THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 63, NOS. 1 AND 2 JANUARY 1 AND 15, 1943

The Relativistic Theory of Excited Spin States of the Proton and the Neutron

V. L. GINSBURG

P. N. Lebedev Physical Institute of the Academy of Sciences of U.S.S.R., Mos.ow, U.S.S.R.

(Received March 31, 1942)

The existing theory gives the cross section for the process of scattering of mesons by heavy particles (i.e., the protons and the neutrons), which increases with energy and is thus contradictory to experimental data as well as to a number of general principles. This increase must be explained as a result of the neglect of the reaction of the proper field of the particle's quasi-magnetic moment on the motion of this moment. If the proper field is taken into account in a quantum mechanical case, we are naturally led to the necessity of supposing that the proton and the neutron possess excited states of spin $\frac{3}{2}$ and even greater. In the present paper, the relativistic theory for the particle capable of being in the state with spin $\frac{1}{2}$ and the rest mass m_1, as well as in the state with spin $\frac{3}{2}$ and the rest mass $m_2>m_1$ is developed. It is shown that the cross section for the scattering of light (of mesons) by the magnetic (by the quasi-magnetic) moment of such a particle increases at first with energy, as in the usual theory. However, for photons with energies $h\nu\gg(m_2-m_1)c^2$ the cross section becomes constant (the recoil for the heavy particle being neglected). Therefore, the introduction of the higher spin states, which is adequate to account for the proper field of the particle's moment, offers the possibility of treating consistently the interaction of the magnetic moment with radiation by the usual perturbation theory methods.

1. INTRODUCTION

As is well known, the cross section for the scattering of mesons by heavy particles (the protons and the neutrons), as calculated in the usual way, increases with energy indefinitely. Such an increase of the cross section, however, is contradictory to experimental data as well as to some very general principles of the theory.[1,2] This situation is due to the fact that the reaction of the proper field of the scattering particle on its motion is not taken into account in the usual quantum mechanical perturbation method. Consequently, the variation of the spin direction of the heavy particle, as well as the variation of its charge, takes place too freely, thus causing an unlimited increase of the scattering cross section.[3] Heisenberg emphasized[3] the occurrence of a similar situation in the classical theory of light scattering by the magnetic moment. In this case, the cross section at first increases as ν^2, ν being the frequency of the scattered radiation. The same result is obtained in a quantum theory of the type of the Pauli theory, in which

$$H=(p^2/2m)-\mu_0(\sigma\mathbf{H})$$

is used for the energy operator, where the vector σ has the usual Pauli spin matrices as components. Finally, a cross section of a similar type results from a non-relativistic approximation of the light scattering by a particle which obeys the Dirac equation including a "true" magnetic

[1] W. Heisenberg, Reports of the Solvay Conference (1939).

[2] L. Landau, J. Phys. (U.S.S.R.) 2, 485 (1940). *Editor's note:* See also H. J. Bhabha and H. C. Corben Proc. Roy. Soc. **A178**, 273 (1941); H. J. Bhabha, Proc. Roy. Soc. **A178**, 314 (1941).

[3] W. Heisenberg, Zeits. f. Physik **113**, 61 (1939).

moment:

$$\gamma_k \frac{\partial \psi}{\partial x_k} + \kappa\psi + \mu_0 \frac{\gamma_i \gamma_k}{2i} F_{ik}\psi = 0.$$

Here γ_i denotes the usual matrices of Dirac's theory and F_{ik} is the electromagnetic field tensor. It is exactly an equation of such a type that is used in the theory of the interaction of mesons with heavy particles (F_{ik} then being replaced by the corresponding meson field tensor).

If, however, the reaction of the magnetic moment proper field on the motion of this moment is taken into account in the classical theory, a constant value of the scattering cross section is obtained for high frequencies. This suggests the possibility of a similar account of the proper field in the quantum mechanics.

We shall begin the investigation of the problem with a more detailed consideration of the self-reaction of the moment in the classical theory.[4, 5] Let us consider the motion of the moment of an uncharged, magnetized particle, its mass center being at rest. Let us assume the magnetization of the particle $\mathbf{M} = \boldsymbol{\mu} D(\mathbf{r})$, where $\boldsymbol{\mu}$ is the total magnetic moment and $D(\mathbf{r})$ the form factor ($\int D(\mathbf{r}) dv = 1$). The set of equations of motion runs as follows:

$$\frac{d\mathbf{s}}{dt} = [\boldsymbol{\mu}\mathbf{H}_{\text{ext}}] + \int [\boldsymbol{\mu}\mathbf{H}(\mathbf{r})] D(\mathbf{r}) dv, \tag{1}$$

$$\square \mathbf{A} = -4\pi \operatorname{rot} \mathbf{M}, \quad \operatorname{rot} \mathbf{A} = \mathbf{H},$$

$\mathbf{H}_{\text{ext}}$ is the external magnetic field, $\mathbf{s}$ is the angular momentum of the particle.

It is possible to eliminate the field $\mathbf{H}$ from Eq. (1) in the general case. If one neglects inessential terms,[4, 5] the following equation for $\mathbf{s}$ results:

$$d\mathbf{s}/dt = [\boldsymbol{\mu} H_{\text{ext}}] - (4\nu_m/3\pi c^3)[\boldsymbol{\mu}(d^2\boldsymbol{\mu}/d_t^2)] + (2/3c^3)[\boldsymbol{\mu}(d^3\boldsymbol{\mu}/dt^3)]. \tag{2}$$

Here ν_m stands for the frequency which cuts off the Fourier spectrum of the proper field acting on the particle. Evidently, $\nu_m \sim c/r_0$, in which r_0 is the radius of the particle. Equation (2) should be supplemented by another one, which connects the vectors $\boldsymbol{\mu}$ and $\mathbf{s}$ with each other. We put, as usual, $\boldsymbol{\mu} = \kappa \mathbf{s}$. It will be seen later on that $\mathbf{s}$ subsequently ceases to play the part of the total angular momentum. The total angular momentum of the particle is the sum of the mechanical angular momentum $\mathbf{s}$ and of the angular momentum $(4\nu_m\kappa^2/3\pi c^3)[\mathbf{s}(d\mathbf{s}/dt)]$, the latter being of magnetic nature (see below). The term $\mathbf{R} = (2/3c^3)[\boldsymbol{\mu}(d^3\boldsymbol{\mu}/dt^3)]$ in Eq. (2) is dissipative and corresponds to the well-known radiation force $(2e^2/3c^3)(d^3\mathbf{r}/dt^3)$, which appears for the translatory motion of the charged particle. The term $\mathbf{L} = -(4\nu_m/3\pi c^3)[\boldsymbol{\mu}(d^2\boldsymbol{\mu}/dt^2)]$, which quantitatively exceeds $\mathbf{R}$ considerably (if $H \sim \cos \nu t$ then $R/L \sim \nu/\nu_m$), does not disturb the conservative character of the motion described by Eq. (2). This term alters essentially the equation of motion for the spin of the particle, as compared to that of which use is ordinarily made. In fact, the equation of motion for the spin is usually taken as

$$(d\mathbf{s}/dt) = \kappa[\mathbf{s}\mathbf{H}_{\text{ext}}] \tag{3}$$

while, if the dissipative term is omitted, Eq. (2) gives:

$$(d\mathbf{s}/dt) = \kappa[\mathbf{s}\mathbf{H}_{\text{ext}}] - (4\nu_m\kappa^2/3\pi c^3)[\mathbf{s}(d^2\mathbf{s}/d_t^2)]. \tag{4}$$

Equation (4) is the intrinsic equation for the symmetrical spinning top. It reduces to Eq. (3) if $\mathbf{L}$ is neglected. It is precisely because of this neglect, characteristic of the usual theory of spin, that the angular momentum equals $\mathbf{s}$ and $\mathbf{s}^2 = \text{const}$. Equation (4) is equivalent to the set of equations

$$d\boldsymbol{\sigma}/dt = \kappa[\mathbf{s}\mathbf{H}_{\text{ext}}]; \quad d\mathbf{s}/dt = -\alpha[\mathbf{s}\boldsymbol{\sigma}]; \quad \alpha = 3\pi c^3/4\nu_m\kappa^2 s^2. \tag{5}$$

Hence, considering the case of the absence of the field, we may conclude that for the particle obeying Eq. (4) the total angular momentum is not $\mathbf{s}$ but $\mathbf{s} + (4\nu_m\kappa^2/3\pi c^3)[\mathbf{s}(d\mathbf{s}/dt)]$. It may be readily shown that in the presence of an external magnetic field the quantity σ^2 is not constant, and it is the quantity $\sigma^2 - (2\kappa/\alpha)(\mathbf{s}\mathbf{H})$ which is conserved. In the case of scattering of light, Eq. (3) leads to the cross section which increases as ν^2 (for detailed calculations see I). Of course, the

[4] V. L. Ginsburg, J. Phys. 5, 47 (1941).

[5] V. L. Ginsburg, Comptes rendus Acad. Sci. U.R.S.S. 31, 319 (1941). Further it is referred to as I.

same result may be obtained from Eq. (4), unless the term **L** ceases to be small. If, for example, we put $\kappa = e/mc$, $\nu_m = c/r_0 = mc^3/e^2$, $\mu = \kappa h$ this term becomes of the same order as ds/dt when $\lambda = c/\nu \sim \hbar/mc$. For the same suppositions concerning κ, ν_m, and μ, the term R becomes of the order of ds/dt only when $\lambda \sim e^2/mc^2$. If $\lambda \ll \hbar/mc$ the cross section attains a constant value which is approximately r_0^2.

The above results show that the account of the proper field of the magnetic moment automatically leads to the substitution of Eq. (3) by Eq. (4), i.e., to the necessity of taking into account the variation of the angular momentum of the particle when it is in the field. Being inessential for low frequencies, this fact changes fundamentally the character of the motion for high frequencies of the perturbing force, as is clear if the scattering is considered as an example.

Going over to the quantum mechanics we conclude that the account of the proper field will be secured if Eq. (4), or the set of Eqs. (5) will be taken as the operator equation for the spin instead of Eq. (3), as is done in the Pauli theory. As is clear from the above considerations, in this case the spin of the particle will no longer be constant. On the contrary, the particle will be considered as a spinning top, and in the case of half-integral spin it will acquire the ability of being in the states with spins $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, etc. Owing to the presence of the field, transitions between states of different spins must take place. Hence, the account of the proper field of the particles' moment, with no assumption *ad hoc*, leads to the conclusion that the elementary particles possess states with different spins. As should be expected, the quantum mechanical theory (see I), based on this supposition, results in the cutting off of the cross section for the scattering and therefore is adequate for the basic classical ideas. After obtaining these results, the author found out that the hypothesis of higher spin states had already been proposed by Heitler and developed by Heitler and Ma.[6] The vectors $\boldsymbol{\sigma}$ and **s**, introduced by these authors, satisfy Eqs. (5) if the natural substitution of $\{[\mathbf{s}\boldsymbol{\sigma}]-[\boldsymbol{\sigma}\mathbf{s}]\}\cdot\frac{1}{2}$ for $[\mathbf{s}\boldsymbol{\sigma}]$ is performed. These equations themselves may be obtained from the Hamiltonian

$$H = \tfrac{1}{2}\alpha\sigma^2 - \kappa(\mathbf{sH})$$

if the commutation rules accepted by Heitler and Ma are used.

The above considerations as well as the Heitler and Ma theory are non-relativistic, and consequently can have only heuristic significance. Therefore, the theory of excited spin states asks for a relativistic formulation. The development of such a relativistic theory comprises the purpose of the present work.

It has already been pointed out in I that there are two ways possible. The first one is to attempt to generalize relativistically Eqs. (4) in the classical case and then perform the quantization (see I). The second way is to suppose, as seems natural, the excited states nearest to the fundamental state to be of the greatest importance and therefore to try to construct the Lagrange function for "the particle of variable spin" at once.

It is mainly the second possibility that the author has followed.[5,7] Accordingly the Lagrange function for a particle capable of being in the state with spin $\frac{1}{2}$ and the rest mass m_1 and in the state with spin $\frac{3}{2}$ and the rest mass $m_2 > m_1$ will be given below.[8]

2. ON THE THEORY OF THE PARTICLE HAVING SPIN 3/2

The equations of the particle of spin $\frac{3}{2}$ must necessarily be used when constructing the theory of the "particle $\frac{1}{2}$–$\frac{3}{2}$." The theory of the particle of spin $\frac{3}{2}$ has been recently developed by Fierz and Pauli[9] in spinor form. The investigation and utilization of the equations for this particle are, however, considerably simplified if the wave functions are written in vector-bispinor form, as has been proposed by Ig. Tamm and A. Davydov.[10,11]

[6] W. Heitler, Nature **145**, 29 (1940); W. Heitler and S. T. Ma, Proc. Roy. Soc. **A176**, 368 (1940).

[7] V. L. Ginsburg, Comptes rendus Acad. Sci. U.R.S.S. **31**, 857 (1941); V. L. Ginsburg and P. E. Nemirovsky, J. Phys. (in press).

[8] Later on we refer to this particle as to the "particle $\frac{1}{2}-\frac{3}{2}$."

[9] M. Fierz and W. Pauli, Proc. Roy. Soc. **A173**, 211 (1939).

[10] The paper by Ig. Tamm and A. S. Davydov is not as yet published.

[11] V. L. Ginsburg, J. Phys. (1941, in press). It is referred to as II. *Editor's note:* See also W. Rarita and T. Schwinger, Phys. Rev. **60**, 61 (1941). In the following formula the spinor indices, which are usually dotted, will be written as bracketed indices.

In the absence of a field, the equations for the particle of spin $\frac{3}{2}$ have the form:[9]

$$\begin{aligned} p^{(\nu)\alpha} \overset{(\mu)}{a}_{\alpha\tau} &= \kappa b_\tau^{(\nu)(\mu)}, \\ p_{\alpha(\nu)} b_\tau^{(\nu)(\mu)} &= \kappa \overset{(\mu)}{a}_{\alpha\tau}, \end{aligned} \tag{6}$$

$$\begin{aligned} \overset{(\mu)}{a}_{\alpha\tau} &= \overset{(\mu)}{a}_{\tau\alpha}, \\ b_\tau^{(\nu)(\mu)} &= b_\tau^{(\mu)(\nu)}, \end{aligned} \tag{7}$$

$$\overset{\alpha}{p}_{(\mu)} \overset{(\mu)}{a}_{\alpha\tau} = 0, \quad \overset{\tau}{p}_{(\mu)} b_\tau^{(\mu)(\nu)} = 0. \tag{8}$$

Applying to Eq. (6) on the left the matrices $\overset{\kappa\tau}{\sigma}_{(\mu)}$ ($k=1, 2, 3, 4$) and contracting over spin indices τ and (μ) we get the equations in which the spinor indices are replaced by one vector index:

$$\begin{aligned} p^{(\nu)\alpha} \overset{k}{a}_{(\nu)} &= \kappa b^{k(\nu)}, \\ p_{\alpha(\nu)} b^{k(\nu)} &= \kappa \overset{k}{a}_{\alpha}. \end{aligned} \tag{9}$$

We see, moreover, that the Dirac Eq. (9) holds for each pair of vector components of the functions of the spin $\frac{3}{2}$ particle. Introducing the notations: $a_1{}^k = A_1{}^k$, $a_2{}^k = A_2{}^k$, $b^{k(1)} = A_3{}^k$, $b^{k(2)} = A_4{}^k$, we can write down Eqs. (9) in the usual form:

$$\gamma_k(\partial A^m/\partial x_k) + \kappa A^m = 0, \tag{10}$$

where

$$A^k = \begin{pmatrix} A_1{}^k \\ A_2{}^k \\ A_3{}^k \\ A_4{}^k \end{pmatrix}$$

and γ_k are the well-known Dirac matrices, operating on the spinor components of the vector-bispinor A^k. The symmetry conditions[7] may be written in the form

$$\gamma^k A^k = 0. \tag{11}$$

Because of the conditions of Eq. (11)

$$\gamma^m \gamma^k (\partial A^m/\partial x_k) = 2(\partial A^m/\partial x_m),$$

and therefore, applying to Eq. (10) the operator γ_m we obtain

$$\partial A^m/\partial x_m = 0. \tag{12}$$

Equation (12) is equivalent to Eqs. (8). For the adjoint functions[12] $A^{\dagger m} = A^{*m}\gamma^4$ we obtain:

$$[(\partial A^{\dagger m}/\partial x_k)\gamma_k] - \kappa A^{\dagger m} = 0, \tag{10$'$}$$

$$A^{\dagger m}\gamma^m = 0, \tag{11$'$}$$

$$\partial A^{\dagger m}/\partial x_m = 0. \tag{12$'$}$$

Equations (10) and (12) may be obtained also from a variation principle.[9,11] The corresponding Lagrange function is

$$L_{3/2} = A^{\dagger m}\gamma_k \frac{\partial A^m}{\partial x_k} + \kappa A^{\dagger m} A^m - i\left(A^{\dagger m}\frac{\partial C}{\partial x_m} - \frac{\partial C^\dagger}{\partial x_m} A^m\right) - \tfrac{3}{2} C^\dagger \gamma^k \frac{\partial C}{\partial x_k} + 3\kappa C^\dagger C. \tag{13}$$

Here C is a bispinor (or undor of the first rank). Variation of (13) with respect to $A^{\dagger m}$ with the subsidiary condition (11′), as well as with respect to $C^\dagger$, gives

$$\begin{aligned} \gamma_k \frac{\partial A^m}{\partial x_k} + \kappa A^m - \tfrac{1}{2}\gamma^m \frac{\partial A^k}{\partial x_k} - i\frac{\partial C}{\partial x_m} + \frac{i}{4}\gamma^m\gamma^k \frac{\partial C}{\partial x_k} &= 0, \\ \gamma_k \frac{\partial C}{\partial x_k} - 2\kappa C + \tfrac{2}{3} i \frac{\partial A^k}{\partial x_k} &= 0. \end{aligned} \tag{14}$$

As may be shown (very similar calculations are given below), it follows from Eqs. (14) that

$$C = 0. \tag{15}$$

Therefore, these equations are equivalent to Eqs. (10) and (12). The external field may be introduced in the function (13) and in Eqs. (14) in the usual manner. In the presence of the field the quantity C is no longer equal to zero.

The current vector is:

$$\begin{aligned} s_k &= ei\{A^{\dagger m}\gamma_k A^m - \tfrac{3}{2}C^\dagger\gamma_k C - i(A^{\dagger k}C + C^\dagger A^k)\} \\ \rho &= \frac{s_0}{e} = \frac{s_4}{ie} = \{A^{*m}A^m - \tfrac{3}{2}C^*C + (A^{*0}\gamma^4 C + C^*\gamma^4 A^0)\}. \end{aligned} \tag{16}$$

[12] One should keep in mind that the fourth component is taken to be imaginary $(x_4 = ict)$ and therefore $A^{*\alpha} = \bar{A}^\alpha (\alpha = 1, 2, 3)$, $A^{*4} = -\bar{A}^4$, where $\bar{A}^m$ is the complex conjugate to A^m.

In the presence of the field the energy-momentum tensor is:

$$T_{ik}=\hbar c A^{\dagger m}\gamma_k \frac{\partial A^m}{\partial x_i}. \tag{17}$$

Further details may be found in II, where the conservation laws, the form of the proper functions, the perturbation theory for the case of presence of the field, and other problems are treated. Only a few simple results made use of below will be given here.

Introducing matrices $\alpha_k=i\gamma_4\gamma_k$, $\beta=\gamma_4$ and substituting iA^0 for A^4 we obtain Eqs. (10)–(12) in the form:

$$\begin{aligned}
&i\hbar(\partial A^m/\partial t)=HA^m,\\
&\boldsymbol{\alpha}\mathbf{A}=A^0,\\
&\operatorname{div}\mathbf{A}+(1/c)(\partial A^0/\partial t)=0,\\
&H=c(\mathbf{p}\boldsymbol{\alpha})+mc^2\beta,\quad p=-i\hbar\nabla,\\
&\mathbf{A}=(A^1,A^2,A^3).
\end{aligned} \tag{18}$$

The eigenfunctions of the particle of spin $\frac{3}{2}$ when at rest are as follows (for the general case see in II):

(1) The energy $W=\kappa\hbar c$, the spin component along Z axis for different states equals $\pm\frac{3}{2}$, $\pm\frac{1}{2}$

$$\begin{aligned}
&\overset{1}{A}_{3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}; \quad \overset{2}{A}_{3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\\0\\0\end{pmatrix}; \quad \overset{3}{A}_{3/2}=0;\\
&\overset{1}{A}_{-3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}; \quad \overset{2}{A}_{-3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-i\\0\\0\end{pmatrix}; \quad \overset{3}{A}_{-3/2}=0;\\
&\overset{1}{A}_{1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}; \quad \overset{2}{A}_{1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\i\\0\\0\end{pmatrix}; \quad \overset{3}{A}_{1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}-2\\0\\0\\0\end{pmatrix};\\
&\overset{1}{A}_{-1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}; \quad \overset{2}{A}_{-1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}-i\\0\\0\\0\end{pmatrix}; \quad \overset{3}{A}_{-1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\2\\0\\0\end{pmatrix}.
\end{aligned} \tag{19}$$

According to Eq. (18) the fourth component of A^k for the particle at rest vanishes. The functions (19) are normalized by means of the condition

$$A_\alpha{}^{*m}A_\beta{}^m=\delta_{\alpha\beta}. \tag{19'}$$

The indices $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ designate the value of the projection of spin on Z axis in the given state.

(2) The energy $W=-\hbar c\kappa$

$$\begin{aligned}
&\overset{1}{A}_{3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}; \quad \overset{2}{A}_{3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\i\\0\end{pmatrix}; \quad \overset{3}{A}_{3/2}=\overset{3}{A}_{-3/2}=0;\\
&\overset{1}{A}_{-3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}; \quad \overset{2}{A}_{-3/2}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\-i\end{pmatrix}; \quad \overset{1}{A}_{1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}; \quad \overset{2}{A}_{1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\0\\0\\i\end{pmatrix};\\
&\overset{3}{A}_{1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\0\\-2\\0\end{pmatrix}; \quad \overset{1}{A}_{-1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}; \quad \overset{2}{A}_{-1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\0\\-i\\0\end{pmatrix}; \quad \overset{3}{A}_{-1/2}=\frac{1}{\sqrt{6}}\begin{pmatrix}0\\0\\0\\2\end{pmatrix}.
\end{aligned} \tag{20}$$

3. WAVE EQUATIONS FOR "THE PARTICLE 1/2–3/2"

It seems natural to try to form the Lagrange function for "the particle $\frac{1}{2}$–$\frac{3}{2}$" by means of connecting, in some relativistically invariant manner, the Lagrange functions of particles of spins $\frac{1}{2}$ and $\frac{3}{2}$. For the spin $\frac{1}{2}$ the Lagrange function is:

$$L_{1/2}=\psi^\dagger\gamma_k(\partial\psi/\partial x_k)+k\psi^\dagger\psi. \tag{21}$$

If no new terms connecting the quantities A^k and C with each other are introduced, the most general expression of this kind is [see Eqs. (13) and (21)]

$$\begin{aligned}L=L_{3/2}+L_{1/2}&+\alpha C^\dagger\psi+\alpha^*\psi^\dagger C\\&+\beta C^\dagger\gamma_k(\partial\psi/\partial x_k)+\beta^*\psi^\dagger\gamma_k(\partial C/\partial x_k)\\&+\epsilon A^{\dagger m}(\partial\psi/\partial x_m)+\epsilon^*(\partial\psi^\dagger/\partial x_m)A^m.\end{aligned} \tag{22}$$

Variation of this function with respect to A^m [the condition (11′) being kept in mind], and with respect to $C^\dagger$ and $\psi^\dagger$ gives:

$$\begin{aligned}&\gamma_k(\partial A^m/\partial x_k)+\kappa A^m-\tfrac{1}{2}\gamma_m(\partial A^k/\partial x_k)\\&\quad-i(\partial C/\partial x_m)+(i/4)\gamma_m\gamma_k(\partial C/\partial x_k)\\&\quad+\epsilon(\partial\psi/\partial x_m)-(\epsilon/4)\gamma_m\gamma_k(\partial\psi/\partial x_k)=0,\\&\gamma_k(\partial C/\partial x_k)-2\kappa C+\tfrac{2}{3}i(\partial A^k/\partial x_k)\\&\quad-\tfrac{2}{3}\alpha\psi-\tfrac{2}{3}\beta\gamma_k(\partial\psi/\partial x_k)=0,\\&\gamma_k(\partial\psi/\partial x_k)+k\psi+\alpha^*C\\&\quad+\beta^*\gamma_k(\partial C/\partial x_k)-\epsilon^*(\partial A^k/\partial x_k)=0.\end{aligned} \tag{23}$$

Operating on the first of these equations by the operator $\partial/\partial x_m$ and on the second one by $\gamma_m(\partial/\partial x_m)$, one obtains an equation which, after multiplying by some easily found factors and their mutual addition gives

$$\frac{\partial A^m}{\partial x_m}-\frac{3}{2}i\gamma_m\frac{\partial C}{\partial x_m}-\frac{\alpha i}{2\kappa}\gamma_m\frac{\partial\psi}{\partial x_m}+\left(\frac{3\epsilon}{2\kappa}-\frac{\beta i}{2\kappa}\right)\Box\psi=0.$$

Combining this equation with the second of Eqs. (23) we are led to the conclusion that

$$6\kappa C+2\alpha\psi+\left(2\beta+\frac{\alpha}{\kappa}\right)\gamma_k\frac{\partial\psi}{\partial x_k}+\left(\frac{3i\epsilon}{2\kappa}+\frac{\beta}{\kappa}\right)\Box\psi=0. \tag{24}$$

If Eq. (22) had not contained the function ψ, i.e., if we had dealt with a particle having spin $\frac{3}{2}$, Eq. (15) would immediately follow from Eq. (24), as must necessarily be. If we put $2\beta+(\alpha/\kappa)=(3i\epsilon/2\kappa)+(\beta/\kappa)=0$, Eqs. (23) split up into two independent equations describing the particles with spins $\frac{1}{2}$ and $\frac{3}{2}$. In the general case Eqs. (23), besides the solutions corresponding to spin $\frac{3}{2}$, have several solutions corresponding to states with spin $\frac{1}{2}$ and different values of the rest mass. This result may be easily understood. In fact, the total number of components of the functions A^m, ψ, and C is 24. Condition (10) reduces this number to 20. On the other hand "the particle $\frac{1}{2}$–$\frac{3}{2}$" must be described by a function with 12 components. Therefore, Eqs. (23) will describe such a particle if 8 subsidiary conditions will follow from these equations.[9,13] This is the case if we put $\beta=\epsilon=0$. It is just such a variant of the theory which is possibly not unique that will be treated below. Here, comparing Eqs. (24) and (23_3) we have

$$C=\alpha a\psi,\quad a=(k-2\kappa)/(6\kappa^2-\alpha\alpha^*). \tag{25}$$

By making use of Eqs. (23_2), (23_3), and (25) one may readily see that

$$\frac{\partial A^m}{\partial x_m}=-i\delta\psi,$$

$$\delta=\alpha\left(\frac{b}{2\kappa}+\frac{3}{2}ab\right)=\alpha(3\kappa a+1+\tfrac{3}{2}ab), \tag{26}$$

where

$$b=k+\alpha\alpha^*a. \tag{27}$$

Eliminating C and $\partial A^k/\partial x_k$ from Eqs. (23) with $\beta=\epsilon=0$ by means of Eqs. (25)–(27) we get finally:

$$\begin{aligned}&\gamma_k(\partial A^m/\partial x_k)+\kappa A^m+i(g/\hbar c)\gamma_m\psi\\&\qquad-i\alpha a(\partial\psi/\partial x_m)=0,\\&\gamma_k(\partial\psi/\partial x_k)+b\psi=0,\\&\qquad g=(\hbar c/4)[(b/\kappa)+2ab]\alpha.\end{aligned} \tag{28}$$

The general expressions for the current four-vector, the energy-momentum tensor, and the

[13] In the case of the particle of spin $\frac{3}{2}$ such conditions are found in Eqs. (11) and (12).

angular momentum tensor will not be given here. They are easily found by the usual methods from Eq. (22). In the absence of a field which was assumed above, and by taking into account Eq. (25) we obtain for the particle density and the energy density:

$$\rho=\frac{s^4}{ie}=\{A^{*m}A^m+(1-\tfrac{3}{2}\alpha\alpha^*a^2)\psi^*\psi+a(\alpha A^{*0}\gamma_4\psi+\alpha^*\psi^*\gamma_4A^0)\}. \tag{29}$$

$$-T_{44}=i\hbar\left\{A^{*m}\frac{\partial A^m}{\partial t}+(1-\tfrac{3}{2}\alpha\alpha^*a^2)\psi^\dagger\frac{\partial\psi}{\partial t}+a\left(\alpha A^{*0}\gamma_4\frac{\partial\psi}{\partial t}-\alpha^*\frac{\partial\psi^*}{\partial t}\gamma_4A^0\right)\right\}. \tag{30}$$

The set of Eqs. (28) may be solved without difficulty. One group of solutions, corresponding to spin $\frac{3}{2}$, results from Eq. (28) if ψ is put equal to zero. In this case the equations for A^m coincide with the equations for the particle having spin $\frac{3}{2}$. The second group of solutions which corresponds to spin $\frac{1}{2}$[14] results if $\psi\neq 0$. Here ψ must be determined from Eq. (28_2) and subsequently A^m may be found by means of Eqs. (28_1). The rest mass in the state with spin $\frac{3}{2}$ is $m_2=|\kappa|\hbar/c$, while in the state with spin $\frac{1}{2}$ $m_1=|b|\hbar/c$. The moduli of κ and b are taken since both signs are possible.

Keeping in mind the future application to the theory of the proton-neutron, it seems natural to consider m_1 as a quantity of the same order as m_2, and therefore to put for the "excitation energy"

$$\Delta=(m_2-m_1)c^2\ll m_1c^2\sim m_2c^2. \tag{31}$$

The cutting off for the process of scattering must take place for energies of the photon (or meson) of order $\hbar\nu\sim\Delta$.[5,6] Therefore owing to Eq. (31) we may, in a preliminary investigation, in order to clarify the general character of the energy dependance of the cross section, consider the case of $\hbar\nu<m_1c^2$. Evidently, it is then permissible to neglect the proton recoil and to assume the heavy particle as being at rest. The non-relativistic approximation, resulting therefrom, corresponds to the non-relativistic theory of Heitler and Ma and of the present author.[5] There is, however, a fundamental difference of principle, since the fulfillment of the requirements put forward by relativity is obvious in the present theory.

Equations (20)–(28) are more suitable to handle if they are written in the following form[15]

$$\begin{aligned}
i\hbar(\partial\mathbf{A}/\partial t)&=\{c(\boldsymbol{\alpha}\mathbf{p})+\kappa\hbar c\beta\}\mathbf{A}+\{g\boldsymbol{\alpha}+\alpha a\beta\mathbf{p}\}\psi,\\
i\hbar(\partial A^0/\partial t)&=\{c(\boldsymbol{\alpha}\mathbf{p})+\kappa\hbar c\beta\}A^0+\{g+\alpha ai\hbar(\partial/\partial t)\}\psi,\\
i\hbar(\partial\psi/\partial t)&=\{c(\boldsymbol{\alpha}\mathbf{p})+b\hbar c\beta\}\psi,\\
\operatorname{div}\mathbf{A}&+(1/c)(\partial A^0/\partial t)=-i\delta\psi,\quad \boldsymbol{\alpha}\mathbf{A}=A^0.
\end{aligned}\tag{32}$$

Let us find the eigenfunctions of the particle at rest. For $\psi=0$ these functions were already written down in Section 2. There was given a set of formulae (19) which corresponds to the mass $\kappa\hbar/c$, while set (20) corresponds to the mass $-\kappa\hbar/c$.

The solutions of Eqs. (32) describing the state with spin $\frac{1}{2}$ are:[16]

(1) The energy $W=b\hbar c$

$$\begin{aligned}
&\psi_+=u\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad A_+{}^0=\frac{\delta}{b}\psi_+;\quad A_+{}^1=\frac{\delta u}{3b}\begin{pmatrix}0\\0\\0\\1\end{pmatrix};\quad A_+{}^2=\frac{\delta u}{3b}\begin{pmatrix}0\\0\\0\\i\end{pmatrix};\quad A_+{}^3=\frac{\delta u}{3b}\begin{pmatrix}0\\0\\1\\0\end{pmatrix};\\
&\psi_-=u\begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad A_-{}^0=\frac{\delta}{b}\psi_-;\quad A_-{}^1=\frac{\delta u}{3b}\begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad A_-{}^2=\frac{\delta u}{3b}\begin{pmatrix}0\\0\\-i\\0\end{pmatrix};\quad A_-{}^3=\frac{\delta u}{3b}\begin{pmatrix}0\\0\\0\\-1\end{pmatrix}.
\end{aligned}\tag{33}$$

[14] This is clear from the investigation of the expression for the tensor of angular momentum density. We do not write it here explicitly.

[15] $\boldsymbol{\alpha}$ and β are Dirac matrices. It should be borne in mind that the letter α also designates the constant included everywhere. However, it is hardly possible to confuse the α_κ matrices with this constant.

[16] When performing the calculations the relation $\hbar c\delta(b+\kappa)/g=3$ may be useful.

(2) The energy $W=-bhc$

$$\psi_+=u\begin{pmatrix}0\\0\\1\\0\end{pmatrix};\quad A_+{}^0=-\frac{\delta}{b}\psi_+;\quad A_+{}^1=-\frac{\delta u}{3b}\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad A_+{}^2=-\frac{\delta u}{3b}\begin{pmatrix}-i\\0\\0\\0\end{pmatrix};\quad A_+{}^3=-\frac{\delta u}{3b}\begin{pmatrix}0\\-1\\0\\0\end{pmatrix};$$

$$\psi_-=u\begin{pmatrix}0\\0\\0\\1\end{pmatrix};\quad A_-{}^0=-\frac{\delta}{b}\psi_-;\quad A_-{}^1=-\frac{\delta u}{3b}\begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad A_-{}^2=-\frac{\delta u}{3b}\begin{pmatrix}0\\i\\0\\0\end{pmatrix};\quad A_-{}^3=-\frac{\delta u}{3b}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}. \tag{34}$$

Here the index $+$ designates the functions describing the state with spin projection on the Z axis equal to $\frac{1}{2}$, the index $-$ corresponds to the projection $-\frac{1}{2}$. The functions (33) and (34) are normalized by means of the condition $\rho=1$ [see Eq. (29)]. They are mutually orthogonal. From the normalization condition we have:

$$|u|^2=\frac{6\kappa^2}{6\kappa^2-\alpha\alpha^*}.$$

Therefore the inequality

$$6\kappa^2>\alpha\alpha^* \tag{35}$$

must necessarily hold. If this inequality were not to hold, ρ could not be normalized with a determined sign. The particle in the state with spin $\frac{1}{2}$ could then have another sign of the charge than in the state with spin $\frac{3}{2}$.

Condition (35) being fulfilled, the total charge of the field under investigation is always definite, which is necessary for quantization of the theory in accordance with the Pauli principle. The energy, on the contrary, is not definite, as happens in all cases of half-integral spin.

4. INTERACTION WITH A FIELD. THE PERTURBATION THEORY

The whole theory has been formulated in the Lagrange form and therefore the generalization for the case of the presence of a field may be readily performed. Indeed, inclusion of the interaction with an electromagnetic field, consistent with the requirements of gauge invariance, is secured by changing $\partial/\partial x_k$ in Eqs. (22) or (23) into $\pi_k=(\partial/\partial x_k)-(ei/\hbar c)\phi_k$, ϕ_k is the field potential. However, we are mostly interested [see (38)] in the case when the equations include terms depending on the field strength explicitly, i.e., terms containing the tensor $F_{ik}=(\partial\phi_k/\partial x_i)-(\partial\phi_i/\partial x_k)$. The interaction with any field, with the meson field for instance, is described by the introduction of the corresponding relativistically invariant terms into the Lagrange function. If the terms of Eqs. (23_1)–(23_3), containing the field variables, are designated by V_m, U, and W, respectively, the equations acquire the form ($\beta=\epsilon=0!$):

$$\begin{gathered}\gamma_k\frac{\partial A^m}{\partial x_k}+\kappa A^m-\tfrac{1}{2}\gamma_m\frac{\partial A^k}{\partial x_k}-i\frac{\partial C}{\partial x_m}+\frac{i}{4}\gamma_m\gamma_k\frac{\partial C}{\partial x_k}+V_m=0,\\ \gamma_k\frac{\partial C}{\partial x_k}-2\kappa C+\tfrac{2}{3}i\frac{\partial A^k}{\partial x_k}-\tfrac{2}{3}\alpha\psi+U=0,\\ \gamma_k\frac{\partial\psi}{\partial x_k}+k\psi+\alpha^*C+W=0.\end{gathered} \tag{36}$$

In the force free case, when $V_m=U=W=0$, the conditions (25) and (26) hold. It seems therefore natural to introduce instead of c the quantity

$$D=C-\alpha a\psi. \tag{37}$$

In the absence of a field $D=0$. When a field is present we obtain, reproducing the calculations that led us to Eq. (25),

$$D=\frac{1}{6\kappa^2-\alpha\alpha^*}\left\{\alpha W+3\kappa U+\tfrac{3}{2}\gamma_k\frac{\partial U}{\partial x_k}-2i\frac{\partial V_m}{\partial x_m}\right\}. \tag{38}$$

Instead of (26) we have:

$$\frac{\partial A^k}{\partial x_k}=-i\delta\psi+\tfrac{3}{2}i\left\{U-\alpha aW+\gamma_k\frac{\partial D}{\partial x_k}-(2\kappa+\alpha\alpha^*aD)\right\}. \tag{39}$$

This equation corresponds to the Eq. (36_2).

Equations (36_1) and (36_3) acquire the form:

$$
\begin{aligned}
&\gamma_k(\partial A^m/\partial x_k)+\kappa A^m+(ig/\hbar c)\gamma_m\psi\\
&-i\alpha a(\partial\psi/\partial x_m)-i(\partial D/\partial x_m)\\
&-(i/2)\gamma_m\gamma_k(\partial D/\partial x_k)+(i/2)(3\kappa+\alpha\alpha^* a)\gamma_m D \qquad (40)\\
&\qquad +V_m-\tfrac{3}{4}i\gamma_m U+(i/2)\alpha a\gamma_m W=0,\\
&\qquad \gamma_k(\partial\psi/\partial x_k)+b\psi+\alpha^* D+W=0.
\end{aligned}
$$

In the absence of a field these equations give Eqs. (28), which was to be expected. Keeping in mind the future application to the radiative processes we shall develop here the non-stationary perturbation theory for the set of Eqs. (38)–(40). Similarly to the case of spin $\frac{3}{2}$ some difficulties are encountered in this way.[11] For example, for the particle at rest in the state with spin $\frac{3}{2}$ $A^4=0$ always holds (see Section 2). Therefore, expanding the wave function of the particle at rest in a field into the functions of the non-perturbed problem we always obtain for the fourth component the zero value. This, however, cannot be the case. In fact, for the state with spin $\frac{3}{2}$, when a field is present, $\partial A^k/\partial x_k\neq 0$ [see (39)]. However, it follows from Eqs. (38), (39) that $\partial A^k/\partial x_k$ and $A^4=iA^0$ are small quantities of the first order of magnitude. Because of this circumstance, as may be shown, in the perturbation theory assuming V_m, U, and W to be small, the development of the wave function in a series of eigenfunctions of unperturbed problems is still possible. We put therefore

$$
\begin{aligned}
A^m&=\sum_n b_{n\mathrm{II}}(t)\overset{m}{A}_{n\mathrm{II}}(\mathbf{x},t)+\sum_n b_{n\mathrm{I}}(t)\,\overset{m}{A}_{n\mathrm{I}}(\mathbf{x},t),\\
\psi&=\sum_n b_{n\mathrm{I}}(t)\,\psi_{n\mathrm{I}}(\mathbf{x},t),
\end{aligned}
\qquad (41)
$$

where $\overset{m}{A}_{n\mathrm{II}}$ are the solutions of the unperturbed problem which correspond to the state with spin $\frac{3}{2}$, and $(\overset{m}{A}_{n\mathrm{I}},\psi_{n\mathrm{I}})$ are those corresponding to the state with spin $\frac{1}{2}$.

Keeping in mind the future applications we confine ourselves to the case of the particle at rest and we neglect the recoil. Let us introduce Eq. (41) into Eq. (46). Multiplying the expressions arising thus by $\overset{\dagger m}{A}_{n\mathrm{II}}$ or by $\overset{\dagger m}{A}_{n\mathrm{I}}$ and by $\psi_{n\mathrm{I}}$, respectively, summing with respect to m and making use of Eqs. (11′), (19), (19′), and (33)–(34) we get:

$$
i\hbar(db_{n\mathrm{II}}/dt)=\hbar c(\overset{\dagger m}{A}_{n\mathrm{II}}V_m), \qquad (42)
$$

and

$$
\begin{aligned}
&\frac{db_{n\mathrm{I}}}{dx_4}\{(\overset{*m}{A}_{n\mathrm{I}}\overset{m}{A}_{n\mathrm{I}})-i\alpha a(\overset{\dagger 4}{A}_{n\mathrm{I}}\psi_{n\mathrm{I}})\}\\
&\qquad\qquad -i\left(\overset{\dagger 4}{A}_{n\mathrm{I}}\frac{\partial D}{\partial x_4}\right)+(\overset{\dagger m}{A}_{n\mathrm{I}}V_m)=0,\\
&\qquad \frac{db_{n\mathrm{I}}}{dx_4}(\overset{\dagger}{\psi}_{n\mathrm{I}}\psi_{n\mathrm{I}})+\alpha^*(\overset{*}{\psi}_{n\mathrm{I}}D)+(\overset{*}{\psi}_{n\mathrm{I}}W)=0,\\
&\frac{db_{n\mathrm{I}}}{dx_4}\{\tfrac{2}{3}i(\overset{*4}{A}_{n\mathrm{I}}\overset{4}{A}_{n\mathrm{I}})+\alpha a(\overset{\dagger 4}{A}_{n\mathrm{I}}\psi_{n\mathrm{I}})\}\\
&\qquad +\left(\overset{\dagger 4}{A}_{n\mathrm{I}}\frac{\partial D}{\partial x_4}\right)-2\kappa(\overset{*4}{A}_{n\mathrm{I}}D)+(\overset{*4}{A}_{n\mathrm{I}}U)=0.
\end{aligned}
$$

The last of these equations is obtained from Eq. (36_2) or (39) after multiplying by $\overset{*4}{A}_{n\mathrm{I}}$. Now, taking into account the properties of functions (33)–(34), we eliminate the quantity D and obtain:

$$
\begin{aligned}
&i\hbar\eta\frac{db_{n\mathrm{I}}}{dt}=\hbar c(\overset{*m}{A}_{n\mathrm{I}}V_m)\\
&\qquad\qquad +i\hbar c(\overset{*4}{A}_{n\mathrm{I}}U)+\frac{2\kappa i\hbar c}{\alpha^*}(\overset{*4}{A}_{n\mathrm{I}}W),\\
&\qquad \eta=-\frac{2\kappa\delta^*}{\alpha^* b}|u|^2=-\frac{6\kappa^2}{6\kappa^2-\alpha\alpha^*}(1+3a\kappa). \qquad (43)
\end{aligned}
$$

It may be easily seen that the probabilities of transitions from state I into state II and vice versa are not equal to each other when η is not equal to unity. In contrast to the usual cases, in the theory developed here the principle of detailed balance does not follow from the equations of motion.* This circumstance seems to be very interesting and requires additional investigations. In order to secure the validity of the principle of detailed balance which seems to be necessary, the requirement $\eta=1$ must be fulfilled. This requirement reduces the number of alternative constants involved in the theory. We shall assume below that $b>0$ and, therefore, $m_1=b\hbar/c$. From the condition $\eta=1$ we get for $|\alpha|^2$:

$$
|\alpha|^2=6\kappa^2
$$

or

$$
|\alpha|^2=6\kappa^2+3\kappa b. \qquad (44)
$$

* *Editor's note:* See note added at end of paper.

The unavoidable condition (35) is satisfied only if the case (44) is assumed, and if, moreover, the negative value of κ is chosen, this choice being at our disposal. In the theory of the proton the mass m_1 for the ground state is known, and the single new constant involved in the theory is the mass m_2 of the particle in the excited state. In fact

$$b = m_1 c/h, \quad \kappa = -m_2 c/h, \quad m_2 > m_1 > 0, \qquad |\alpha|^2 = (3c^2/h^2)(2m_2^2 - m_2 m_1). \tag{45}$$

The remaining constants introduced above [see Eqs. (25)–(27)] are:

$$|u|^2 = -2\kappa/b = 2m_2/m_1, \qquad a = \frac{b-2\kappa}{6\kappa^2} = \frac{h(m_1+2m_2)}{6m_2^2 c}, \qquad \left|\frac{\delta}{b}\right|^2 = \left|\frac{\alpha b}{4\kappa^2}\right|^2 = \frac{3m_1^2}{2m_2^2}\left(1 - \frac{m_1}{2m_2}\right). \tag{46}$$

5. THE SCATTERING CROSS SECTION

Now we go over to the calculation of the cross section for the scattering of light by the "particle $\frac{1}{2}$–$\frac{3}{2}$." As is clear from what has been said in Section 1, we are interested in the interaction energy terms involving F_{ik}. These are, for example,

$$A^{\dagger m} \frac{\gamma_i \gamma_k}{2i} A^m F_{ik}, \quad \psi^\dagger \frac{\gamma_i \gamma_k}{2i} \psi F_{ik}, \quad i(\psi^\dagger \gamma^i A^k + A^{\dagger k} \gamma^i \psi) F_{ik},$$

$$A^{\dagger m} \frac{\gamma_i \gamma_m}{2i} A^k F_{ik} = \frac{2}{i} A^{\dagger i} A^k F_{ik}, \text{ etc.}$$

In distinction to the Dirac theory we introduce also the terms containing $\partial F_{ik}/\partial x_m$. A fuller analysis of all the possible interactions of the particle with different tensor fields encounters no difficulties. However, it would go beyond the limits of the present article. Here we should like only to establish the theory of excited states and to prove that this automatically prevents the unlimited increase of the scattering cross section due to the magnetic moment. Because of the complete similarity of this case to the case of scattering of, for example, vector mesons by the quasi-magnetic moment of the heavy particle, no special investigation of the latter case is necessary. When the mesons are charged it is only essential to introduce[6,17] the neutron-proton states with the charges $-e$ and $2e$ in order to obtain a constant scattering cross section for high frequencies. Let us now, therefore, as an example, consider the light scattering by a neutron-proton, the interaction energy being assumed as

$$\frac{\mu_1}{hc} A^{\dagger m} \frac{\gamma_i \gamma_k}{2i} A^m F_{ik} + \frac{\mu_2}{hc} \psi^\dagger \frac{\gamma_i \gamma_k}{2i} \psi F_{ik}. \tag{47}$$

The cross section for the elastic scattering of light having the frequency ν is

$$\sigma = \frac{2\pi}{hc} \frac{\nu^2 d\Omega}{(2\pi)^3 c^3 h} |H|^2,$$

$$H = \sum \left\{ \frac{\overset{\mathrm{I}}{v}_{n'0} \overset{\mathrm{I}}{v}_{kn'}}{h\nu} - \frac{\overset{\mathrm{I}}{v}_{n''0} \overset{\mathrm{I}}{v}_{kn''}}{h\nu} + \frac{\overset{\mathrm{II}}{v}_{m'0} \overset{\mathrm{II}}{v}_{km'}}{h\nu - \Delta} - \frac{\overset{\mathrm{II}}{v}_{m''0} \overset{\mathrm{II}}{v}_{km''}}{h\nu + \Delta} \right\}, \tag{48}$$

where v_{ik} are the matrix elements of the interaction energy. The index I denotes the transitions between states with spin $\frac{1}{2}$, the index II, those for which in the intermediate states the spin equals $\frac{3}{2}$. The particle is assumed to be at rest, and the excitation energy is put equal to $\Delta = (m_2 - m_1)c^2$. The eigenfunctions of the initial and the final states are determined according to Eq. (33), the eigenfunctions of the intermediate states with spin $\frac{3}{2}$ are given by Eqs. (20). Here we have $\kappa < 0$ and therefore $W = -hc\kappa = m_2 c^2 > 0$.

For the sake of definitiveness, as the initial state the state $(\psi_+^{\mathrm{I}}, A_+^{m\mathrm{I}})$ [see Eq. (33)] will be taken. The magnetic field of the light wave we shall assume to be directed along the Z axis. The unit vector along this direction will be denoted by $\mathbf{e}_0$. From the usual quantization of the electromagnetic field one readily shows that the matrix element of the magnetic field strength for a one-photon transition is

$$(1|\mathbf{H}|0) = (2\pi h\nu)^{\frac{1}{2}} \mathbf{e},$$

where the inessential phase factor has not been written out. Omitting the simple calculations of

[17] Similarly to the higher spin-excited states the "higher" charge states must evidently be considered as a natural consequence of the reaction of the heavy particle proper charged field. *Editor's note:* See also H. J. Bhabha, Phys. Rev. **59**, 100 (1941).

the matrix elements for the interaction energy we shall give here only the final result.

For the process involving no change in spin direction, i.e., if the final state coincides with the initial state $(\psi_+^{\mathrm{I}}, A_+^{m\mathrm{I}})$:

$$|H|^2 = B^2 \frac{16\Delta^2 e_z^2}{\{(\hbar\nu)^2 - \Delta^2\}^2}, \qquad (49)$$
$$B^2 = (8/27)^2 \pi^2 \hbar^2 \mu_1^4 |\delta u/b|^4 \nu^2.$$

Here e_z denotes the Z component of the magnetic field of the scattered wave $(|e|=1)$. The formula (49) is a result only of transitions to the intermediate states with spin $\frac{3}{2}$.

For the process involving a change of the spin direction, i.e., when the final state is the one which is denoted by $(\psi_-^{\mathrm{I}} A_-^{m\mathrm{I}})$, connecting the values of the constants μ_1 and μ_2 by the assumption

$$\frac{4}{27}\mu_1^2 \left|\frac{\delta u}{b}\right|^2 = \left\{\mu_2 |u|^2 - \frac{10}{9}\mu_1 \left|\frac{\delta u}{b}\right|^2\right\}^2,$$

we have:

$$|H|^2 = B^2 \left\{\frac{2}{\hbar\nu} - \frac{1}{\hbar\nu - \Delta} - \frac{1}{\hbar\nu + \Delta}\right\}^2 (e_x^2 + e_y^2)$$
$$= \frac{4B^2\Delta^4}{(\hbar\nu)^2\{(\hbar\nu)^2 - \Delta^2\}^2}. \qquad (50)$$

The total cross section is found by summing over the final state, i.e., by adding the results of the substitution of Eqs. (49) and (50) in Eq. (48). If $\hbar\nu \ll \Delta$, only the first term in the brackets [in Eq. (50)] is of importance, and we obtain, as usual, for the light scattering by a magnetic moment a cross section, which increases with the frequency:

$$\sigma = \text{const. } \nu^2.$$

If, on the contrary, $\hbar\nu \gg \Delta$, the part of the cross section which is not connected with a change of the spin direction, approaches a constant value independent of the frequency.

The cross section which is connected with a change of the spin direction decreases at high frequencies as $1/\nu^2$, this decrease being in full accordance with classical arguments. In fact, the change of moment direction must necessarily be hampered, if the reaction of the proper field is taken into account.

In the above calculations the heavy particle has been assumed to be at rest, this being permissible if $h\nu < m_1c^2$. If the recoil of the heavy particle were taken into account, the scattering cross section when $\hbar\nu \gg \Delta$ would evidently decrease with the energy, probably as $1/\nu$.

It seems hardly advisable to perform any fuller calculations of the scattering cross section in different cases which are distinguished by the assumptions concerning the interaction energy, by the neglect or the inclusion of recoil, etc. before the connection with the theory of nuclear forces is established. Therefore, the purpose of further investigation is to consider the nuclear forces within the bounds of the relativistic theory developed above. In order to do this, the interaction with various meson fields must be included in the equations. Here various possibilities arise. Independently of this problem however, the above results seem to make the hypothesis of excited proton states very probable.

Unlimited growth of the cross section results also when the light scattering by the meson with spin 1 is calculated. This result is explained apparently as in the case of the scattering by a proton, by the neglect of the proper field of the meson magnetic moment.[4, 5, 18] According to this point of view it seems to be of interest to consider the particle capable of occupying states with spin 1 and rest mass m_1 and with spin 2 and the rest mass $m_2 > m_1$. One may believe that the cross section for the light scattering by such a "particle 1–2" for high energies will not increase with the energy. However, since in this case the cross section must be calculated with the motion of the particle taken into account, some difficulties may be encountered.

In conclusion I wish to express my sincere gratitude to Professor Ig. Tamm for his encouragement and advice.

ADDED NOTE

In the above article it was erroneously concluded that for the particle transitions $(\frac{1}{2} \rightleftarrows \frac{3}{2})$ the principle of detailed balance does not hold. The erroneousness of this conclusion follows from the

[18] In distinction to the case of a proton, no terms including F_{ik} are introduced. Therefore, that part of the scattering which is connected with the magnetic moment becomes noticeable only at high energies. At low energies the scattering by the meson charge predominates.

fact that the fundamental equations of the theory [Eqs. (36)] are invariant with respect to the change of the sign of time. Meanwhile Eq. (43), which was derived by the perturbation method and indeed led to the above conclusion concerning the lack of symmetry of transition, does not actually follow from the Eq. (39) since the development (41) is not complete. As a matter of fact, for the unperturbed system in the state of spin $\frac{3}{2}$ we have $\psi=0$. However, in the presence of the field even in this state $\psi\neq 0$, while the development (41) does not take into account that part of the function ψ which in the presence of the field corresponds to the state of spin $\frac{3}{2}$.

Therefore the formula (43) is wrong, while the relations (44)–(46) are not necessary. The whole general theory, of course, remains unchanged. The fundamental result of the paper—the cutting off of the cross section for the scattering of light by the particle's moment—also remains valid. This is easily shown by the evaluation of the cross section for the scattering by means of the correspondence principle instead of the incorrect perturbation theory, as was done in Section 5. Thus the result of Section 5 remains true, i.e., the cross section for the scattering, which in the beginning increases as the square of the photon energy, ceases to depend on energy for $E\gg\Delta$.

I should like also to point out the following: In order to avoid the change of the number of independent functions describing the particle $(\frac{1}{2}-\frac{3}{2})$ when the field is put on, it suffices that the right-hand side of Eq. (38) involves no time derivatives of A^m, D, and ψ. This requirement restricts to some extent the choice for the expression of the energy of interaction of the particle with the field. The usual interaction with the electromagnetic field, expressible by means of potentials, satisfies this requirement. The interaction described by Eq. (47) does not satisfy it. It is, however, very easy to find another type of interaction energy which does and which moreover leads to the cutting off of the cross section as well.

The equations for the particle capable of being in the states of spins 1 and 2, mentioned at the end of the paper, have been recently obtained and investigated. The calculation of the scattering of light by the particle (1–2) in the relativistic case, which is only of interest according to the arguments given at the end of the paper, turns out to be very cumbersome, although not very difficult and has not yet been performed.

JANUARY 1 AND 15, 1943 PHYSICAL REVIEW VOLUME 63

High Centrifugal Fields and Radioactive Decay

SIMON FREED, A. H. JAFFEY, AND M. L. SCHULTZ
George Herbert Jones Laboratory, University of Chicago, Chicago, Illinois
(Received October 6, 1942)

Previous experiments to detect some influence on the decay process of a naturally radioactive substance by means of centrifugal fields (20,000 g) had yielded no effect within 0.1 percent. It seemed of value to re-examine such a possibility in view of the high centrifugal fields (order of 1,000,000 g) now available, the new types of decay processes in artificial radioactivity, and the non-electromagnetic forces now recognized as operating in atomic nuclei. Except for fission an example of each type of radioactive decay was investigated. Two methods were employed, one of which made use of a toroidal tube counter surrounding the spinning rotor. An ionization chamber furnished a precision of several tenths of a percent and the counter allowed a precision of several percent. No definite effect was found. However, in Br^{80} where a transition occurs between isomers by emission of gamma-rays rather than the usual transformation within the nucleus involving charged particles, there was some sign of systematic deviation from the accepted half-lives in a centrifugal field of 632,000 g but it may be within the total experimental error.

MANY experiments, very diverse in character, have all failed to effect any measurable change in the decay processes of naturally radioactive substances.[1] The discovery of artificial radioactivity throughout the periodic system, the appearance of new types of nuclear transformations and the new basic particles, the neutron and the positron, have brought to the fore the need of attempting once again to influence the decay processes and thus to link

[1] F. W. Kohlrausch, "Radioaktivität," *Handbuch der Experimental Physik* (Leipzig, 1928), Vol. 15, p. 669.