SATURATION VALUE OF THE INTENSITY OF MAGNETIZATION AND THE THEORY OF THE HYSTERESIS LOOP.

BY E. H. WILLIAMS.

RECENTLY, Weiss[1] has shown that an alloy composed of iron and cobalt combined in relative amounts given by the expression Fe_2Co gives a much higher value of the intensity of magnetization than either iron or cobalt taken alone. Furthermore, it has been shown by Mr. D. T. Yensen[2] that the magnetic properties of pure iron can be greatly improved by melting the iron in a vacuum and it was hoped that the magnetic properties of Fe_2Co could be improved by treating it in like manner. The object of the present paper was not only to make a careful study of the saturation value of the intensity of magnetization of Fe_2Co prepared under various conditions but to use the data thus obtained in a test of the theory of the hysteresis loop as developed by J. Kunz.[3]

In his paper, Kunz tests the theory with the data then available. The results show the discrepancy between theory and experiment to be very great. If all quantities involved are obtained from the same sample, the test of the theory will be more satisfactory.

PREPARATION OF SAMPLES.

Iron and cobalt were taken in the proportion indicated by the formula Fe_2Co, melted in a vacuum furnace under pressures varying from 5 mm. to 0.5 mm. Hg, allowed to cool slowly and then forged into long bars from which samples were turned. In most cases enough of the material was taken to make both an ellipsoid and a rod—the ellipsoid for the determination of the saturation value of the intensity of magnetization and the rod for the determination of the hysteresis loop. The hysteresis data were taken by Mr. D. T. Yensen on a magnetic testing apparatus similar to those used by the Bureau of Standards. Mr. Yensen has made a study of the samples from the viewpoint of the engineer. His results are to be published soon in the General Electric Review.

[1] P. Weiss, Compt. Rend., 156, p. 1970, 1913.
[2] D. T. Yensen, Bul. No. 72, Eng. Exp. Sta., Univ. of Illinois, Urbana, Ill.
[3] J. Kunz, Phys. Zeit., XIII., p. 591, 1912.

Ellipsoids.

A great deal of trouble was experienced in making ellipsoids that were accurate. The form of the ellipsoids was tested by projecting an image of the ellipsoid on the figure of an ellipsoid drawn to the desired proportions. Finally the accuracy was tested by comparing the volumes obtained by calculation, using the dimensions of the ellipsoid, with those obtained by immersion in distilled water at known temperature. No ellipsoid was used where the difference in volume differed by more than 2 per cent. and most of them differed by less than 1 per cent. The ellipsoids were about 1.19 cm. in length and about .56 cm. in diameter.

The author wishes to express his thanks to P. Weiss for samples of his material which he kindly sent. This material, when received, was porous and had apparently been melted and cast at atmospheric pressure. One ellipsoid was turned from the material just as received. A second ellipsoid was turned from a portion of the material which had been forged into a small rod, after which the remainder of the sample was remelted in a vacuum furnace under a pressure of .5 mm. of Hg. It was then forged into a rod from which an ellipsoid was turned. The results obtained with these ellipsoids are included in Table I.

The field inside an ellipsoid is uniform and is given by

$$H = H_0 - NI, \tag{1}$$

where H_0 is the external field applied, I the intensity of magnetization, H the resultant field within the ellipsoid and N a constant depending on the dimensions of the ellipsoid.

The field H_0 was produced by a large electromagnet the pole pieces of which were 3.2 cm. apart and bored to receive a glass tube 9 mm. in diameter. On this tube was wound an induction helix. The field, H_0, between the poles of the magnet was calibrated by two methods—by means of a flip coil and with a magnetic balance. The mean of the two calibrations, which differed in no case by more than one half of one per cent., was taken to plot the calibration curve. The intensity of magnetization I was obtained by suddenly removing the ellipsoid from the induction helix between the pole pieces and noting the change of flux as indicated by a ballistic galvanometer. From the constants of the apparatus the value of I could be calculated.

If, in equation (1), NI is greater than H_0, H becomes negative while H_0 and I are still positive, *i. e.*, the field within the ellipsoid is opposite in direction to the field outside. The ellipsoids used in this work were such as to produce this result, so that when a hysteresis loop was taken with one of the ellipsoids a very peculiar S-shaped form was obtained.

RESULTS FOR I_m.

The results for the saturation values of the intensity of magnetization are summarized in Table I. In this table also, values obtained by other experimenters as well as by the author are given for comparison. Annealing these samples at 900° C. and 1100° C. produced practically no change in the values of I_m. Analysis of the first two samples of Fe_2Co listed in Table I., were made, the first showing 33.36 per cent. Co and the second 33.33 per cent. Co. From these results we see that com-

TABLE I.

Values of I_m (t = 20° C.)

Commercial steel (Williams)	1,751
Swedish wrought iron (Ewing)	1,690
Bessemer steel (.4 per cent. C.)	1,770
Electrolytic iron (melted under pressure of 3 mm. of Hg (Williams)	1,798
Cobalt (1.66 per cent. Fe) (Ewing)	1,310
Cobalt (pure) (Stifler)	1,421
Cobalt (melted under pressure of 1 mm. Hg (about 99 per cent. pure) (Williams)	1,504
Fe_2Co melted under pressure of 3 mm. of Hg without being forged (Williams)	1,791
Same—hand forged (Williams)	1,962
Same—forged with steam hammer (Williams)	1,977
Fe_2Co melted under pressure of 1 mm. of Hg. Forged with steam hammer (Williams)	2,050
Fe_2Co melted under pressure of 0.5 mm. of Hg. Forged with steam hammer (Williams)	2,056
Fe_2Co melted and cast at atmospheric pressure (sample received from P. Weiss) (Williams)	1,752
Same—forged as received (Williams)	1,977
Same—remelted under .5 mm. pressure and forged (Williams)	2,038

bining pure iron for which the value of I_m is 1,800, when the iron is melted in a vacuum, with cobalt for which the value of I_m is 1,500 when melted under the same conditions, we obtain an alloy for which I_m is 2,050, or 14 per cent. higher than pure iron itself. Weiss, in the paper referred to above, states that if one takes into account the difference in atomic weight, the temperature at which ferromagnetism disappears and the densities, one finds that at ordinary temperatures ferro-cobalt has a magnetization at saturation 10 per cent. higher than that of iron, so that the extra 4 per cent. is probably due to the fact that the alloy in the present case was melted in a vacuum. This conclusion is substantiated by the difference between the last two results of Table I (material received from P. Weiss). This difference is undoubtedly due to melting under greatly reduced pressure since all other conditions are as nearly equal as it was possible to make them.

Photomicrographs of the first two samples of Fe_2Co given in Table I. are shown in Figs. 1, 2, 3 and 4. Fig. 1 is of the first sample after being forged and Fig. 2 is of the same sample after being annealed at 900° C. and cooled uniformly at the rate of 30° C. per hour. Fig. 3 is of the second sample of Fe_2Co listed in Table I. after the same had been forged and Fig. 4 is the same sample after being annealed at 900° C.

Hysteresis Theory.

In the article by J. Kunz referred to above, the author obtains the following expression for the energy of the hysteresis loop:

$$W = 2I_mH_c^2\left[\frac{1}{H_c\sqrt{\dfrac{I_m}{I_m+I_1+\Delta I_1}}} - \frac{1}{H_1-\Delta H}\right] + I_mH_c^2\Delta H\left[\frac{1}{H_c^2\dfrac{I_m}{I_m+I_1+\Delta I_1}} - \frac{1}{(H_1-\Delta H)^2}\right],$$

where I_m is the saturation value of the intensity of magnetization, H_c the coercive force, I_1 the intensity of magnetization corresponding to the magnetizing field H_1, and where

$$H = H_c\left(1 - \frac{1}{\sqrt{2}}\right)$$

and

$$I_1 = I_m\left[\left(\frac{H_c}{H_1-\Delta H}\right)^2 - \left(\frac{H_c}{H_1}\right)^2\right].$$

According to this theory the hysteresis loss per cycle experienced when the field alternates between the values $+H_1$ and $-H_1$, producing the intensities of magnetization $+I_1$ and $-I_1$, can be calculated directly if one knows the values of I_m and H_c for the material concerned.

As pointed out above, the test given this theory proved very unsatisfactory and seemed to indicate that the theory was of very little practical importance.

It seemed desirable to give the theory a thorough test by the careful determination of the four quantities I_m, H_c, H_1 and I_1 with the same sample. The results for the hysteresis loss, W, calculated and the hysteresis loss, W', as measured from the hysteresis loops are given in

Tables II., III. and IV. Table II. is for a sample of Fe_2Co before being annealed; Table III. for the same sample after being annealed at 900° C.,

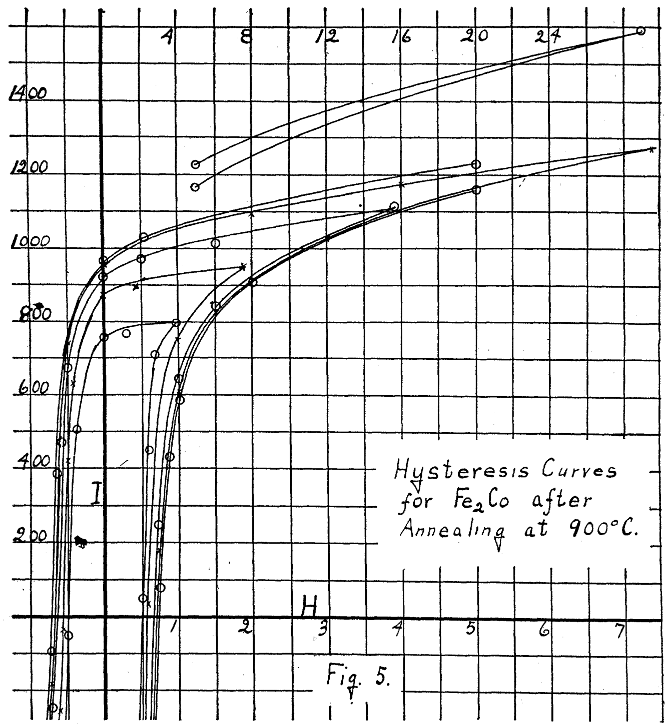

Fig. 5.

and Table IV. for a sample of pure iron after being annealed at 900° C. The hysteresis curves from which the values of W' in Table III. were measured are shown in Fig. 5.

Table II.

$I_m = 2{,}050$; $H_c = 6.4$; $\Delta H = 1.876$.

H_1	I_1	ΔI_1	W	W'
14	415	143.5	19,580	18,685
24.5	798	24.2	28,700	28,600
56.5	1,179	2.27	35,620	41,825
146.	1,588	.06	45,630	56,245

The results in Tables II., III. and IV. show fairly good agreement between the values for the hysteresis loss as calculated by the above formula and those obtained by measurement of the hysteresis curves.

If the theory were modified to take into account the curvature as the

Fig. 1.

Fe_2Co melted under 3 mm. pressure and forged.

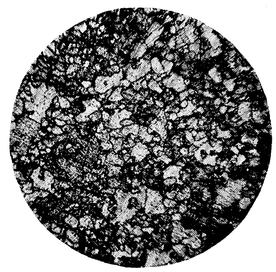

Fig. 2.

Same as Fig. 1, annealed at 900° C.

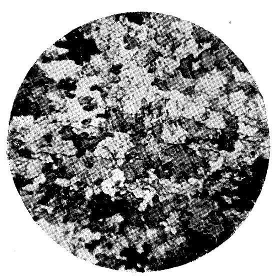

Fig. 3.

Fe_2Co melted under 1 mm. pressure and forged.

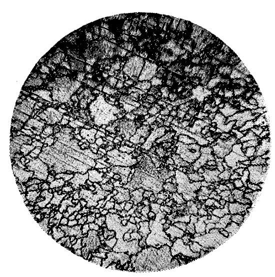

Fig. 4.

Same as Fig. 3, annealed at 900° C.

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TABLE III.

$I_m = 2{,}050$; $H_c = 0.65$; $\Delta H = 0.19$.

H_1	I_1	ΔI_1	W	W'
.95	796	537.	1,513	1,488
1.82	955	64.4	2,727	2,340
3.82	1,115	6.35	3,434	3,020
7.33	1,273	.902	3,777	3,792
29.	1,590	.013	4,166	4,930

TABLE IV.

$I_m = 1{,}800$; $H_c = 0.36$; $\Delta H = 0.105$.

H_1	I_1	ΔI_1	W	W'
.6	955	306.	973	970
0.9	1,115	81.	1,364	1,353
1.5	1,195	16.	1,651	1,552
6.5	1,264	.2	1,940	1,918

coercive force is being applied, it would probably agree with experiment even more accurately than it now does. But even as it stands the results for ordinary fields are accurate enough to make the theory of great value in calculating losses where they cannot be measured directly.

Physics Department,
University of Illinois,
May, 1915.

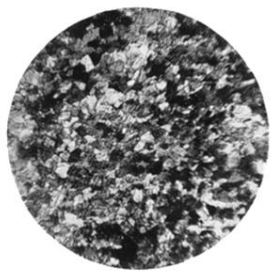

Fig. 1.

Fe_2Co melted under 3 mm. pressure and forged.

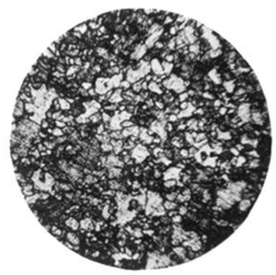

Fig. 2.

Same as Fig. 1, annealed at 900° C.

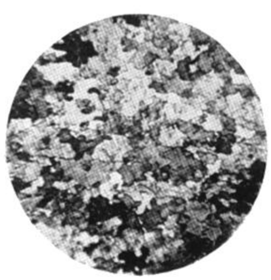

Fig. 3.

Fe_2Co melted under 1 mm. pressure and forged.

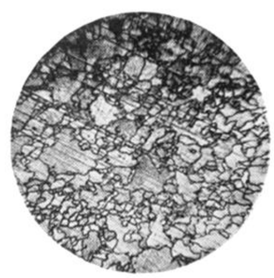

Fig. 4.

Same as Fig. 3, annealed at 900° C.