On the Excited States of Nuclei Beyond Oxygen

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If spin-orbit coupling and the difference between neutron-neutron, neutron-proton, and proton-proton forces are neglected, the energies of the ground states of the nuclei beyond oxygen can be expressed in terms of the symmetry character of the wave functions and a smoothly varying function of atomic weight. With the same assumptions, the energies of the excited states of these nuclei can be expressed in similar terms. Thus the difference in energy of the ground states and the excited states of the same symmetry would be expected to vary smoothly as a function of mass number. Since each state is a supermultiplet of states which will be split under the influence of spin-orbit coupling, it will not in general be possible to observe this behavior of the energy levels. However, in the case of the alpha-particle nuclei, the low-lying excited states are singlets so the regularity should be observable. Unfortunately there are insufficient experimental data on the excited states of these nuclei to make a positive statement as to agreement, but those data which are available do not disagree with the suggested behavior.

IT has been shown by Wigner[1] that the energies of the ground states of the nuclei beyond oxygen can be calculated by means of a simple expression involving the symmetry character of the wave function in the ground state and a smoothly varying function of atomic weight. This result is obtained by neglecting the spin-orbit coupling and the difference between neutron-neutron, neutron-proton, and proton-proton forces except for the average effect of the Coulomb interaction. Barkas[2] has compared the theoretical masses of the nuclei obtained in this way with the experimental masses and has found reasonably good agreement between them.

If the energies of the excited states are calculated with the same approximations, they are expressions of the same type as those for the ground states. Thus the difference in energy of the ground states and the excited states of the same symmetry character would be expected to vary smoothly as a function of the mass number of the nuclei. However, each state is a supermultiplet[3] of states which will be split into its many components under the influence of the spin-orbit coupling. In addition it is likely that states belonging to different multiplets will be mixed. Therefore, it will in general be very difficult to make any correlation between the experimentally observed excited states and those predicted in first approximation by the Wigner theory. The only type of state for which it seems feasible to make this correlation is that belonging to a singlet supermultiplet[3] of the (0,0,0) type. Even these states may give trouble if the influence of the tensor spin-orbit coupling is important[4] since this interaction mixes the (0,0,0) state with other multiplets. However, the general success of the Wigner theory in giving the masses of the nuclei seems to indicate that the influence of the tensor interaction on the ground states is not very great. Therefore, it may not be a factor of great importance in determining the energies of the low excited states.

The low states of the alpha-particle nuclei are expected to be (0,0,0) multiplets, so the difference in energy between these states and the ground states would be expected to behave as a smoothly varying function of atomic number if the influence of the tensor force is small. An evaluation of this smooth function for these nuclei would make it possible to determine what the energies of the excited states of other nuclei in the same region of the periodic table would be if spin-orbit coupling were negligible. Comparison with the actual experimental energies of these excited states would then lead to information concerning the spin-orbit coupling.

[1] E. Wigner, Phys. Rev. **51**, 947 (1937).
[2] W. H. Barkas, Phys. Rev. **55**, 691 (1939).
[3] E. Wigner, Phys. Rev. **51**, 106 (1937).
[4] A rough calculation by E. Gerjuoy and J. Schwinger [Phys. Rev. **61**, 138 (1942)] indicates that the influence of the tensor interaction on the energy of the ground state of He^4 may not be negligible. The influence on the energy of an excited state would probably be even more important.

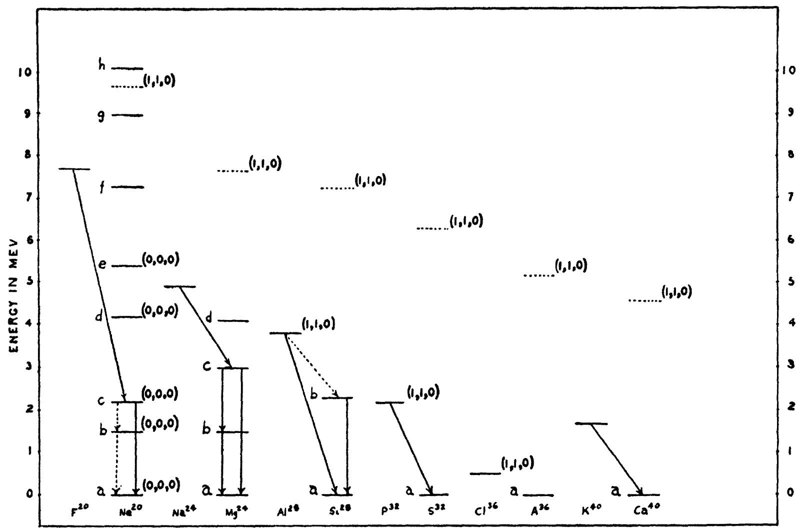

FIG. 1. The solid lines indicate experimentally observed energy levels and transitions. The transitions associated with the broken lines are predicted but not yet observed. The horizontal broken lines are the positions of the lowest (110) levels of the corresponding nuclei as calculated by means of a formula given by Barkas (reference 2).

The purpose of the present note is to point out that the known data on the excited states of the nuclei[5] Ne^{20}, Mg^{24}, Si^{28}, S^{32}, A^{36}, and Ca^{40} do not disagree with the suggested behavior. Unfortunately there are insufficient data to make any positive statement as to agreement, but it is to be hoped that calling attention to these facts will stimulate experimental work which will complete the picture.

The observed energy levels of Ne^{20}, Mg^{24}, Si^{28}, S^{32}, A^{36}, Ca^{40}, and the beta- and gamma-transitions leading to them are indicated in Fig. 1 by the solid lines. The levels *b*, *d*, *e*, *f*, *g*, *h* of Ne^{20} have been obtained by Bonner[6] by measuring the neutron energies in the $F^{19}(d,n)Ne^{20}$ reaction. His high energy maxima are too broad to allow a separation of the levels *b* and *c*. The beta- and gamma-assignments are taken from Curran and Strothers.[7] The $b \rightarrow a$ and $c \rightarrow b$ transitions have not been observed.

The beta- and gamma-ray spectra assigned to Na^{24} are somewhat different from the latest given by Curran, Dee, and Strothers.[8] They assume that the 3-Mev and 1.5-Mev gammas are in cascade. However, such an arrangement of levels would lead to a mass for Na^{24} which would be much higher than expected on the basis of present ideas about nuclei. The best evidence for the level scheme given in Fig. 1 is that the levels *b* and *c* are in excellent agreement with those found by Wilkins[9] who measured the energies of protons which had been inelastically scattered from Mg. He assigned the level *d* to Mg^{26}, but it may be a level of Mg^{24} corresponding to a gamma-ray of approximately 4.5 Mev tentatively reported by Curran, Dee, and Strothers[8] for the reaction $Na^{23}(p,\gamma)Mg^{24}$.

The data on Al^{28}-Si^{28} are taken from Livingston and Bethe.[10] According to them, the beta-transi-

[5] Only these nuclei have been considered because the nuclei at either end of this series involve a change from one closed shell to another.

[6] T. W. Bonner, Proc. Roy. Soc. **A174**, 339 (1940).

[7] S. C. Curran and J. E. Strothers, Proc. Camb. Phil. Soc. **36**, 252 (1940).

[8] S. C. Curran, P. I. Dee, and J. E. Strothers, Proc. Roy. Soc. **A174**, 546 (1940).

[9] T. R. Wilkins, Phys. Rev. **60**, 365 (1941).

[10] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937). This assignment has recently been contradicted by the coincidence experiments of Watase [Proc. Physico-Math. Soc. Jap. **23**, 618 (1941)]. He found no evidence for a complex beta-spectrum and therefore assumes that the 3.3-Mev beta-transition is followed by the 2.3-Mev gamma-transition, thus increasing the mass of Al^{28} by 2.3 Mev. This mass is in complete disagreement with the theoretical value obtained by Barkas and may indicate that the tensor force is particularly important in determining the mass of Al^{28}. It is hoped that this coincidence experiment will be repeated within a short time.

tion to state b must be weak since the beta-spectrum of Al^{28} appears to be simple. The maximum energy of the beta-spectrum of P^{32} is taken from the results of Lawson.[11] No gamma-radiation has been observed. The levels of Cl^{36} and A^{36} are those given by Barkas.[2] The beta-transition appears to be energetically forbidden. The energy difference K^{40}-Ca^{40} has been taken from the results of Henderson.[12]

Evidently there is a correspondence between those levels in Fig. 1 which carry the same label. Thus it would be assumed that the energies of the levels of type b are a smooth function of atomic weight. Apparently the level c continues to rise so that it is above the ground state of Al^{28} and P^{32}, thereby causing the beta-transition to follow a different path. Similarly the level b is rising rather rapidly at S^{32} so it is above the ground state of P^{32} and the only possible beta-transition is that to the ground state of S^{32}. Direct energy level determinations for Si^{28}, S^{32}, A^{36}, and Ca^{40} are essential for a verification of this behavior.

All of the lower levels of the alpha-particle nuclei are (0,0,0) states. However, the ground states of F^{20}, Na^{24}, etc. are (1,1,0) states and there are corresponding states of the same multiplet for the alpha-particle nuclei. The positions of these levels have been estimated by means of formulae given by Barkas[2] on the basis of the Wigner theory. They are indicated by the horizontal broken lines in Fig. 1. Apparently levels g and h and possibly f belong to the (110) multiplet.

Information concerning the angular momenta and parity of the states can be obtained by considering the product of the lifetime t for the beta-transition and a function, $I\ (E_{max})$, of the maximum beta-energy which gives the dependence of the lifetime on this energy. For an allowed transition this product should be of the order of magnitude of[13] 10^3 and increases roughly by a factor of 100 for each order in which the transition is forbidden. The values of $It/\log 2$ are given in Table I under the assumption that the observed beta-spectra are essentially simple. The fact that all values are at least of the order of 10^5 agrees with the fact that a (110)→(000) transition is forbidden at least in first order for either Fermi or Gamow-Teller selection rules.[13] Apparently the P^{32}→S^{32} transition is even more strongly forbidden, as one would expect if the symmetry of the ground state of P^{32} is the same as that of F^{20} and Na^{24}. The data on the decay of K^{40} were not included since this nucleus has one neutron outside of a closed shell so that no correlation between its ground state and those of the other beta-active nuclei can be expected.

TABLE I. Values of $It/\log 2$ for the given radioactive elements.

Element	F	Na	Al	P
$10^{-5}It/\log 2$	8.00	8.62	1.44	672

No attempt has been made to assign the angular momenta of the levels because there are many possibilities depending on the parity, the spin in the lowest state of the (110) multiplet of the radioactive nuclei, and the particular type of selection rule (Fermi or Gamow-Teller) assumed. We can say that there is no contradiction if one assumes that corresponding levels have the same parity and orbital angular momenta. The apparent contradiction in the case of Al^{28}→Si^{28} may be due to an inversion of the (110) multiplet in Al^{28}.

[11] J. L. Lawson, Phys. Rev. **56**, 131 (1939).
[12] W. J. Henderson, Phys. Rev. **55**, 238 (1939).
[13] E. Wigner, Phys. Rev. **56**, 519 (1939).