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The Solid Angle Subtended by the Main and Shadow Cones of Cosmic Radiation

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For the calculation of the longitude and latitude effects, as well as the time variations, of charged primary cosmic radiation, the knowledge of the solid angle subtended by the main and shadow cones for all latitudes and energies is essential. This solid angle is calculated over a wide range of energy and latitude from the results of Lemaître and Vallarta, and from those of Schremp, and the results given in the form of several graphs.

IT is well known that the intensity of charged primary cosmic radiation of a given energy depends on the solid angle subtended at a given point of the earth by the allowed cone for that energy,[1] and also on the energy distribution of the primaries. As a consequence of Liouville's theorem the intensity, defined as the number of primaries of a given energy arriving at the observer per unit time, is given by the product of the number of particles of that energy multiplied by the aperture of the allowed cone at the given point and for the given energy. The knowledge of the aperture of the allowed cone is thus a matter of importance.

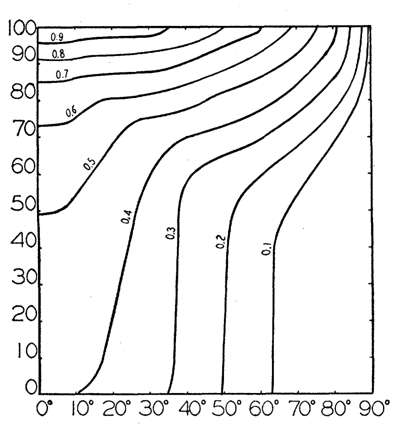

FIG. 1. The solid angle subtended by the shadow cone. Ordinates—percent; abscissas—latitude.

The theory of the allowed cone has now been pushed sufficiently far that accurate values for the aperture of the main and shadow cones can

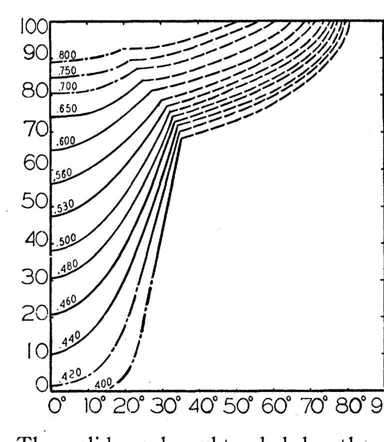

FIG. 2. The solid angle subtended by the main cone. Ordinates—percent; abscissas—latitude.

[1] G. Lemaître and M. S. Vallarta, Phys. Rev. **43**, 87 (1933); M. S. Vallarta, *An Outline of the Theory of the Allowed Cone of Cosmic Radiation* (University of Toronto Press, 1938).

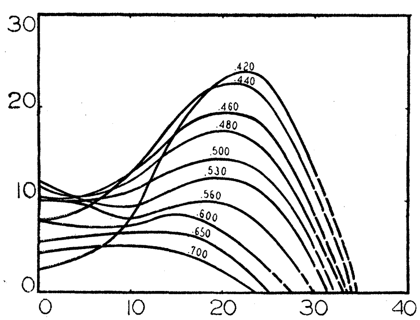

FIG. 3. The difference between the shadow cone and the main cone. Ordinates—percent; abscissas—latitude.

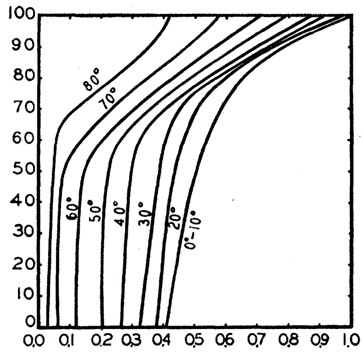

FIG. 4. The shadow cone as a function of energy for fixed latitudes, as shown. Ordinates—percent; abscissas—energy in störmers.

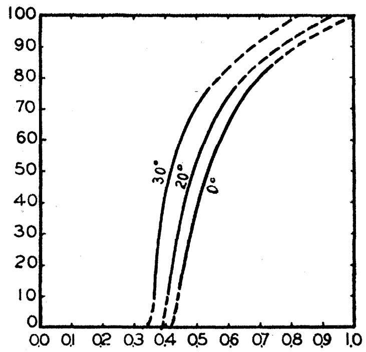

FIG. 5. The main cone as a function of energy for fixed latitudes, as shown. Ordinates—percent; abscissas—energy in störmers.

be given over a wide range of energy and latitude. An observer at a given point of the earth may define the direction of arrival of a cosmic-ray particle by using the angle θ between the tangent to the trajectory at the point of arrival and its projection on the meridian plane, and the angle η between this projection and the zenith. The solid angle subtended at the vertex by either the main or the shadow cone is then given by

$$\omega = \int\int \cos \theta d\theta d\eta,$$

the integration being extended over the boundary of the cone. This integration can be performed graphically or mechanically. The boundary of the main cone has been determined by Lemaître and Vallarta,[2] and that of the shadow cone by Schemp.[3] From their results the solid angle ω may be readily determined.

The results of this integration are shown in Figs. 1 and 2. Figure 1 gives the solid angle subtended by the shadow cone as a function of geomagnetic latitude and for different energies, the latter being expressed in Störmers.[4] It is worth recalling at this point that the shadow cone gives the upper limit of the aperture of the allowed cone for a given latitude and energy. Similarly Fig. 2 gives the solid angle subtended by the main cone as a function of latitude for different energies. The main cone gives the lower limit of the allowed cone for a given latitude and energy. The discontinuity of the curves in Fig. 2 is due to the fact that the main cone for a given energy is continued beyond a certain latitude by the shadow cone. The lowest part of certain curves (dot-dash) has been interpolated.

The difference between the main and shadow cones gives the region of penumbra. This difference is plotted in Fig. 3 for several energies as a function of latitude. The dashed part of each curve is extrapolated and doubtful. It is seen that the penumbra vanishes at high latitudes for energies higher than about 0.44 störmer. It is also worth noting that the maximum contribution of the penumbra is displaced to higher latitudes as the energy decreases.[5]

Finally, Figs. 4 and 5 give the solid angle subtended by the shadow and the main cones, respectively, as a function of energy and for a given latitude. In Fig. 4 the curve for $\lambda = 10°$ is omitted because it would be practically superposed on that for $\lambda = 0°$. These curves are very useful for the calculation of the latitude and longitude effects.

I wish to thank Professor M. S. Vallarta, who suggested this problem, for his guidance and encouragement. I am also indebted to Dr. R. A. Hutner and Dr. O. Godart for frequent discussions.

[2] G. Lemaître and M. S. Vallarta, Phys. Rev. **50**, 493 (1936).
[3] E. J. Schremp, Phys. Rev. **54**, 158 (1938); also his Ph.D. Thesis, M. I. T., 1937.
[4] For the equivalence between energy in störmers and in electron volts, see Lemaître and Vallarta, reference 1, Table VI, p. 90. For the definition, see M. S. Vallarta, reference 1, p. 7.

[5] See also R. A. Hutner, Phys. Rev. **55**, 15 (1939), Fig. 4.