### On the Dynamics of Complex Fission

We wish to report preliminary results of an investigation of the large amplitude distortions of a nucleus undergoing fission. The nucleus is assumed to have a uniform charge and mass density and a constant volume for all distortions. The radius vector $r$ is expanded in a series of Legendre polynomials $r=R(1+\Sigma a_n P_n)$, and the values of $a_0$ and $a_1$ are adjusted to insure constancy of the volume and center of gravity. An estimate of the quantum excitation energies shows that only the first five harmonics need be considered. The potential energy of distortion, including all terms so far evaluated, is given by

$$\begin{aligned}\Delta E=4\pi R^2O\{&0.4(1-x)a_2^2+(0.7143-0.4082x)a_3^2\\&+(1-0.3704x)a_4^2+(1.2727-0.3306x)a_5^2\\&-(0.03810+0.07619x)a_2^3-(0.07619+0.2503x)a_2a_3^2\\&-(0.1143+0.3429x)a_2^2a_4-(0.05195+0.2226x)a_3^2a_4\\&-(0.1732+0.6702x)a_2a_3a_5-(0.2171-0.2563x)a_2^4\\&-(1.3615-1.0934x)a_2^2a_3^2+(0.09350+0.03206x)a_2^5\},\end{aligned}$$

where $2x=(3z^2e^2/5R)/(4\pi R^2O)$ and $O$ is the surface tension.

The following conclusions may be drawn from this formula. (1) There are large coupling terms between odd and even harmonics. These provide a possible explanation of the marked asymmetry of fission.[1] Bohr and Wheeler[2] have neglected these coupling terms. (2) The coupling between $a_2$ and $a_4$ is such as to give an equatorial bulge to the drop when it is on the road to fission.[3] The drop is pinched in near both poles; this would lead to ternary fission were it not for the excitation of the third harmonic which causes one of the preformed droplets to come off first, leaving a residual fragment which has not a sufficiently high excitation energy for its low charge number to divide further. However, it seems possible that such a ternary fission may occasionally be observed, in view of the fact that the saddle point for this mode of fission corresponds to $a_3=0$, and moreover, appears to lie at very low energies. Thus, whether fission occurs principally through the coupling of $a_2$ and $a_3$ or through the coupling of $a_2$ and $a_4$, in either case the fragments are of unequal sizes. According to the experiments, it is very improbable that the nucleus should divide into nearly equal fragments.[1,4] (3) The coupling between $a_2$ and $a_3$ leads to an energy surface which has been investigated for a wide range in $x$. We have assumed $a_3$ to be of the order of $a_2^{\frac{3}{2}}$ and taken all terms in $\Delta E$ up through the fifth order in $a_2$. For $x=0.75$ we find a saddle point at $a_2=0.66$, $a_3=0.30$ with an activation energy $E_f=13$ Mev (assuming the surface energy of uranium to be 500 Mev); for $x=0.85$ there is a saddle point with $a_2=0.65$, $a_3=0.25$ and $E_f=6$ Mev. For such large amplitudes, the terms considered can give only a rough description of the fission process; moreover, theoretical uncertainties in the values of the nuclear radius and surface tension make any exact estimate impossible. These results are nevertheless of the right order of magnitude to explain the observed fission of uranium. (4) The energy surface for the $a_2-a_4$ coupling has been investigated for terms up through the fourth order in $a_2$ assuming that $a_4$ is of the order of $a_2^2$. For $x=0.75$ there is a saddle point at $a_2=0.47$, $a_4=0.057$ with $E_f=4.4$ Mev and for $x=0.85$ we find one at $a_2=0.31$, $a_4=0.029$ with $E_f=1.1$ Mev. (5) With all the mutual coupling terms for $a_2$, $a_3$ and $a_4$ included, there are two roots of the extremalizing equations which are of interest. One of these corresponds to $a_3=0$ and reproduces the results already stated in (4). The other leads to a reduction of the amplitudes and activation energies evaluated in (3). We have not yet included a sufficient number of terms to give a significant estimate of this effect, but it appears to be of the order of several Mev. In view of this fact and especially because of the modifications that may arise with the inclusion of higher order terms, we cannot conclude that the $a_2-a_4$ mode of fission requires less activation energy than the $a_2-a_3$ mode. (6) An estimate of the particle sizes to be expected from the $a_2-a_3$ mode of fission can be made by using the relative amplitudes of $a_1$, $a_2$ and $a_3$ at the saddle point. The two fission products have approximately 0.4 and 0.6 the mass of the original nucleus, this result being insensitive to the value of $x$. For the $a_2-a_4$ mode of fission the particle sizes are roughly in the ratio of 1:3 for binary fission and 1:2:1 for ternary fission. The first of these is consistent with the experimental energy peaks.[1]

Finally we want to emphasize that certain conclusions of this note are subject to possible modification due to the inclusion of higher order terms which are probably not small. A more detailed discussion of the dynamics of fission will be published shortly.[5]

R. D. PRESENT
J. K. KNIPP

Purdue University,
Lafayette, Indiana,
April 1, 1940.

[1] Booth, Dunning and Slack, Phys. Rev. **55**, 981 (1939); M. H. Kanner and H. H. Barschall, Phys. Rev. **57**, 372 (1940).
[2] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[3] In contradiction to the statement of Bohr and Wheeler, reference 2, p. 433, that $a_4$ contributes a concavity about the equatorial belt such as to lead continuously to a dumb-bell shaped figure. We do not check the higher order coefficients of their Eq. (22) and we note further that Eq. (23) is inconsistent in sign and magnitude with Eq. (22).
[4] L. A. Turner, Rev. Mod. Phys. **12**, 1 (1940), cf. table on p. 23.
[5] Related biophysical problems are being considered.

### Radio Isotopes of Chromium

We have investigated the induced radioactivities of chromium by the aid of our cyclotron. The results are summarized as follows:

(I) *Deuteron bombardment.* The exposure of chromium metal to about 100 microampere hours of 3-Mev deuterons yields chemically identified chromium isotopes of half-lives 1.6 hours and about 14 days.

(II) *Slow neutron bombardment.* In this case activity induced in chromium was very weak. The following periods were established: 2.8 hr., 14 hr., 1.7 hr. The purity of the sample was chemically tested and Al (1.1 percent), Si (0.17 percent), Fe (0.7 percent), Mn(>0.02 percent) were shown as impurities. Therefore the 2.8-hour activity seems to be due to $Si^{31}$ (2.5 hours) or $Mn^{56}$ (2.5 hours), and the 14-hour activity is probably due to $Na^{24}$ produced in aluminum impurity by the reaction Al-$n$-$\alpha$-Na, as a considerable number of fast neutrons were always among slow neutrons. The period 1.7 hours is very near to the

period 1.6 hours observed in the case of deuteron bombardment, and therefore can probably be ascribed to an isotope of chromium, but unfortunately no chemical identification was done because of very weak intensity.

(III) *Fast neutron bombardment.* Periods of 3–4 hours, 14 hours and about 12 days were observed. In this case the activities were very weak. The 3–4-hour activity is very likely due to $V^{50}$ (3.7 hours) according to the reaction $Cr^{50}$-*n*-*p*. The 14-hour activity is probably $Na^{24}$ as in the case of slow neutron bombardment. The period 12 days is probably due to the same activity (14 days), as was observed in the case of deuteron bombardment.

Now chromium has four stable isotopes $Cr^{50}$, $Cr^{52}$, $Cr^{53}$, $Cr^{54}$. According to the above results we can attribute the 1.6-hour activity to $Cr^{55}$, and the longer period (about 14 days) activity to $Cr^{51}$.

A more detailed account of this work will be given later in the *Scientific Papers of the Institute of Physical and Chemical Research.*

In conclusion we wish to express our best thanks to Dr. Y. Nishina for his kind interest.

TOSHIO AMAKI
TAKEO IIMORI
ASAO SUGIMOTO

Nuclear Research Laboratory,
Institute of Physical and Chemical Research,
Tokyo, Japan,
January 10, 1940.

## Spectroscopic Evidence for the $B_2$ Molecule

Modern valence theories lead one to expect that a molecule formed from two boron atoms should be stable.[1] The stability of $B_2H_6$ also suggests that a stable bond between two boron atoms is possible. But no evidence for the existence of the $B_2$ molecule has yet been reported. The observation of a $B_2$ spectrum, apart from showing that $B_2$ exists, also would be of interest because it would make possible a determination of the spin and the statistics of the boron nuclei.

We have therefore tried to observe a $B_2$ spectrum in a discharge through helium of about 10 mm pressure to which a trace of $BCl_3$ was added. Under these conditions we have been able to find between 3300 and 3170A a system of new bands having a simple fine structure (*P* and *R* branches only). The following considerations prove definitely that the emitter of these bands is the $B_2$ molecule whose existence is thus established for the first time.

(1) A clear intensity alternation is observed in the two bands that are least overlapped by others, the even-numbered lines being the strong ones. This shows definitely that the carrier of the spectrum is a homonuclear molecule.

(2) Each band is accompanied by a weaker isotopic band, whose intensity and displacement from the main band is accountable only on the assumption that the main bands are due to $B^{11}B^{11}$, the weaker bands to $B^{11}B^{10}$. The $B^{11}B^{10}$ bands, as is to be expected, do not show any intensity alternation. There are indications also of bands due to $B^{10}B^{10}$.

Only two sequences of the new system, $\Delta v=0$ and $\Delta v=+1$, have thus far been observed. A preliminary vibrational analysis yields the following formula for the band heads:

$$\nu=30546.1+(929.3v-2.75v^2)-(1035.2v''-9.58v''^2).$$

A preliminary rotational analysis yields $B_0''=1.205$ cm$^{-1}$, $B_0'=1.155$ cm$^{-1}$ and consequently for the internuclear distances $r_0''=1.595\times10^{-8}$ cm, $r_0'=1.628\times10^{-8}$ cm.

A rough estimate leads to a value of the intensity ratio of the strong and weak lines between 2 : 1 and 1.5 : 1 which would mean that the spin of the $B^{11}$ nucleus is very probably $I=\frac{3}{2}$. However, more accurate intensity measurements are necessary to fix definitely this value. By such measurements we also hope to determine the spin of the $B^{10}$ nucleus.

The fact that the bands have only one *P* and one *R* branch shows that they represent a $\Sigma-\Sigma$ transition. Since the lines with even values of *J* are the strong ones it follows, if Fermi statistics is assumed for the $B^{11}$ nucleus, that the lower state is a $\Sigma_g$ state and the upper state $\Sigma_u^-$. From the electron configuration one would expect the ground state of $B_2$ to be $^3\Sigma_g^-$ and a $^3\Sigma_u^-\rightarrow{}^3\Sigma_g^-$ transition to be a strong one. It seems very likely that the new bands are this transition, the triplet splitting being too small for detection. It is significant to find that the assumption of Fermi statistics for the $B^{11}$ nuclei leads to reasonable results since it is the statistics one would expect for this nucleus on the basis of modern nuclear theory.

When nitrogen was added to the discharge in He and $BCl_3$ two new band systems were found, one around 3600A, the other around 3225A. The former can be assigned definitely, the latter probably to the BN molecule whose spectrum has not been previously observed. When hydrogen was added two new electronic transitions of the BH molecule were found at 3415 and at 3100A.

A more detailed report of our experiments will be submitted to the *Canadian Journal of Research.*

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A. E. DOUGLAS
G. HERZBERG

Department of Physics,
University of Saskatchewan,
Saskatoon, Saskatchewan, Canada,
March 25, 1940.

[1] See, for example, G. Herzberg, *Molecular Spectra and Molecular Structure I* (Prentice-Hall, New York, 1939), p. 392.