ON THE PRESENT THEORY OF MAGNETISM.

BY JAKOB KUNZ.

THE electron theory seems to account for the magnetic phenomena in a very direct way. Indeed we have only to assume that the molecular currents of Ampere, which form the elementary magnets, are revolving electrons in order to express Ampere's theory of magnetism in terms of the electron theory. Nevertheless it was only on the basis of the researches of P. Curie that P. Langevin was able to account for the difference in diamagnetism and paramagnetism. Curie found that the diamagnetic susceptibility is independent of the temperature while the paramagnetic susceptibility is inversely proportional to the absolute temperature. Langevin concluded that there is a fundamental difference between diamagnetic and paramagnetic properties. In Langevin's theory the diamagnetism is a characteristic property of each atom which contains a certain number of revolving electrons. If the resultant magnetic moment of these electrons for an external point is zero, then the body is diamagnetic, the action of an external magnetic field consists in a change of the orbit, giving rise to the diamagnetic modification of the atom. If the revolving electrons possess a resultant magnetic moment, the body is paramagnetic. Matter in all its forms is diamagnetic; paramagnetism, whenever it appears, covers as it were the diamagnetism, and there is no transition between the two distinct groups. We shall add a short deduction of the diamagnetic susceptibility.

We consider in a diamagnetic gas an atom with an electronic orbit, of radius r, the electron e revolving with velocity v, in a plane perpendicular to the magnetic field. The moment will be equal to $M = \pi r^2(e/T)$ without magnetic field; if a field H is applied, the time of vibration T and the angular velocity will change, so that

$$dM = e\pi\left(\frac{2r}{T}dr - \frac{r^2}{T^2}dT\right),$$

or neglecting the first part

$$= -e\pi r^2\frac{dT}{T^2},$$

$$\frac{mv^2}{r} = f.r \text{ or } f = \frac{mv^2}{r^2},$$

$$\frac{mv^2}{r'} = fr' - \text{Hev}; \frac{mv^2}{r'^2} = f - \frac{\text{Hev}}{r'} = \frac{mv^2}{r^2} - \frac{\text{Hev}}{r'},$$

$$\frac{mv^2}{r'^2} - \frac{mv^2}{r^2} = \frac{\text{Hev}}{r'},$$

$$m2\pi\left[\frac{1}{T'^2} - \frac{1}{T^2}\right] = \frac{He}{T'},$$

$$m4\pi\frac{dT}{T^2} = He; \frac{dT}{T^2} = \frac{He}{4\pi m},$$

$$dM = \frac{-e^2r^2}{4m}H.$$

If there are N orbits per unit volume and if the axes are uniformly distributed in all directions, then we have

$$M = -\frac{e^2r^2N}{12m}H, \text{ or } k = -\frac{e^2r^2N}{12m}.$$

Apparently N and r are independent of the temperature. This theory of Langevin of the diamagnetic susceptibility k is at the same time the theory of the Zeeman effect.

In order to find the expression for the paramagnetic susceptibility fo a gas, we shall use a method quite different from that of Langevin. Let the angle between an external magnetic field H and the direction of the moment M of an elementary magnet be equal to α, the work required in order to rotate the magnetic particle from the direction H into its present direction will be equal to $W = -MH\cos\alpha + C$; the heat of the gas will change by this amount, and in order to keep the temperature constant, we have to add a quantity of heat $Q = -W = MH\cos\alpha$, and the increase of entropy will be equal to $S = (Q/T) = (MH\cos\alpha/T)$; and this entropy will be proportional to the logaithm of the probability P that we find the magnet in the direction α

$$S = R\log P + \text{const.} = \frac{MH\cos\alpha}{T},$$

$$P = e\frac{MH\cos\alpha}{RT},$$

and the number of magnets which are found in an angular interval $d\alpha$ will be proportional to P, or

$$dn = Ke\frac{MH\cos\alpha}{RT}\cdot d\omega,$$

where

$$d\omega = 2\pi \sin\alpha \, d\alpha,$$

$$dn = Ke^{\frac{MH\cos\alpha}{RT}} 2\pi \sin\alpha \, d\alpha,$$

the total number N of molecules per unit volume will be

$$N = 2\pi K \int_{\alpha}^{\pi} e^{a\cos\alpha} \sin\alpha \, d\alpha,$$

where we put a for (MH/RT),

$$N = \frac{4\pi K}{a} \sin ha$$

and the intensity of magnetization $\mathfrak{J}$ becomes:

$$\mathfrak{J} = \int_0^{\pi} M\cos\alpha \, dw$$

$$= MN\left(\frac{\cos ha}{\sin ha} - \frac{1}{a}\right).$$

The maximum value of the intensity of magnetization $\mathfrak{J}_m = MN$, hence

$$\mathfrak{J} = \mathfrak{J}_m\left(\frac{\cos ha}{\sin ha} - \frac{1}{a}\right) = \mathfrak{J}_m\left(\frac{1}{3}a - \frac{2}{90}a^3 + \frac{4}{45\cdot 42}a^5 - \ldots\right).$$

Neglecting the higher powers of $a = MH/RT$ we find

$$\mathfrak{J} = \mathfrak{J}_m \frac{MH}{3RT} = kH,$$

$$k = \frac{NM^2}{3RT},$$

that is, the paramagnetic susceptibility is inversely proportional to the absolute temperature; that is the rule of Curie.

EXPERIMENTAL FACTS.

The phenomena are far more complicated than the theory of Langevin indicates. The investigations of H. DuBois, K. Honda, M. Owen, Kamerlingh Onnes, P. Weiss, A. Perrier and others have revealed a very large variety of phenomena, in which the rules of Curie are altogether exceptions so that we have to extend or abandon the present theories.

The diamagnetic susceptibility should be an atomic property, which is independent of temperature, of a change of state, of a polymorphic transformation or of chemical combination. This is not the case. For instance, the diamagnetic susceptibility of amorphous carbon, of Cu, Zn, Zr, Cd, Sn, Sb, Te, Tl, J, Pb, Bi, decreases with increasing temperature, k in the melting of Ag, Sn, Sb, Ga, Ge, Au, Hg, Tl, Pb, Bi changes

discontinuously. In the polymorphic transformation of C, S, Sn, and Tl the susceptibility changes abruptly, even the sign changes in the polymorphic transformation and during the melting process of tin. In the case of boron (0–400° C.), diamond, silver and iodine (0–114°) the diamagnetic susceptibility increases with the temperature. There are only a few elements whose diamagnetic k remains constant within a certain interval of temperature; the diamagnetic susceptibility of an inorganic compound is not an additive property. Oxygen, for instance, is a strongly paramagnetic element, but if it combines with the paramagnetic elements of Be, Mg, Al, Mo, W, Th, it forms diamagnetic oxides. And in general the diamagnetic and paramagnetic properties depend so much on physical and chemical influences, that one might be inclined to ascribe them to electrons which are revolving on the surface of the atom. In organic compounds, at all events, it has been shown by P. Pascal that the molecular susceptibility X is an additive property of the atomic susceptibility. Oxygen in these compounds may be para- or diamagnetic. In more complicated compounds, the structure has a great influence on X. The diamagnetic constants are on the whole not smaller than the positive paramagnetic values.

The diamagnetic susceptibility of graphite is greater than the paramagnetic susceptibility of such an element as manganese, one of the strongest paramagnetic elements; charcoal, bismuth and antimony have also large negative susceptibilities. Besides in the crystals of graphite and antimony k varies with the direction of the axes. All these facts seem to indicate that what we observe is the difference between a positive and negative magnetism.

A similar variety of phenomena is observed in paramagnetism. Oxygen follows Curie's law at ordinary and at higher temperatures, but at lower temperatures the susceptibility varies inversely as the square root of the absolute temperature and finally probably becomes constant. Over certain intervals of temperature the susceptibility remains constant in the elements: Na, Al, K, V, Cr, Nb, W, Os, and even increases with increasing temperature in the case of Ti, V, Cr, Mn, Mo, Ru, Rh, Ir, Th. The ferromagnetic metals above the critical temperature, where the ferromagnetism disappears, seem to follow Curie's law, probably with the exceptions of the compounds Fe_3O_4 and pyrrhotite.

An extension of Langevin's theory is necessary both for the diamagnetic and the paramagnetic susceptibility. In the first place, in Langevin's theory it is silently assumed that the moments of the elementary magnets are independent of the temperature. This assumption is by no means self-evident. The electrons revolve in the outer layers

of the atoms probably and the moment of a molecule is the resultant of the moments of the atoms. With increasing temperature we have reasons to believe, the atoms share the energy of temperature agitation and the resultant moment of the molecule may be affected. Besides the fact that the diamagnetic susceptibility changes abruptly, in polymorphic transformations, in changes of state, in chemical transformations, indicates, that the diamagnetism is not simply an additive atomic property. In general we have to put:

$$M = M_0 f(T).$$

In the second place Langevin's theory of paramagnetism applies only to gases and dilute solutions. The resultant magnetic moment per unit volume depends only on the directing power of the field and on the "scattering" power of the temperature agitation. The equilibrium between these two effects leads to Curie's rule. But as soon as we consider a more condensed state of aggregation, the molecules will exert an influence on each other, and this influence for crystals will vary in different directions. P. Weiss has indeed extended Langevin's theory to ferromagnetic substances by adding to the external field H an internal or molecular field $N\mathfrak{J}$ which is proportional to the intensity $\mathfrak{J}$ of magnetization. In this way P. Weiss was able to explain a large number of phenomena of ferromagnetic crystals. A similar influence however must exist in paramagnetic solid and liquid substances. The mutual action of the molecules will be a certain function of the temperature $f(T)$ and will in general oppose the tendency of the external field to direct the elementary magnets, just as the temperature agitation; so that the energy of the opposing forces may be written in the form; $RT + f_1(T)$; the parameter a will now be equal to $\frac{M_0 f(T) H}{RT + f_1(T)}$ and $\mathfrak{J}$ will become:

$$\mathfrak{J} = M_0 f(T) N \left[\frac{\cos ha}{\sin ha} - \frac{1}{a} \right]$$

$$\mathfrak{J} = M_0 f(T) N \left[\frac{1}{3} a - \frac{2}{90} a^3 + \dots \right]$$

or approximately:

$$\mathfrak{J} = [M_0 \cdot f(T)]^2 \frac{HN}{3[RT + f_1(T)]},$$

$$k = \frac{[M_0 f(T)]^2 N}{3[RT + f_1(T)]}.$$

if $f(T)$ is a constant, for instance, equal to 1 and $f_1(T)$ also a constant ΘR, then we find

$$k = \frac{M_0^2 N}{3(RT + \Theta R)}$$

or

$$k(T + \Theta) = \text{constant},$$

a result which has been deduced by E. Oosterhuis[1] from Planck's theory of quanta of energy by entirely different considerations. At very low temperatures the specific heat of all substances seems to approach zero, the coefficient of expansion approaches zero also, the electrical conductivity of metals becomes very large, if not infinite, the thermal conductivity increases rapidly. All these properties can be explained by the assumption that at these very low temperatures in the neighborhood of the absolute zero the molecules gradually lose their mobility, and a given substance at absolute zero is a real solid body, as it were one large molecule, where the molecular mobility has disappeared; that means that the influence of the temperature agitation of the individual elementary magnets becomes weaker and weaker and that we can not even define the molecular magnet, because the whole system of magnets is as it were solidified, so that even at the absolute zero saturation of a substance is impossible and that the influence of temperature becomes smaller and smaller or the paramagnetic susceptibility becomes constant. This seems to be the tendency of solid oxygen.

If now at the lowest temperatures the function $([f(T)]^2/RT + f_1(T))$ is a constant, and at rather high temperatures is equal to $1/T$ and changes continuously from one extreme to the other, then it will in intermediate temperatures be approximately equal to $1/T^{\frac{1}{2}}$, and for this interval we shall have:

$$k = \frac{M_0^2 N}{3T^{\frac{1}{2}}},$$

a result which has been obtained by H. Kamerlingh Onnes and A. Perrier for liquid and solid oxygen. The formula $k(T + \Theta) = C$ or $X(T + \Theta) = C$, where X is the molecular susceptibility will be tested by means of measurements made in Leiden[2] on manganese sulfate.

In the next place the free electrons will be considered as contributing to the diamagnetic susceptibility. It has been shown by H. E. Dubois and K. Honda that the diamagnetic susceptibility of amorphous carbon, Cu, Zn, Cd, In, ℑ, Sb, Te, Tb, Pb, Bi, Sb decreases with increasing temperatures. The strongest diamagnetic metals are Sb and Bi, which show also a very large Hall effect. The magnetic field seems to act on

[1] Die Abweichungen vom Curie'schen Gesetz im Zusammenhang mit der Nullpunktsenergie, Phys. Zeitschrift, Vol. XIV., p. 862, 1913.

[2] Leiden, Comm. No. 132e.

$M_nSO_44H_2O$				M_nSO_4			
Tabs.	$X \cdot 10^6$ Abs.	$C10^{10}$	$\Theta = 1.18$	Tabs.	$X \cdot 10^6$	$C10^{10}$	$\Theta = 26.5$
288.7	66.3	1.92		293.9	82.8	2.82	
169.6	111.5	1.905		169.6	144.2	2.83	
77.4	247	1.93		77.4	274.8	2.85	
70.5	270	1.93		64.9	314.5	2.87	
64.9	292	1.93		20.1	603	2.82	
20.1	914	1.94		17.8	627	2.78	
17.8	1,021	1.94		14.4	636	2.60	
14.4	1,233	1.92					

the free electrons so that they move in spirals or circles and produce diamagnetism. E. Schrödinger[1] found for the contribution k of the diamagnetism, made by the free electrons:

$$k = -\frac{1}{3}\frac{e^2}{m}\lambda^2 N,$$

where N is the number of free electrons per unit volume, and λ the mean free path. The effect, calculated in this way, is 100 times too large for silver and copper, which seems to be another argument against the "free" electrons in metals. Nevertheless the action of temperature on diamagnetism and the fact that Sb and Bi have a large Hall effect and a large negative magnetism indicate that the conduction electrons contribute somewhat to the diamagnetism.

The Periodic System of the Elements and Their Magnetic Properties.

The elements may be arranged in series according to the atomic weights in different ways. A certain periodicity between atomic weights and magnetic properties always appears. If the atomic weights are represented by abscissæ and the magnetic susceptibilities as ordinates, the curve obtained is of a most irregular character, representing seven distinct maxima, among which that of the iron group is by far predominating. If only the sign of the magnetic properties is taken into account, one gets the best representation perhaps by the method of the helix due to B. K. Emerson, which is given in Fig. 1.

The strongly magnetic groups appear on a diameter, where we find Fe, Ni, Co, then Pd, Ru, Rh, then Gd, En, Sm, then Pt, Ɣr, Os. Moving on the spiral from iron to the right, we meet Mn and Cr, elements which are paramagnetic, but whose strongly magnetic properties appear only in some of their alloys and compounds such as the Heusler alloys, manga-

[1] Kinetische Theorie des Magnetismus, Sitz. Ber. der K. Akademie der Wissenschaften, II*a*, Bd CXXI., p. 1305, 1912.

nese-antimony, manganese-tin, manganese-zinc, Cr_5O_9. On the right hand side from the ferromagnetic elements, there are paramagnetic elements, on the left hand side the diamagnetic elements. Opposite to the magnetic metals there are the inert gases, which seem to be weakly diamagnetic. On the right-hand side of the inert gases we find the alkali metals whose weakly paramagnetic properties are not yet sufficiently known. The strongly magnetic metals, cobalt, nickel and iron, belong to the elements with minimum compressibility, with most complex

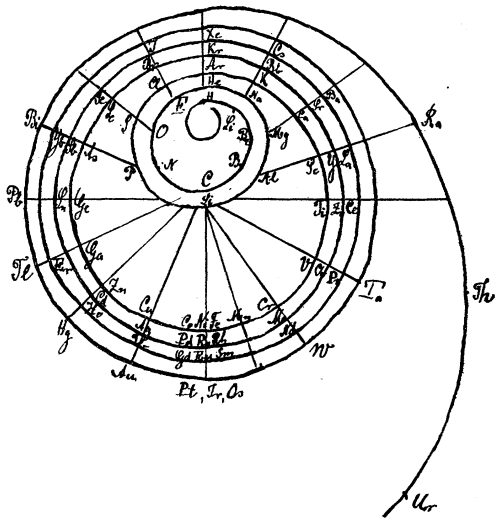

Fig. 1.

spectra, with complex double salts, with great condensation of mass, the heavy metals. Thus it looks as if condensation of electronic orbits were a maximum in these ferromagnetic metals and that the magnetic properties were related directly or indirectly to the mechanical, optical and chemical properties. It is very remarkable that immediately after the strongly magnetic metals there follow the diamagnetic metals:

	k
Cu	−0.66
Ag	−1.4
Tb	
Au	−2.6

On the next diameter we have:

Zn	−0.96
Cd	−1.16
Ho	
Hg	−2.6

When we move outward on a diameter of the spiral, the diamagnetic

susceptibility increases. The same rule is repeated by chlorine, bromine and iodine; sulphur, selenium and tellurium; phosphorus, arsenic, antimony, bismuth. If a represents the atomic weight, α and β two constants, then the atomic susceptibility can be represented for the last three groups by

$$X_a = - C_e^{+\beta a - \alpha}.$$

The same law seems to hold for all the groups of diamagnetic elements which are in the previous representation on the left of the diameter passing through the iron group and the inert gases. Thus, for instance, for zinc, cadmium and mercury we find:

$$X_{Zn} = \frac{0.96 \cdot 65.4}{7.1} 10^{-6} = 8.83 \cdot 10^{-6},$$

$$X_{Cd} = \frac{1.16 \cdot 112.4}{8.6} 10^{-6} = 15.2 \cdot 10^{-6},$$

$$X_{Hg} = \frac{2.6 \cdot 200}{13.6} 10^{-6} = 38.3 \cdot 10^{-6}.$$

If we put

$$x_a = 10^{\beta \cdot a + \alpha}$$

we find from Cd and Hg for α and β the values:

$$\alpha = 0.6146, \ \beta = 0.00502,$$

$\log X_{Zn} = 1.583$, the calculated value $= 1.618$.

The agreement is not so good for the last diamagnetic group of elements: Cu, Ag and Au, here the atomic susceptibilities are as follows:

$$X_{Cu} = 5.29 \cdot 10^{-6},$$
$$X_{Ag} = 14.39 \cdot 10^{-6},$$
$$X_{Au} = 26.55 \cdot 10^{-6}.$$

The copper seems to make an exception. Whether this is due to an inaccurate determination of X or to the fact that this element follows immediately after the iron group, remains an open question. Between the Zn, Cd, Hg group and the P, As, Sb group there are two more groups of diamagnetic elements, namely, those of Ga, In, Tl and Ge, Sn, Pb. The few values of X known for these elements show that this magnetic constant increases toward the periphery along the diamagnetic diameter of the spiral.

If we travel along the spiral from copper towards zinc and from silver toward cadmium, we find the following values for the atomic susceptibilities.

Cu	$5.29 \cdot 10^{-6}$	Ag	$14.4 \cdot 10^{-6}$
Zn	8.83	Cd	15.2
Ga		ℑn	
Ge		Sn	5.95
As	5.8	Sb	77.5
Se	24.0	Te	38.9
Br	21.9	ℑ	46.5

With the exceptions of As and Sn the atomic susceptibility increases from the south toward the north of the graphic representation. While in the strongly magnetic metals the susceptibility decreases as we move on the diameter outward, the diamagnetic susceptibility increases when we travel in the same direction. Oxygen occupies an exceptional position through its paramagnetic properties. Its regular diamagnetic properties appear only in some of the organic and inorganic compounds.

No theory of magnetism is complete, which is unable to account for the exceptionally high magnetic constants of iron, nickel and cobalt and of the other few ferromagnetic substances like the Heusler alloys. All elements can be divided into an electropositive and an electronegative group; all elements are either para- or diamagnetic. Just as we can try to ascribe the forces of affinity to electrical charges in the atom, we might try to reduce affinity to magnetic forces, or magnetons. It is very interesting to note that the strongest positive metals of the alkali group are the weakest paramagnetic elements; and that the most negative elements like F, Cl, Br, ℑ, are rather strongly diamagnetic. While the chemical properties of the most electropositive and electronegative elements may be explained by electrical forces, it seems possible to think that the magnetic forces due to magnetic doublets play a similar rôle in the chemical affinity of the elements with strongly magnetic properties. In this way we should get a periodicity of the magnetic properties as functions of the atomic weight as we have a periodic variation of the electropositive and negative properties of the elements. The graphical presentation of the law of periodicity shows the strongly magnetic metals just opposite to the strongly positive and negative metals. This explanation of the periodic variation of the magnetic properties would obtain strong support if it were possible to prove that all magnetons are identical just as all electrons are identical. But there is very little evidence in favor of the identical nature of all magnetons or elementary magnets as we shall see in the last paragraph. In three investigations[1] published in this journal the moments of the elementary magnets and the charge e have been determined for the following substances.

[1] The Absolute Values of the Moments of the Elementary Magnets of Iron, Nickel, and Magnetite, PHYSICAL REVIEW, Vol. XXX., p. 359, 1910. Stifler, PHYSICAL REVIEW, Vol. XXXIII., p. 268, 1911. P. Gumaer, PHYSICAL REVIEW, Vol. XXXV., p. 288, 1912.

	$m \cdot 10^{20}$	$e \cdot 10^{20}$
Fe	5.15	1.60
Fe_3O_4	2.02	0.90
Ni	3.65	1.54
Co	6.21	1.56
Heusler alloy 1	3.55	1.54
Heusler alloy 2	4.23	2.04

$1.53 \cdot 10^{-20} = e$ (average).

This value of e agrees fairly well with the values obtained by independent methods. On account of the necessary extrapolations it is difficult to obtain higher accuracy.

While I used the Langevin-Weiss theory for the determination of the elementary moment m, P. Weiss himself measured the intensities of magnetization at very low temperatures, and found noticeable deviations between the theory and the experiment in the temperature-intensity curve and he found at the same time a common divisor among the molecular intensities of the ferromagnetic substances. He called that divisor the magneton-gram, for which he gave the value 1,123.5. In addition the paramagnetic susceptibility of Fe_3O_4 above the critical temperature showed discontinuities as function of the temperature, which consisted of four straight lines, each of which led to a new determination of the magneton. Finally P. Weiss applied the equation

$$C_m = X_m T = \frac{{\sigma_{m0}}^2}{3r}$$

to solutions of paramagnetic substances containing iron and to a considerable number of solid salts. It has been shown, however, by Koenigsberger and Meslin that the molecular coefficient of magnetization of dissolved substances is a function of the concentration; at least for some solutions, while for others it seems to be constant. This fact makes it necessary to study solutions infinitely dilute or undissolved substances. The number of magnetons found by Weiss in the various substances is shown by the following series, in which the values coincide nearly with whole numbers.

10.41	28.83
21.89	26.99
21.96	28.94
24.04	29.19
28.03	21.23
27.93	25.05
30.09	17.97
25.99	20.04
27.11	12.12
27.91	20.16
27.69	20.16

If we displace the decimal point by one cypher to the left, we find again approximately whole numbers, which with one exception are almost as exact as the numbers given by P. Weiss. The numbers of magneton per molecule is rather high and it is not very surprising that in dividing for instance, 32,400 of $FeCl_3$ by 1,123.5 one finds approximately a whole number, 28.83 or 2.883 respectively. The large number of magnetons shown by the last two columns raises the question as to why those substances are so weakly magnetic, while nickel, being ferromagnetic, possesses at low temperatures only 3 magnetons. In addition, the numbers given by P. Weiss are based on the assumption that the magnetization of the pure salts and of the solutions varies according to Curie's law down to the absolute zero. As far as I know, this assumption has not yet been tested by experiments. Recently however Auguste Piccard[1] measured with great accuracy the susceptibility of oxygen at 20° C. and found for the moment of the atom: $7.8725 . 10^1$; dividing this number by 1123.4 one gets 7.007, a whole number again. But in this case H. Kamerlingh Onnes and A. Perrier have shown that at low temperatures the susceptibility of oxygen changes according to the law:

$$X_{\text{lig}} = \frac{2284}{\sqrt{T}} 10^{-6},$$

$$X_{\text{sol}} = \frac{1690}{\sqrt{T}} 10^{-6}.$$

If this element does not follow the law of Curie, solid salts and solutions will probably also show deviations, and at the same time the evidence in favor of the magneton will decrease. At all events the number of magnetons seems to vary in the atom of a given element like nickel, which contains 3 magnetons at low temperature, 8 at high temperatures, 9 at the limit of the alloys of iron and nickel, 16 in the solutions. In order to determine the magneton P. Weiss, abandoning the theory, has directly measured the molecular moments of iron, nickel and cobalt; Auguste Piccard, on the contrary, has used the theory in order to find the magneton in spite of the measurements of Kamerlingh Onnes and A. Perrier. If we assume that Curie's law holds down to the absolute zero, we find for the elementary moment of oxygen

$$m = 2.58 \cdot 10^{-20}.$$

The moment of each individual magneton of Weiss on the other hand would be equal to:

$$\frac{1123.5}{6.1 \cdot 10^{23}} = 1.85 \cdot 10^{-21}.$$

[1] Archives de Genève, Tome XXXV., p. 480, 1913.

If we determine the molecular moments by means of Langevin's theory, we find values varying from $2.02 \cdot 10^{-22}$ to $5.15 \cdot 10^{-20}$ for substances so different among each other as oxygen, iron, magnetite and Heusler alloys. These values are of the order of magnitude which we should expect from the theory of quanta by Planck. Let us assume that the kinetic energy of a revolving electron $\frac{1}{2}mv^2$ is equal to a whole number z times hn, then we find:

$$\frac{m\omega^2 r^2}{2} = zhn = \frac{h\omega z}{2\pi},$$

$$\omega r^2 = \frac{hz}{\pi m}.$$

The moment of M of the revolving electron will be equal to:

$$M = iA = \pi r^2 \frac{e}{T} = \frac{e}{m}\frac{hz}{2\pi},$$

if we put $z = 1$, we find $M = 1.83 \cdot 10^{-20}$; for the frequency n we find $1.63 \cdot 10^{15}$, assuming $r = 1.5 \cdot 10^{-8}$. Why are these magnetons not sources of light? Not much importance must be attached to this approximate coincidence of $1.83 \cdot 10^{-20}$ with the moments determined by means of Langevin's theory. We find indeed about the same magnetic moment without the theory of the quanta, by calculating the velocity of the electron by the equation:

$$\frac{mv^2}{r} = \frac{e^2}{r^2};$$

putting $r = 1.5 \cdot 10^{-8}$; we get $n = 1.4 \cdot 10^{15}$; $M = 1.54 \cdot 10^{-20}$. And the question arises again, why does such a magneton not emit light? The difficulty might be removed by admitting a large number of electrons revolving in a circle, instead of one electron. The assumption of one single magneton, identically the same in all substances, seems to require much more experimental support, if it exists at all.

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