# THE CATHODE FALL IN GASES.[1]

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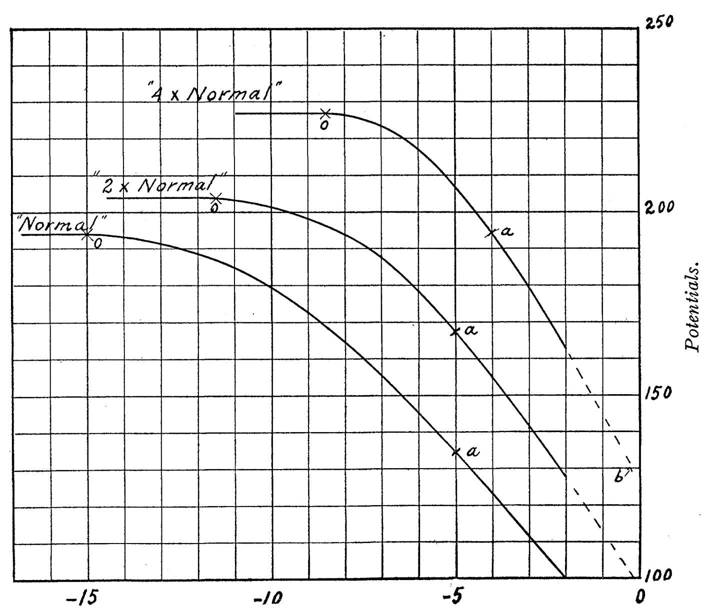
Each column of potentials in the tables (III, IV and V) was obtained with fresh gas at approximately the pressure indicated at its head—the current density always being accurately maintained at the value indicated at the top of the table.

At lower gas pressures the potentials begin to diverge distinctly from the fairly constant values found here. At higher gas pressures the mean free path of the electron is too small to allow of reliable measurements being made with a probe wire of practical size.




*No. M. F. P. from Cathode.*

Fig. 1. Potential Curves in Hydrogen.

On comparing the individual values, given by the various gas pressures, with their means (column 7) we feel justified in concluding that over the range of pressures giving a constant value for the normal cathode fall,

[1] Concluded from the June number, Vol. V, p. 496.

*the potential at a given number of mean free paths from the cathode is simply a function of* $(j\bar{\lambda}^2)$. That is, it is a function of the ratio of the current density to the square of the gas pressure.

The means in columns 7 are plotted as curves in Fig. 1. These potential curves show the interesting fact that the potential gradient approaches a constant maximum value as it nears the cathode.[1] This brings out a distinct similarity in the form of the gradient on the two sides of the negative glow—rising gradually as it does to a constant value in the unstriated positive column.

*Test of the Theory by the Potential Curves.*—From the expressions in (18) (l. c., p. 488) we may write for any point $x_1$, having a potential $V_1$,

$$V_0 - V_1 < \left\{ K^{2/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_1 - x_0}{\bar{\lambda}}\right)\right\}^{3/2}$$
$$> \left\{ K^{2/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_1 - x_0}{\bar{\lambda}}\right) - \epsilon^{2/3}\right\}^{3/2} + \epsilon \tag{24}$$

from which

$$K > \frac{1}{(j\bar{\lambda}^2)^{1/2}} \cdot \frac{(V_0 - V_1)}{\left(\frac{x_1 - x_0}{\bar{\lambda}}\right)^{3/2}}$$
$$< \frac{1}{(j\bar{\lambda}^2)^{1/2}} \cdot \left\{ \frac{(V_0 - V_1 - \epsilon)^{2/3} + \epsilon^{2/3}}{\left(\frac{x_1 - x_0}{\bar{\lambda}}\right)} \right\}^{3/2}. \tag{25}$$

The values for these upper and lower limits of $K$ were calculated by substituting from the means of Tables III., IV. and V. the values at 2 m.f.p. the locations of $x_0$ being obtained from the curves of Fig. 1. The magnitudes thus obtained for $K$ were

| | "Normal." | 2 × "Normal." | 4 × "Normal." |
|---|---|---|---|
| Lower limit of $K$ | 10.3 | 9.5 | 9.8 |
| Upper limit of $K$ | 12.8 | 12.1 | 12.8 |

These agree in indicating, as expected, that $K$ does not change with current density.

Their mean value 11.5 was assumed as the correct magnitude, and therefrom the two limiting potential values calculated for each current

[1] This is very different from the impression one gets from using the much less reliable method of measuring the gradient with two parallel wires, usually several mean free paths apart. One then is led to indicate the gradient as a continuous curve rising to a very large magnitude in front of the cathode.

## TABLE III.

*Potential Difference between Cathode and Points in the Gas at a Given Number of Mean Free Paths from it. For Normal Current Density: ($\bar{j}\lambda^2$ = .038).*

| No. of m. f. p. | Gas Pressures. | | | | | Means. |
|---|---|---|---|---|---|---|
| | 1.1 mm. | 1.6 mm. | 2 mm. | 2.5 mm. | 3 mm. | |
| 2 | 101 volts | 99.5 | 98 | 101.5 | 98 | 99.5 |
| 3 | 114.5 | 114 | 112.5 | 111 | 109.5 | 112 |
| 4 | 128 | 127.5 | 121.5 | 125 | 120 | 123 |
| 5 | 141.5 | 142 | 134.5 | 135 | 127 | 136 |
| 7 | 165 | 164 | 154.5 | 155.5 | 145 | 155 |
| 9 | 181.5 | 181 | 171 | 167 | 163 | 173 |
| 11 | 191 | 187 | 183 | 177 | 176.5 | 183 |
| 15 | 200 | 196.5 | 193 | 189 | 192 | 194 |

## TABLE IV.

*Potential Difference between Cathode and Points in the Gas at a Given Number of Mean Free Paths from it. For Twice Normal Current Density: ($\bar{j}\lambda^2$ = .076).*

| No. of m. f. p. | Gas Pressures. | | | | | Means. |
|---|---|---|---|---|---|---|
| | 1.1 mm. | 1.6 mm. | 2 mm. | 2.5 mm. | 3 mm. | |
| 2 | 120 volts | 130 | 124 | 131 | 130 | 127 |
| 3 | 138.5 | 145 | 141.5 | 144 | 144.5 | 143 |
| 4 | 154.5 | 161 | 152 | 157 | 157 | 156 |
| 5 | 170 | — | — | 165 | — | 167 |
| 6 | — | 185 | 177 | 177 | 172 | 178 |
| 7 | 191 | — | — | 184 | — | 187 |
| 8 | — | 198 | 193 | 192.5 | 190 | 193 |
| 9 | 203 | | | | | |
| 10 | | 203 | 201 | 198 | 200 | 200 |
| 12 | 207 | 204.5 | 203 | 201 | 205.5 | 204 |

## TABLE V.

*Potential Difference between Cathode and Points in the Gas at a Given Number of Mean Free Paths from it. For Four Times Normal Current Density: ($\bar{j}\lambda^2$ = .152).*

| No. of m. f. p. | Gas Pressures. | | | | | Means. |
|---|---|---|---|---|---|---|
| | 1.1 mm. | 1.6 mm. | 2 mm. | 2.5 mm. | 3 mm. | |
| 2 | 149.5 volts | 160 | 162.5 | 168 | 174 | 163 |
| 3 | 173 | 177 | 180 | 184 | 186 | 180 |
| 4 | 191 | 195 | 192.5 | 198 | 202 | 196 |
| 5 | 205.5 | 208 | — | 206 | — | 206 |
| 6 | — | — | 218 | — | 216 | 217 |
| 7 | 224 | 223 | — | 221 | — | 223 |
| 10 | 226 | 225 | 229 | 222 | 232 | 227 |

density by substitution in their respective expressions on the right hand side of (24). The observed potentials at two mean free paths from the cathode were used in each case as the starting point, and the number of mean free paths from the cathode to the negative glow (point of zero gradient) were obtained from the curves of Fig. 1.

TABLE VI.

*Comparison of Calculated with Observed Potentials.*

| Dist. from Cath. (m. f. p.) | "Normal" Current Density. | | | "Twice-Normal" Current Density. | | | "Four-Normal" Current Density. | | |
|---|---|---|---|---|---|---|---|---|---|
| | Calc. | Limits. | Obs. | Calc. | Limits. | Obs. | Calc. | Limits. | Obs. |
| .2 | 77 | 78 | — | 100 | 102 | — | 129 | 131 | — |
| 1 | 88 | 88 | — | 112 | 113 | — | 146 | 146 | — |
| 2 | 100 | 100 | 100 | 127 | 127 | 127 | 163 | 163 | 163 |
| 3 | 112 | 109 | 112 | 142 | 139 | 143 | 179 | 177 | 180 |
| 4 | 123 | 119 | 123 | 155 | 150 | 156 | 194 | 189 | 196 |
| 5 | 134 | 128 | 136 | 167 | 161 | 167 | 208 | 199 | 206 |
| 6 | 144 | 136 | — | 179 | 170 | 178 | 220 | 206 | 217 |
| 7 | 154 | 144 | 155 | 190 | 178 | 187 | 229 | — | 223 |
| 8 | 164 | 152 | — | 199 | 184 | 193 | — | — | — |
| 8.5 | — | — | — | — | — | — | 238 | 219 | 227 |
| 9 | 172 | 158 | 173 | 207 | — | — | | | |
| 10 | 180 | 163 | — | 214 | — | 200 | | | |
| 11 | 187 | 168 | 183 | — | — | — | | | |
| 11.5 | — | — | — | 220 | 199 | 204 | | | |
| 12 | 193 | 169 | | | | | | | |
| 13 | 199 | — | | | | | | | |
| 14 | 203 | — | | | | | | | |
| 15 | 205 | 181 | 194 | | | | | | |

These theoretical values are given in Table VI. along with the corresponding observed magnitudes—the means of Tables III., IV. and V. The observed values, with very slight exceptions, are consistent with the calculated limits.

*Mobility of the Positive Ions.*—The quantity $K$ will obviously serve for calculating $\overset{+}{\mu}$, the mobility of the positive ions. As used in (17)—l. c., p. 488—this mobility is defined as the velocity of the ions in unit electrostatic field. In calculating the above value of $K$ the potentials were measured in volts, the current in milliamperes, and the mean free path of the electrons in millimeters. The theory is developed however with the c.g.s.e.s. system of units as a basis. Using this system for the electrical units and $\bar{\lambda}$ in centimeters we obtain for the corresponding values of $K$.

| | In Practical Units. | In c.g.s.e.s. Units. |
|---|---|---|
| Mean of lower limits of $K$............ | 9.9 | $2 \times 10^{-4}$ |
| Mean of upper limits of $K$............ | 12.6 | $2.4 \times 10^{-4}$ |
| $K$ as used.......................... | 11.5 | $2.2 \times 10^{-4}$ |

Using the c.g.s.e.s. values for $K$ it follows from (17), (17$a$) and (24) that

$$\overset{+}{\mu} = \frac{16\pi\bar{\lambda}}{9K^2}.$$

This gives for a gas pressure of one millimeter ($\bar{\lambda}$ = .072 cm.) the values of $\overset{+}{\mu}$ corresponding with those of $K$ in the preceding table.

| | In Unit Electrostatic Field. | In Field of One Volt per Cm. |
|---|---|---|
| Lower limit of $\overset{+}{\mu}$ | $6.7 \times 10^6$ cm./sec. | $2.2 \times 10^4$ cm./sec. |
| Upper limit of $\overset{+}{\mu}$ | $10.8 \times 10^6$ | $3.6 \times 10^4$ |
| Mean ($K = 2.2 \times 10^{-4}$) | $8.2 \times 10^6$ | $2.7 \times 10^4$ |

These values of $\overset{+}{\mu}$ are about six times the magnitude as customarily obtained, with, however, very much smaller potential gradients per mean free path.[1]

There is no doubt that the mobility of the electrons increases at low pressures much more rapidly than their mean free path. The above is however the first experimental evidence that the same occurs with the positive ions.

If we take into account the velocity as imparted by the field between collisions the high value for the mobility given here does not appear unreasonably large. The field strengths under which the above measurements were made run up to 2 and 3 volts per mean free path of the positive ion—or molecule. To a freely accelerated molecule of hydrogen with univalent charge, potentials of these magnitudes would impart velocities respectively of $1.4 \times 10^6$ cm./sec. and $1.7 \times 10^6$ cm./sec. The square root of the mean square velocity of the hydrogen molecules at ordinary temperature is about $1.7 \times 10^5$ cm./sec., or only one-eighth to one-tenth the accelerated velocity imparted between collisions (the ion considered as starting from rest after each collision). Where the accelerated velocity of the ion is larger than the temperature velocity of the molecules, we should expect a relatively larger mobility than where it is small compared with the temperature velocity—as has been the case with previous measurements.

*Test of the Theory of Rebound.*—From the fact that the difference of potential between the cathode and any definite number of mean free paths distance from it is simply a function of ($j\bar{\lambda}^2$) it follows that the immediate fall at the cathode is also simply a function of the same quantity—it being assumed to be over a distance of one m.f.p. of the

[1] From the value given in Winklemann for atm. pressure that at one mm. should be $4.3 \times 10^3$ cm./sec. with one volt per cm.

positive ion—or molecule. This requires that the factor $P$ in (22) and (23)—l. c., pp. 492, 493—be either *constant* with varying current, or, that it be a *function also of* $(j\bar{\lambda}^2)$. If it is a constant (22) requires the fall over the region of rebound to be proportional to $(j\bar{\lambda}^2)^{2/3}$. The theoretical values in table VI agree closely in fixing the values of $V_b$ (at .2 m.f.p. of the electron from the cathode) for the different current densities. The following table taken from these serves to show up the nature of $P$.

| $V_b$ | Ratios. | $(j\bar{\lambda}^2)^{\frac{2}{3}}$ | Ratios. | $(j\bar{\lambda}^2)^{.38}$ | Ratios. |
|---|---|---|---|---|---|
| 77 | 1 | .113 | 1 | .289 | 1 |
| 101 | 1.31 | .180 | 1.59 | .375 | 1.30 |
| 130 | 1.69 | .285 | 2.53 | .488 | 1.69 |

These prove distinctly that $V_b$ is not proportional to $(j\bar{\lambda}^2)^{2/3}$, but instead is, for the few current densities used, very closely proportional to $(j\bar{\lambda}^2)^{.38}$, which requires that $P$ *decrease* with increasing $V_b$. The theory, as already stated, requires it to *increase*.

*Magnitude of the Electron Current from the Cathode.*—From equations (4) and (8)—l. c., p. 486—the ratio of the electron current leaving the cathode $j_k$ to the total current $\bar{j}$ may be derived.

$$\frac{\bar{j}_k}{\bar{j}} = \frac{\bar{\mu}}{\bar{\mu} + \overset{+}{\mu}} \cdot 2^{-\left\{\frac{x_k - x_a}{\bar{\lambda}} + \frac{V_0 - V_a}{\epsilon}\right\}}.$$

Where $\bar{\mu}$ and $\overset{+}{\mu}$ are the mobilities in the very low gradient region of the negative glow, $[(x_k - x_a)/\bar{\lambda}]$ the number of m.f.p. from the cathode to where the potential gradient per m.f.p. drops below $\epsilon$, and $(V_0 - V_a)$ the fall of potential between this point and the negative glow.

The following table gives these magnitudes as obtained from the curves in Fig. 1. It may be noted that an error in locating $x_a$ has no appreciable effect on the result; also, that $\bar{\mu}$ and $\overset{+}{\mu}$ being assumed to have the same value does not affect the results as much as the natural errors in the exponential quantities.

| Current Density. | $\left(\frac{x_k - x_a}{\bar{\lambda}}\right)$ | $\left(\frac{V_0 - V_a}{\epsilon}\right)$ | $\left(\frac{\bar{j}_k}{\bar{j}}\right)$ Calc. |
|---|---|---|---|
| "Normal" . . . . . . . . . . . . . | 5 | 5.3 | $4 \times 10^{-4}$ |
| 2 × "Normal" . . . . . . . . . | 5 | 3.4 | $15 \times 10^{-4}$ |
| 4 × "Normal" . . . . . . . . . | 4 | 2.9 | $42 \times 10^{-4}$ |

These results are enlightening in that they show, under the theory here advanced, what an extremely small part of the current is furnished by the initial ionizing agency.[1]

The foregoing table shows too that the ratio of electron emission to the current density increases with the latter. It is interesting to know to what magnitudes this ratio may rise. We have carefully measured the extent of the cathode dark space in hydrogen and found that it remains constant—about 6.5 m.f.p. from the cathode—over an extreme range of current densities. As the current density increases, the point of zero gradient approaches closer and closer to the edge of this dark space but never quite reaches it. This suggests, that with the highest current densities (there being a correspondingly abrupt rise in the potential gradient from its zero value in the negative glow) the number of ionizations between cathode and negative glow could not drop much below *six*—giving an upper limit of about .01 for $(\bar{j}_k/j)$.

In nitrogen, on the other hand, the cathode dark space decreases considerably with increasing current density—*e. g.*, with a pressure as low as 2 mm. it can, by increasing the current, be made to almost disappear. This suggests that $(\bar{j}_k/j)$ may rise to a distinctly larger magnitude in nitrogen than in hydrogen.

*Explanation of the " Normal " Cathode Fall.*—The curves of Fig. 1 and calculated values in Table VI. both show that while the immediate fall at the cathode increases with the current density, the fall over the rest of the distance to the negative glow decreases. This decrease in the gas arises from the increased ratio of the electron current from the cathode to the total current, which, by the relative increase in ionization it produces, reduces the distance to the point of minimum gradient.

Conversely then, as the current density is decreased, a certain point should be reached where the rate of *decrease* in the immediate cathode fall is just balanced by the rate of *increase* in the rest of the cathode fall. At that current density the cathode fall would be a minimum.

An experimental verification of this view was incidentally obtained with an aluminium cathode in hydrogen at a pressure of .5 mm. The normal current density was sought by starting at a higher value then gradually reducing the current and simultaneously observing the cathode fall. This latter dropped to a magnitude of about 232 volts, then rose again with a further reduction of the current—being carried to a value of about 240 volts without the luminosity on the cathode indicating an uneven distribution of the current over its face. The observations were

[1] The above results refer of course to an aluminium cathode. With a steel cathode in hydrogen, and normal current density $(\bar{j}_k/j)$ comes to about $5 \times 10^{-5}$—which is only one eighth the ratio for aluminium. This is consistent with the well known fact that aluminium is a much more copious source of cathode rays than the heavier metals.

repeated with the same result. The current density at which this minimum cathode fall was observed proved to be of the same magnitude as the normal current density obtained at higher gas pressures.

We may conclude therefore that the " normal " cathode current density is that giving a minimum cathode fall, which condition (neglecting small effects beyond the negative glow) corresponds to a minimum loss of electrical energy in sending a given current through the gas.

Similar conclusions may be drawn from (23)—l. c., p. 493—by enquiring under what current densities the expressions on the right hand side have a minimum value. Inasmuch however, as this enquiry cannot be made to result in quantitative results, at present at least, it does not appear wise to burden this article with the discussion.

I wish to express my indebtedness to Dr. A. Q. Tool for his assistance in the experimental work and also for valuable suggestions on the theoretical part.

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February, 1915.