-clockwise from 0 to π.

The probability $P(1)$ that the electron shall reach the collector by a single deflection is given by

$$P(1)=\sqrt{2}nh\Omega f(\pi/2).$$

Hence the ratio of plural to single scattering at 90° on the side of incidence will not be less than

$$[P(2)]/[P(1)]=2nh\frac{\int_0^\pi f^2(\theta)d\varphi}{f(\pi/2)}. \qquad (2)$$

It is not difficult now to obtain the corresponding ratio for the electrons emerging at 90° with the direction of incidence from the surface of the foil opposite to that on which they are incident. In Fig. 1, the direction of motion of the twice deflected electron in this case is opposite to PB. Consequently the angle of the second deflection, instead of being equal to the angle of the first deflection, is supplementary to it. The equation in this case, corresponding to Eq. (2) in the other, is therefore,

$$\frac{P(2)}{P(1)}=2nh\frac{\int_0^\pi f(\theta)\cdot f(\pi-\theta)d\varphi}{f(\pi/2)}. \qquad (3)$$

While the limits of φ in the integrals of Eqs. (2) and (3) are 0° and 180°, those of θ are 45° and 135°. Within these limits, and for the energies of the electrons here considered, Rutherford's equation, with the relative mass, will give a tolerable approximation for the present requirements, which are only qualitative. Hence we take

$$f(\alpha)=b^2\csc^4(\alpha/2), \qquad (4)$$

where

$$b=[Ze^2(1-v^2/c^2)^{\frac{1}{2}}]/[2mv^2],$$

Z is the atomic number of the scattering element, and e, m, and v are, respectively, the charge, rest mass, and speed of the electron. Equations (2) and (4) give for the scattering on the side of incidence

$$[P(2)]/[P(1)]=\tfrac{1}{2}nhb^2\int_0^\pi \csc^8(\theta/2)d\varphi. \qquad (5)$$

Equations (3) and (4), with a little transformation, give similarly for the scattering to the side opposite that of incidence,

$$[P(2)]/[P(1)]=8nhb^2\int_0^\pi \csc^4\theta d\varphi. \qquad (6)$$

Approximate numerical values can be found without much trouble for the integrals of Eqs. (5) and (6). The equations then become

$$[P(2)]/[P(1)]=500nhb^2 \qquad (7)$$

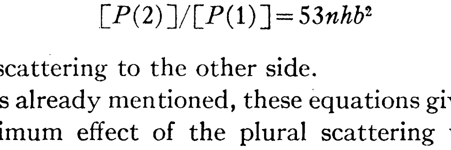

FIG. 2. Scattering within foil.

for scattering to the side of incidence and

$$[P(2)]/[P(1)]=53nhb^2 \qquad (8)$$

for scattering to the other side.

As already mentioned, these equations give the minimum effect of the plural scattering which involves the combination of two deflections of the same order of magnitude. They are sufficient to show that this effect may be much greater than that of the plural scattering which results from the combination of small-angle scattering with one large deflection.

In the experiment of Chase and Cox already referred to, composite foils were used, made by evaporating aluminum on thin collodion films. The thickest foil used was equivalent in scattering power to an aluminum foil probably a little more than 10^{-5} cm thick. The electrons were accelerated by 50 kv. Incidence was at 45°. The plural scattering given by Eq. (8) for the electrons passing through the foil and scattered at 90° is in this case 0.3 percent of the single scattering. This is about the same as their estimate of the increase in scattered intensity caused by the combination of small- and large-angle scattering. Since Eq. (8) gives a minimum estimate, it is likely that even on this side of the foil the scattered intensity is increased more by the combination of large deflections than by the process usually considered responsible.

For the plural scattering on the side of the foil from which the electrons were incident, Eq. (7) gives a value of about three percent of the single scattering. Actually it was observed that the scattered intensity at 90° was, with this foil, 20 percent higher than on the other side. Thus Eq. (7) does not account for the whole of this excess. On the other hand, this excess scattering was observed to increase with the thickness of the foil, as would be expected if it were due to plural scattering, and the difficulties of any other explanation are considerable, in view of the precautions taken in the experiment. For these reasons it seems likely that the excess was principally or wholly due to plural scattering of the kind considered here.

The agreement found by Chase and Cox between their observations and Mott's theory of the nuclear scattering of electrons is not appreciably affected by this conclusion. For the electrons passing through the foil, the effect of plural scattering should be within their estimate of the experimental uncertainty, which was five percent in relative intensity and 20 percent in absolute intensity. Most of their observations were made on this side of the foil. For the others, their method of correction, while arbitrary, would scarcely have introduced a serious error.

An experiment in which the type of plural scattering here considered would seem to have been more serious is that in which Dymond[3] looked for the asymmetry predicted by Mott in the scattering by a second foil of electrons previously scattered at 90°. From Dymond's diagram of his apparatus, it appears that all the electrons used were scattered to the side of the foils from which they were incident. The voltages used in this experiment were 90, 125, and 160 kv. The foils were of gold and varied from 2.5×10^{-5} cm to $\sim$4 or 5×10^{-6} cm, the thicker foils being used at the higher voltages to yield a measurable scattered intensity. Equation (7) gives a value for the plural scattering equal to about 15 percent of the single scattering for the highest voltage and the thickest foil used, and to about 30 percent for the lowest voltage and the thinnest foil. The equation is not so applicable here as with a lighter element than gold, since Rutherford's equation is a poor approximation at higher atomic numbers. But if it be assumed (and observations of Shull, Chase, and Myers[4] indicate that the assumption is reasonable) that the actual plural scattering, as in the experiment of Chase and Cox, is some six times greater than indicated by the equation, then it must be concluded that half or more of the scattering in Dymond's experiment was plural in spite of the precautions taken against it. The polarization in this event would have been much reduced, and it seems possible that it may have been so much diminished as to escape detection.

We are indebted to Dr. Theodore Holstein for the suggestion, made in a discussion of the experiment of Shull, Chase, and Myers,[4] that with oblique incidence enough electrons might be deflected to directions giving them long paths in the foil that the probability of plural scattering would be materially increased.

Note added in proof.—The problem of this paper seems to have been considered by Petukhov and Vyshinsky.[5] We have been unable to see the original article, and the abstract does not describe the method of their calculation.

[3] E. G. Dymond, Proc. Roy. Soc. **A145**, 657 (1934).
[4] Shull, Chase, and Myers, Phys. Rev. **63**, 29 (1943).
[5] V. A. Petukhov and I. A. Vyshinsky, J. Phys. U.S S.R. 5, 137–139 (1941); Phys. Abs. 2369 (Sept. 1942).