A Critical Analysis of the Classical Experiments on the Relativistic Variation of Electron Mass

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(Received February 14, 1938)

In the course of a previous mathematical analysis of an improved method for the determination of the ratio e/m it was suggested that the generally accepted interpretations of the classical experiments of Bucherer and of Neumann might be subject to serious limitations. A similar analysis for the unmodified Bucherer-Neumann experiment is therefore carried out in the present paper. By this means it is discovered that Neumann's supposed velocity filter, even for the case of negligible scattering of electrons along the condenser plates, must have been completely broken down on the high velocity side, for values of $\beta > 0.7$; and that, even for the lower velocities observed, the resolution width was approximately *as great as that equivalent to the whole relativistic mass effect.* Besides, it is found from this analysis that, with a certain choice of geometrical constants, a very poor resolving power may be concealed by spurious focusing effects—and that it was just this choice that Neumann, by trial, found it necessary to make in order to obtain sharp lines. It is also seen that, when the filter is open on the high velocity side, one may still obtain an apparent line, which may be considerably shifted from the position predicted by the simple theory, and which in extreme cases may form the edge of a heavily fogged region extending inward to the central zero line corresponding to $\beta = 1$. This fogging can actually be seen very clearly in Neumann's photographs for $\beta = 0.8$. Further, a consideration of the possible effect of scattering on such experiments shows that, when the resolving power is very near the critical limit at which the filter breaks down, the probability of leakage of high velocity scattered electrons through the filter becomes relatively large, especially when the spectrum of the source is distributed largely amongst velocities above that to be observed. Under these latter circumstances it is suggested that one might obtain seriously shifted lines, and even mistake for the expected line a spurious maximum due to the source distribution itself. Since the interpretations of Bucherer and of Neumann are based on such doubtful premises, a short discussion is given of other similar experiments designed to distinguish between the Abraham and the Lorentz electrons. So far as is known to the authors, it appears that, at least for the higher velocities, no very satisfactory experimental distinction between the two types of electron has as yet been made by direct electric and magnetic deflection methods. In view of the fundamental importance of such experiments it seems that much is left to be desired.

IN a recent article[1] the authors described a method for the determination of the ratio e/m for beta-particles. This method was developed for the purpose of determining the specific charge of *primary* beta-particles, as distinguished from the secondaries; and it consists essentially in a modification of the Bucherer-Neumann experiment in which the positions of the source and the detector are effectively interchanged and as detector a Geiger counter is used in a fixed position. Preliminary measurements showed a main central peak and two side peaks, the origin of which was not quite clear, since they could have been due either to the characteristics of the ideal instrument, or to scattering, or to a bona fide mass effect. On examining the literature it was found that only an incomplete discussion was given of the resolution limitations of the classical experiments of Bucherer and of Neumann. Bestelmeyer had at the time raised objections along these lines by questioning whether or not the finite resolving power of the apparatus might cause shifts in the density maximum of the observed photographic line. In partial answer to this objection Bucherer made only a very special resolution calculation for the case $\beta = 0.3$; but, surprisingly, does not seem to have made similar tests for the cases of the higher velocities—although he found it impossible to obtain sharp lines for $\beta > 0.8$. In fact, the general behavior of a condenser in crossed fields seems not to have been clearly understood at the time.

Since a more complete understanding of the behavior of the condenser in such experiments was necessary for a careful interpretation of the results obtained by the above-mentioned modified method, it was decided to make a mathematical analysis of the instrumental behavior. Although the system of rays involved in experiments of this type is rather complicated, it was found not too difficult to determine the resolution characteristics for a particular observation. In

[1] C. T. Zahn and A. H. Spees, Phys. Rev. 52, 524 (1937). See also: C. T. Zahn and A. H. Spees, Phys. Rev. 53, 357 (1938).

this way a clear idea of the resolution limitations of the experiment was obtained; and it appeared that the modified method had, in this regard, very decided advantages over the original method. Closely connected with the effective resolving properties of the condenser is the possible presence of radiation scattered from the condenser plates, since the resolving power for the scattered radiation decreases very rapidly as the point of scattering approaches the condenser exit. Another advantage consisted in the fact that the scattering was largely eliminated. (For a more complete discussion of this point see the abovementioned article.[1])

In studying the system of rays involved in the modified method certain general relations were obtained for the behavior of electrons in the crossed fields between the plates of a shallow condenser. Since the latter problem was found not too difficult, it was decided to apply the same general method of analysis to the case of the Bucherer-Neumann experiment. This seemed to be particularly desirable, since from time to time questions have been raised in the literature as regards the interpretation of Neumann's results, especially at the higher velocities where the results should be more significant.

First, as has been mentioned, Bestelmeyer raised the question of the finite resolving power of the instrument. This question still does not seem to have been answered satisfactorily, although it is not so very difficult a matter to obtain a considerable amount of information in this connection. One would perhaps not expect very serious difficulties so long as one worked at velocities well below the limit at which resolution becomes impossible; but it should be remembered that the classical experiments were extended up to this limit. Besides, an examination of Neumann's original paper[2] shows that there were certain peculiar practical limitations on the ratio of the lengths of the electron path inside and outside of the condenser, which limitations were mentioned by Neumann, but left unexplained. Also the effect of scattering may be of great importance because of its effect on the resolving power. To take it into account quantitatively would be a very complicated problem in the case of the Bucherer experiment, but it should be of considerable value at least to determine the limitations of the apparatus for the idealized experiment where no scattering appears, and thereby to obtain a lower limit for the effect of the finite resolving power.

A further question which might be raised is the following. For a given experiment, with given values of β and a given relationship between mass and velocity, it is always possible to calculate the over-all spread in the momentum beam transmitted by the condenser. If this spread is of the order of magnitude of, or greater than the width of discrete peaks in the momentum distribution of the source, one must consider the possibility of false or shifted peaks due to the source itself. Except for the one particular case, $\beta=0.3$, no test along these lines seems to have been applied to the classical data.

It is not desired here to imply doubts as to the validity of the theory of relativity in this connection; but rather to point out that there are still certain rather serious difficulties, in the correct interpretation of Neumann's data, which make it impossible to state within what limits of error and over what region of velocities the theory of relativity has actually been verified experimentally by Neumann. Until a complete understanding of the resolution limitations is had and the effect of possible scattering is taken into account, such a statement will not be possible. In an experiment of such fundamental importance it seems especially desirable to have definite information as to the exact range of energies over which the theory of relativity has actually been verified. It is therefore the purpose of the present discussion to answer as many as possible of the above questions.

Theory of the Bucherer Experiment

In the development of the theory for the modified experiment[1] there were obtained certain general relations, which apply to the motion of the unscattered beam of electrons transmitted by a shallow condenser in crossed fields:

$$k_i=k(1-v_0/v), \tag{1}$$

where k_i is the curvature of the trajectory inside the condenser; k, that in the magnetic field alone; v, the velocity of the electron; v_0, the value of v

[2] G. Neumann, Ann. d. Physik **45**, 529 (1914).

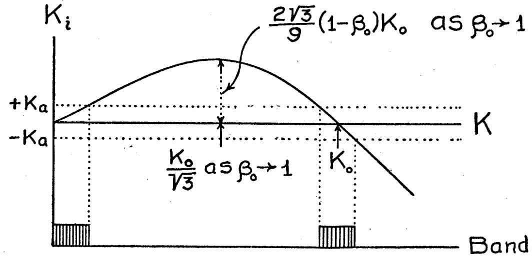

FIG. 1. Conditions for potential transmission of Lorentz electrons.

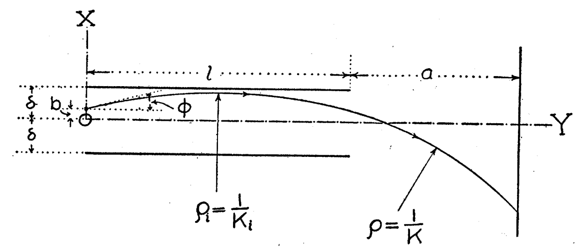

FIG. 2. Initial conditions imposed by the position of the source.

for which the electric and magnetic forces would be compensated and k_0 the curvature of an electron having a velocity v_0 in the magnetic field alone. When the relation between the mass and the velocity is given, it is possible to obtain an expression in terms of curvatures alone, which expression, for the case of Lorentz electrons, is:

$$k_i = k[1-((A^2+k^2)/(A^2+k_0^2))^{\frac{1}{2}}], \qquad (2)$$

where $A = eH/m_0c$. For the case of nonrelativity electrons this would have been:

$$k_i = k(1-k/k_0). \qquad (3)$$

It has been pointed out that for small spreads the resolution characteristics depend on $(\partial k_i/\partial k)_{k_0}$, and therefore on the particular relationship between k_i and k. One will therefore assume here that the theory of relativity is valid, and hence obtain the resolution limitations which would exist if the theory were true. The relativity relationship (2) is represented graphically in Fig. 1, where $k_a = 16\delta/l^2$ is the maximum curvature of circles passing freely between the plates of the shallow condenser. The value k_a represents a geometrical limitation, and the band of velocities shown in the figure represents the only velocities which could be freely transmitted under any, circumstances. Transmission will, however, be further contingent on the proper initial conditions of the trajectories. As was pointed out in the previous article,[1] when either k_a becomes too large, or k_0 too small (i.e., v_0 too large) one reaches a condition where the two bands spread out to meet each other and cover the whole region of velocities greater than v_0, as well as a small range of velocities slightly less than v_0. This indicated very serious limitations on the Bucherer experiment for the higher velocities.

In Fig. 2 is shown a diagram representing the conditions of Neumann's experiment. The source may be imagined to be distributed over the plane $y=0$ at the left end of the condenser at values $x=b$ and with $|b| < \delta$. For each value of the source position b the condenser will behave slightly differently. The initial conditions for a "ray" are specified by the two geometrical variables b and ϕ, and the two variables k_i and k, which latter two give the curvatures of the trajectory inside and outside the condenser, respectively. In terms of these variables the equation of the trajectory may be written as follows, for small values of x. (For the purpose of resolution calculations it should suffice to make this approximation for the region outside as well as that inside the condenser.)

$$x = b + \phi y - y^2 k_i/2 \qquad \text{for } 0 \overline{<} y \overline{<} l, \qquad (4)$$

$$x = b + \phi(a+l) - l(l+2a)k_i/2 - a^2k/2 \qquad \text{for } y = a > l. \qquad (5)$$

Cut-off conditions: In the previous article the general conditions for cut-off were expressed in terms of the three variables b, ϕ, and k_i. It was found that all rays *not* falling in the category defined by the following inequalities would be transmitted freely:

(I) $\qquad +\delta \overline{<} b \overline{<} -\delta,$

(II) $\qquad +\delta \overline{<} b + \phi l - l^2 k_i/2 \overline{<} -\delta,$

(III) Simultaneously:

$$0 \overline{<} \phi/k_i \overline{<} l,$$
$$+\delta \overline{<} b + \phi^2/2k_i \overline{<} -\delta.$$

Condition (I), in the case of the Bucherer experiment, requires simply that the source lie between the condenser plates. One may then consider the effect of the conditions (II) and (III) for various

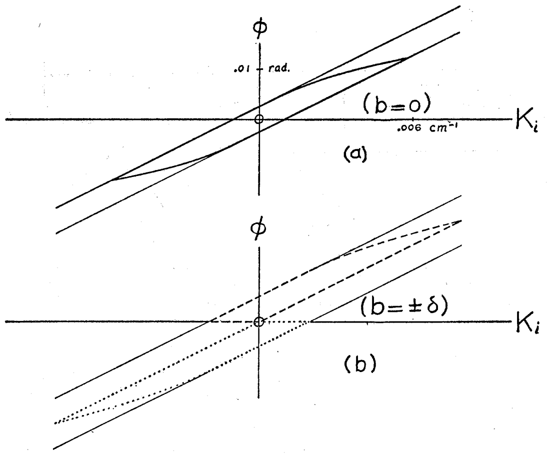

FIG. 3. Restricted areas in the ϕ, k_i plane.

values of b between $+\delta$ and $-\delta$. With a fixed value of b these conditions involve only the two variables ϕ and k_i; and restrict the transmitted rays to a certain area, in the ϕ, k_i plane, bounded by two parallel straight lines (end cut-off) and by two simple parabolae, each of which is tangent to one of the lines. In Fig 3a is shown the restricted area for the case of a source in the central plane $b=0$; and in Fig. 3b, the cases $b=+\delta$ (dotted lines) and $b=-\delta$ (broken lines). It may be noted in general that changing the sign of b corresponds to reflection of the restricted area through the origin.

For any given value of k_i the permissible spread in ϕ is given by the vertical section of the restricted area at a distance k_i from the origin; and, if the source is isotropic, this length represents the distribution function of the transmitted radiation in terms of k_i, to within a constant factor. (This, of course, will be true exactly, only if the spread is so narrow that the variations of the source distribution function are negligible.) It will also be obvious from the above-mentioned property of symmetry, that the integrated distribution function for the transmitted radiation from a source extending uniformly from $+\delta$ to $-\delta$ will be symmetrical around $k_i=0$. It may be noted further that as b changes from $b=0$ to $b=\pm\delta$ the maximum permissible values of $\pm\phi$ and $\pm k_i$ increase only by ratios $\pm 8/4.82$ and $\pm 16/11.65$, respectively. It is also rather easily seen that the effect of distributing the source over the distance 2δ is to give a beam in k_i somewhat sharper at the center, even if somewhat wider at the extreme ends. Hence the curve for $b=0$ may be regarded as giving a good qualitative picture of the resolution limitations.

The latter analysis determines the bundle of rays transmitted by the condenser, as a function of k_i. In order to obtain the spread in x, the measured deflection on the photographic plate, it is necessary to consider the equation of the trajectory *after the electron leaves the condenser*:

$$x=b+\phi(a+l)-l(l+2a)k_i/2-a^2k/2.$$

Now if the spread in k_i is small compared to k_0 one can set:

$$k_i=B(k-k_0),\quad \text{where}\quad B=(\partial k_i/\partial k)_{k_0}=-(1-\beta_0^2)$$

for Lorentz electrons. Then one can write:

$$x=-a^2k_0/2+b+\phi(a+l)-[l(l+2a)+a^2/B]k_i/2.$$

For the central compensated ray $b=\phi=k_i=0$, and $x=-a^2k_0/2\equiv x_0$; and setting $x-x_0\equiv\Delta x$, one may write:

$$\Delta x=b+\phi(a+l)-[l(l+2a)+a^2/B]k_i/2,$$

where Δx represents the departure from the ideal central line.

An examination of the latter equation shows that, if the constants of the apparatus be so chosen that for a given value of β_0 the coefficient of k_i vanish, then there will be energy focusing of the rays, and the line width will be due entirely to the spreads in ϕ and b. The condition for such energy focusing may be written:

$$l(l+2a)=a^2/(1-\beta_0^2)$$

or

$$l/a=((2-\beta_0^2)/(1-\beta_0^2))^{\frac{1}{2}}-1.$$

In Table I are shown the appropriate values of l/a for different values of β_0, under the condition for energy focusing.

Under the condition for energy focusing the maximum spreads in ϕ, corresponding to

TABLE I.

β_0	l/a
0.0	0.41
0.5	0.53
0.7	0.72
0.8	0.94
0.9	1.50
0.95	2.36

$b=\pm\delta$, are easily seen to be $\pm 8\delta/l$, and the corresponding limits in x are given by:

$$\Delta x=\pm(8a+7l)\delta/l. \qquad (6)$$

Furthermore it is obvious from Fig. 3b that the distribution in Δx is symmetrical around $x=x_0$ and that this may be true even if the line is rather wide, unless so wide that $B=\partial k_i/\partial k$ is no longer practically constant over the whole width. Therefore, in order to insure that the line is symmetrical, it would be necessary to determine the limits in k_i involved, and to test for uniformity of B over the allowed interval.

The latter condition for the energy focus permits a rather wide spread in x; but if one notes that the integrated spread in ϕ, as a function of k_i, is symmetrical around the line $\phi=lk_i/2$ and for a given value of b. ϕ lies between the lines $\phi=lk_i/2\pm\delta/l-b/l$, one sees that one can over-focus the energies in such a way as largely to compensate for the variations in x due to the spread in ϕ, and thus obtain a best quasi-focus. This may be seen by writing the latter limiting values of ϕ into the equation for Δx. Then the limits of Δx are:

$$\Delta x=-(a/l)b\pm(a+l)\delta/l-a(l+a/B)k_i/2,$$

which shows that, if one chooses the ratio l/a in such a way that $(l+aB)=0$, or $l/a=1/(1-\beta_0^2)$, one obtains the best quasi-focus with: $\Delta x=-(a/l)b\pm(a+l)\delta/l$; or if one writes in the limiting cases $b=\pm\delta$, then:

$$\Delta x=\pm(1+2a/l)\delta. \qquad (7)$$

One sees from the expressions (6) and (7) that for values of l of the same order as a, the spread Δx for the energy type of focusing is about five times as large as that associated with the best quasi-focus. In Table II are shown the values of the ratio l/a for the various values of β_0 under the conditions for the best quasi-focus.

It is noteworthy that for the energy focus one must have $l<a$ for values of $\beta_0\lessapprox 0.8$, whereas for the quasi-focus one must have $l>a$, always. Before considering the actual numerical values involved in Neumann's experiments, it is of interest to recall certain statements made by Neumann as regards the particular practical conditions he found necessary in order to obtain sharp lines. He mentioned that the determination of deflections was extremely difficult; and that so long as he confined himself to small velocities ($\beta_0=0.4\rightarrow 0.6$), and small distances a (4 cm), he obtained very fine sharp lines; and that for $\beta_0=0.6$ the lines were already indistinct for $a=5$ cm, and for still greater velocities they were very broad even with $a=4$ cm. No attempt was made at an explanation of these rather peculiar facts, but they are easily seen to be in harmony with the predictions of the foregoing analysis, if one remembers that for Neumann's condenser $l=5$ cm. The condition for the quasi-focus requires that $a<l$, and this is just the condition, roughly expressed, that Neumann found necessary for sharp lines. Now Neumann's plate separation $2\delta=0.25$ mm and his deflections were $5\rightarrow 10$ mm. From (7) one calculates that the *total* width of the quasi-focus should be about 0.7 mm, approximately; but since the line may be sharpened at the center because of the vertical extension of the source, it may appear somewhat narrower than this value. If, on the other hand, the condition for energy focusing had held, the line width should have been about five times this latter value, or about 3.7 mm, which would have been very wide, indeed, in consideration of the fact that the total

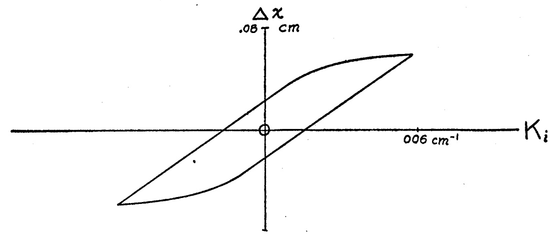

FIG. 4. Limiting values of the spread Δx.

TABLE II.

β_0	l/a
0.0	1.00
0.5	1.33
0.7	1.96
0.8	2.78
0.9	5.26

TABLE III. *Selected cases from Neumann's data.*

No.	H (GAUSS)	V (VOLTS)	β_0	DEFL. (CM)	a (CM)	ρ_0 (CM)
40	106.8	407.9	0.507	0.9714	4.1905	9.05
39	128.2	580.8	0.602	0.8961	4.1905	9.78
54	128.2	693.2	0.718	0.6427	4.2013	13.65
37	128.2	772.8	0.801	0.5004	4.1905	17.55

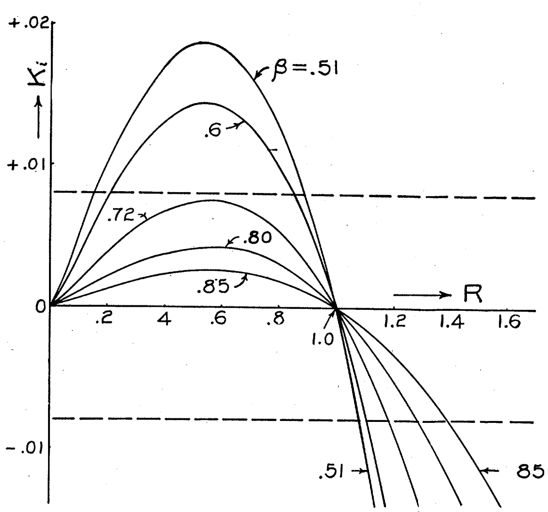

FIG. 5. Resolution conditions for Neumann's typical cases.

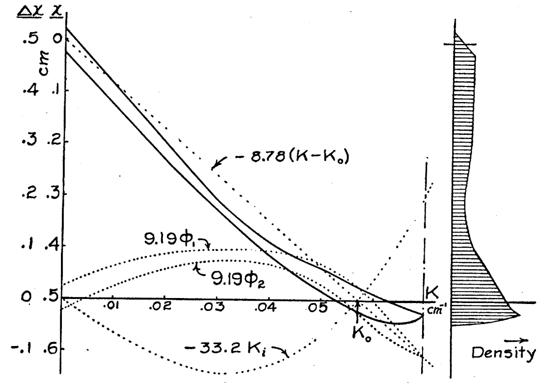

FIG. 6. Photographic density to be expected for Neumann's case $\beta=0.8$, if the source distribution function were uniform in terms of k, and if there were no scattering.

deflection was only 5→10 mm. It is therefore obvious that Neumann was by trial finding approximately the condition for the best quasi-focus, without knowing how poor the velocity resolution of his condenser really was.

NEUMANN'S DATA

In Table III are given some typical cases taken from Neumann's table. On the basis of these data and the foregoing analysis one can calculate the expected spreads in the various quantities involved. The general equation for the spread in x, not assuming any particular type of focus, is:

$$\Delta x = b + \phi(a+l) - [l(l+2a) - a^2/(1-\beta_0^2)]k_i/2 \quad (8)$$

and for Neumann's case No. 37, $\beta_0=0.801$ one can calculate the spreads for $b=0$, and also the restricted area, as shown in Fig. 3a. From these one can further calculate the limiting values of Δx corresponding to the contour of the restricted area and then plot against k_i, as shown in Fig. 4. In this way one finds a *total* expected line width of about 1.2 mm, which is about 25 percent of the total deflection 5 mm. This great line width need not of itself cause any serious difficulty, since it has been shown that the line should be symmetrical, provided that the derivative $\partial k_i/\partial k$ be practically constant over the allowed interval, and also that the k distribution function for the source be practically constant over the same interval. Therefore, it remains only to ascertain whether the actual spread in k is sufficiently small to justify the assumption of the latter conditions. It is of importance to recognize the fact that the focussing properties of the apparatus may conceal a condition where the spread in k could be very large, indeed.

In order to test for the width of the interval in k one can obtain a rough idea, at least, of the order of magnitude of the spread by assuming, as in the latter analysis, that the spread is actually small; that is, $\Delta k = \Delta k_i/B = \pm k_a/(1-\beta_0^2)$. In this way one obtains for the total spread in k a value which is actually as much as 78 percent of k_0 for Neumann's case $\beta_0=0.8$. It is therefore immediately obvious that the assumption of small spreads in this case cannot be valid, and that the observed line cannot be symmetrical. In fact, the spread in k is so large as to suggest that the velocity filter may have "broken down" completely on the side of the higher velocities. In any case, it will *not* be sufficient to calculate Δx from the equation (8). One must rather return to the original equation in which k_i and k both appear, and use the actual relation (2) between these two curvatures. This gives:

$$\Delta x = b + \phi(a+l) - l(l+2a)k_i/2 - a^2(k-k_0)/2. \quad (9)$$

In order to obtain an accurate idea of the way in which the ideal velocity filter should behave for Neumann's cases one can draw the actual graphs representing the relation (2) in k_i and k; or one can transform the relation to a more convenient form as follows. If, instead of k, one introduces the dimensionless variable $k/k_0 \equiv R$, and if one notes that in general $k = A(1-\beta^2)^{\frac{1}{2}}/\beta$, or more specifically $k_0 = A(1-\beta_0^2)^{\frac{1}{2}}/\beta_0$, then one can write (2) as:

$$k_i = k_0 R[1 - \{1 + (R^2-1)(1-\beta_0^2)\}^{\frac{1}{2}}], \quad (10)$$

which enables one to determine k_i as a function of R for different particular experimental conditions specified by given values of $k_0 \equiv 1/\rho_0$ and β_0. With this method of representation the compensated condition is represented by $R=1$ for all values of k_0 and β_0, and the percentage spreads can be read directly from the graphs. In Fig. 5 are shown the graphs of (10) for Neumann's four typical cases (Table III) and also for the case later discussed by Schaeffer ($\beta_0 = 0.85$; $\rho_0 = 21.45$ cm; plate No. 50). The horizontal lines $k_i = \pm k_a = \pm 16\delta/l^2 = \pm 0.008$ correspond to the maximum and the minimum curvatures allowed by Neumann's condenser.

These curves show very definitely that the velocity filter behaves so poorly for $\beta_0 > 0.7$ that *some* radiation of all velocities $\beta > \beta_0$ may be transmitted. One may then ask why it was possible at all for Neumann to obtain apparent lines for the higher velocities β_0. By means of the relation (10), shown graphically in Fig. 5 for $\beta_0 = 0.8$, together with the cut-off conditions shown in Fig. 3a for the same case and for $b=0$, one can by using Eq. (9) calculate the allowed values of Δx for all values of k. The results of such a calculation, for the case of a line source in the central plane of the condenser, are shown in Fig. 6. From (9) it is easily seen that over a wide range in k the value of x is determined chiefly by the last term which represents the effect of the magnetic field after the electron has left the condenser space; since the first three terms are always restricted to small values, while k can vary from zero to a value somewhat greater than k_0. Hence one sees that the deflections $(-x)$, to be expected, will range all the way from a small negative value (due to the spread in ϕ) to a value somewhat greater than $(-x_0)$. In Fig. 6 are also shown, for the same case ($\beta_0 = 0.8$; $b=0$), the various individual contributions to Δx of the several terms of (9), all plotted against k, and drawn with dotted lines. For the term in ϕ there are two curves corresponding to the two limiting values ϕ_1 and ϕ_2 for a given value of k. Finally the limiting values of x, or of Δx are shown by the solid curve. The horizontal section of the area enclosed by the solid curve represents approximately the line density to be expected on the photographic plate for a source of uniform distribution in k. (For an actual source one must therefore multiply by the actual distribution function in terms of k.) These values are plotted on the right of the figure as a function of x. This density curve shows that even with the filter broken down there is a small tendency to transmit more radiation in the vicinity of $k = k_0$, and hence there may be an apparent line superimposed on the general fogging. One would expect the maximum density to occur at a value of x about 5 percent greater than x_0 when shifts or spurious peaks due to the variation in the k distribution, are neglected. The error introduced, *in this particular case*, by assuming that this point of maximum density corresponds to $x = x_0$ would be of such a nature as to make the mass appear less than the true value by about 5 percent. It is interesting to note that Neumann's value of e/m_0 for this case indicates a deviation from his other values by about 2.5 percent in the same direction. However, it should be remembered that the latter analysis was carried out for the case $b=0$, and that the variation of the source distribution density in k was neglected; and hence the results give only an approximate idea of the magnitude of the various spreads. In order to calculate the actual expected shift one would have to take into account the vertical extension of the source between the condenser plates. In principle this would be possible, even if somewhat laborious; but, as stated, the possibly important effect of the source distribution has also been neglected in the latter analysis. Since the velocity filter is "open" for $\beta_0 = 0.8$ over a wide range in k, as indicated by Fig. 6, one must then take the momentum distribution of the source into account. Actually, if the source has sharp maxima, one may expect the instrument under these conditions to behave rather as a momentum spectrograph; and consequently spurious, or seriously shifted maxima may appear on the photographic plate. Neumann stated that his source had a maximum at $\beta = 0.83$, which would definitely tend to produce a spurious maximum or a shifted maximum in the region of β_0. With this state of affairs it may easily be understood how by chance considerable errors could compensate each other. For the case $\beta_0=0.85$, later discussed by Schaeffer, the maximum of the source at $\beta=0.83$ and that due to the velocity filter would by chance lie very close to each other, and one would again have a similar condition. It might then seem that Neumann was able to obtain lines at the higher velocities, at least partially because of the chance condition that in his experiments β_0 was near the value where his source distribution function had a maximum value.

As regards further qualitative predictions of the present analysis, the density curve of Fig. 6 indicates that the photographic plate should be considerably fogged between the zero line and the apparent line, and that the blackening should fall off rather suddenly at the outer edge of the line, with no fogging beyond it. On examining Neumann's photographs for $\beta_0=0.8$ one sees that this is actually the case, since the background inside the line is definitely darker than that outside. In the other photographs, for $\beta_0=0.7$, for example, this fogging is not pronounced. One might have expected fogging for this latter case also; but it should be remembered that the actual intensity must be obtained by a consideration of graphs of the type shown in Fig. 6 and of the source distribution function, and that the expected fogging for this case might be considerably less than that for the former case $\beta_0=0.8$. It may, however, be noted that Neumann, in his original paper, stated that small departures from the relativity theory were obtained for values $\beta_0>0.7$, which is just the value at which the present theory predicts complete breakdown of the velocity filter on the side of the higher velocities.

The above analysis was not intended to give an accurate correction for Neumann's data; but rather to indicate the general behavior of the condenser when it no longer selects a narrow band around k_0. While the actual density maximum of the apparent line is shifted by an amount of the order of only 5 percent, it is important to note that the various individual contributions to the spread Δx, as represented by the dotted curves of Fig. 6, are much greater, and because of the focusing tendency partially compensate each other to produce the resulting 5 percent. Without a detailed analysis for each individual case it would be impossible to state how large the errors might be, in general. These calculations then show how very important and necessary such an analysis really is, and how fallacious it may be to assume that the mere existence of fairly sharp photographic lines is a good criterion for the proper functioning of the velocity filter. This difficulty appears much more serious, when one remembers that the original object of the Bucherer-Neumann experiments was to distinguish between the two cases of the rigid Abraham electron and the contracting Lorentz electron, which two cases themselves differ by only a small amount.

One may now consider the spreads for the other cases of Neumann's data. As regards the spread Δk around k_0, if it is small, it should suffice to calculate it from the differential relation $(\partial k_i/\partial k)_{k_0}=-(1-\beta_0^2)$, taking the limiting values of k_i equal to $\pm k_a=\pm 16\delta/l^2=\pm 0.008$. In this way one obtains for the total fractional spread: $\Delta k/k_0=2k_a/(1-\beta_0^2)k_0=0.016/k_0(1-\beta_0^2)$ for Neumann's condenser. If the spread is large it must be obtained from the relation (2), or from the graphs of Fig. 6. The values of the *total* spread $\Delta k/k_0$ obtained in the above two ways are shown in Table IV. In the fourth column are given the approximate values of the percentage spread obtained from the derivative of relation (2) around k_0; and in the fifth column, the accurate values taken from the graphs of Eq. (10), including the individual spreads on the positive and negative sides of k_0. (The difference between the latter two individual values is an indication of the amount of dissymmetry, and 100 percent on the left hand side indicates complete breakdown on the high velocity side of k_0). One sees here that for the last three cases the filter has broken

TABLE IV. *Total spread $\Delta k/k_0$ from typical cases of Neumann's data.*

No.	β_0	$\rho_0=1/k_0$	TOTAL SPREAD $\Delta k/k_0$ APPROX.	TOTAL SPREAD $\Delta k/k_0$ EQUATION (10)	$(m-m_0)/m_0$
40	0.507	9.05	20%	11+ 9=20%	16%
39	0.602	9.78	24	15+11=26	25
54	0.718	13.65	45	100+18=118	44
37	0.801	17.55	78	100+29=129	67
50	0.85	21.45	124	100+39=139	90

down on the high velocity side; and that even for the smallest velocity, $\beta_0=0.5$, the spread is by no means small, in fact, as much as 20 percent. In consideration of the complexity of Neumann's source one might have expected, in this interval of 20 percent, to observe resolved peaks due to the source distribution, unless they were smoothed out either by the vertical extension of the source, by scattering, absorption, and deceleration at the condenser plates, or in the source itself, or by the spreads in the angle ϕ—in which case one would possibly expect just one wide peak. In the last column of Table IV is given the percentage excess of the relativistic mass over the rest mass m_0; that is, the whole percentage effect of the relativistic variation of mass. This reveals the astonishing and disturbing fact that the actual spreads in the beam transmitted by the condenser, even under the best conditions, were of approximately the same value as the momentum variation caused by the whole relativistic change of mass. It need not be emphasized, under these circumstances, that a special detailed analysis of the structure of the observed lines is necessary before attempting to interpret such data. An analysis of this sort is possible along the lines followed in the above discussion, for the case of negligible scattering; but it seems quite possible that scattering actually played a considerable, if not a predominant role in Neumann's experiments. Now, from the foregoing treatment it was seen, that, even apart from the consideration of scattering, the usual interpretations are subject to serious criticism. It may well be that the shifts mentioned above always compensate each other approximately, but it definitely requires proof; and such a proof seems difficult even for the case of the direct beam, not to mention the effect of scattering.

It may be recalled again that the experiments of Bucherer and of Neumann were designed to make a quantitative measurement of the relativistic variation of the electron mass with velocity and thereby to distinguish between the Abraham and the Lorentz electrons. If one examines Neumann's table (reference 2, p. 571) one finds that the values of e/m_0 calculated on the Lorentz theory are practically constant, as required by the theory, while those calculated on the Abraham theory vary by about 10 percent amongst themselves. Hence the validity of Neumann's interpretations rested on the accurate measurement of a 10 percent effect. Now if one considers the very large spreads actually present, the fact that they may be concealed by focusing effects, and the further serious consequence of possible scattering, it is not in any sense inconceivable, even if possibly improbable, that the corresponding uncertainties could have produced errors of as much as 10 percent and in such a way as to have masked a real variation of the Lorentz values and at the same time to have produced an apparent variation in possibly truly constant Abraham values.

On the basis of the foregoing discussion it seems fair to say that the Bucherer-Neumann experiments actually proved very little, if anything more than the Kauffmann experiments, which indicated a large qualitative increase of mass with velocity. One might say that their experiments gave a slight suggestion in favor of the relativity electron, but that the uncertainties of interpretation are so great as to give one very little feeling of certainty as regards a 10 percent effect. Indeed, it seems remarkable that they were able to obtain lines at all for the higher velocities in consideration of the exceedingly poor performance of their velocity filters. What seems very surprising is the fact that Bestelmeyer actually raised objections along the lines of the present treatment, but still it was not discovered how very poor the resolution of the velocity filter really was.

Although by tedious calculations it would in principle be possible to carry out a more thorough analysis of the system of rays for the direct radiation, the uncertainty of scattering makes it seem almost hopeless to arrive at any reliable conclusions on the basis of Neumann's data. Since Neumann's difficulties seemed to set in at just the velocity expected on the present theory *for the unscattered radiation*, it might seem that scattering was of no importance. On the other hand there are so many possibilities of spurious lines and shifts, and hence of compensating errors, that it would be dangerous to draw such a conclusion. In fact, it would seem to be extremely hazardous in general to draw conclusions from experiments on Bucherer's type of apparatus without a complete knowledge of the resolution characteristics and of the scattering. For example, it seems possible that the scattering might be negligible for experiments at a value of β_0 in the upper part of the source spectrum; but for experiments at a value of β_0 in the lower part of the spectrum, the scattering might become the predominant factor, since as scattering occurs the electrons are progressively decelerated and there may be a very much greater chance of electrons finally reaching the allowed interval around β_0. It even seems possible that one could make experiments at a low velocity, with a spectrum largely composed of higher velocities—and, because of the relatively low distribution density for the low velocity, miss the expected line altogether—and at the same time, because of an open filter for the scattered radiation, obtain, instead, a rough momentum spectrogram somewhat like that of the source itself, and hence a spurious maximum. This seems all the more possible when one remembers that the density curve of Fig. 6, for example, must be multiplied by the distribution function in terms of k, and that the latter is obtained from the ordinary $H\rho$ distribution function by multiplying by ρ^2, which accentuates the higher energies and suppresses the lower energies by considerable factors.

If one should mistake this general maximum of the scattered spectrum for the theoretical line, one would come to the conclusion that the electrons were heavier than normal. On the authority of the long accepted results of Bucherer and Neumann, and without a knowledge of the critical conditions of the Bucherer experiment, it is easy to understand how one might unwittingly come to such a conclusion. In the previously mentioned investigation of continuous beta-rays a first attempt was made with the Bucherer method, and with a source of artificially radioactive phosphorus obtained by bombardment in the Michigan cyclotron. Fortunately, however, the intensity of the source was not sufficient to give visible blackening in a reasonably short time; and, as a result, an improved method was developed for the determination of the ratio e/m. Amongst other advantages of this method, the uncertainties of scattering were largely eliminated. This modified method also offers a means of measuring e/m more accurately than was possible by the original method; and it promises the possibility of extending the experimental study to velocities considerably greater than those previously reached in experiments of this general type.

Since the Bucherer-Neumann interpretations are based on such doubtful premises, a search was made in the literature for other experiments of the same general kind and for discussions of such experiments. In 1924 Doi[3] presented an analysis in which he claimed that the Bucherer experiments gave more support to a mass formula of his own than to that of Lorentz. Doi's basis of contention was that his own formula led to the correct curvatures for the Bucherer deflection traces, whereas the Lorentz formula did not. He also pointed out that the photographic traces were longer than expected from the theory. In a subsequent paper Schott[4] gave a criticism of Doi's analysis, directed chiefly at the latter's failure to consider the non-compensated rays, which falsify the traces near the ends.

Also Lewis[5] gave an analysis showing that the extension and fogging of the ends of the traces was due to the noncompensated rays. His calculations, in which scattering was neglected, gave the intensity distribution to be expected on the photographic plate, and he concluded that the Bucherer traces were in satisfactory agreement with his theory. It thus appears that none of the above-mentioned authors carried out calculations which revealed the resolution limitations of the method.

As regards experimental investigations, beside those of Bucherer and of Neumann, several others are to be found in the literature. Hupka,[6] working with high velocity cathode rays of velocities up to $\beta=0.5$, obtained relative values of the electron mass at different velocities in order to decide between the formulae of Lorentz and of Abraham. His experimental method required a precise determination of accelerating potentials; and Heil[7] calculated that in Hupka's experiment a $\frac{1}{4}$ to $\frac{1}{2}$ percent error in the potential would have allowed a decision in favor of the Abraham theory, whereas Hupka claimed an accuracy of

[3] U. Doi, Phil. Mag. **49**, 434 (1925).
[4] G. A. Schott, Phil. Mag. **50**, 389 (1925).
[5] T. Lewis, Proc. Roy. Soc. **A107**, 544 (1925).
[6] E. Hupka, Ann. d. Physik **31**, 176 (1910).
[7] W. Heil, Ann. d. Physik **31**, 519 (1910) and **33**, 403 (1910).

only about $\frac{1}{4}$ percent. Hence these measurements do not provide the basis for a definite conclusion

Guye, Ratnowski and Lavanchy,[8] working with cathode rays of velocities $\beta=0.2$ to $\beta=0.5$, also used a comparison method; and they stated that their results spoke in favor of the Lorentz theory.

At higher velocities, Tricker[9] made observations on several *discrete* beta-ray lines from Ra B and Ra C, with velocities ranging from $\beta=0.363$ to $\beta=0.794$. Some of the above-mentioned uncertainties of the Bucherer type of experiment seem to have been recognized by Tricker, but he did not investigate them quantitatively as has been done in the present treatment. For his own experiments Tricker used a method which seems to be free from the latter type of uncertainty; but he stated that he could measure his photographic traces to within an accuracy of 2 percent, whereas the difference between the two types of electron amounts effectively to only 5 percent. Hence even Tricker's results do not provide a very satisfying basis for distinction between the Abraham and the Lorentz electrons.

In conclusion the authors wish gratefully to acknowledge support from the Horace H. Rackham School of Graduate Studies of the University of Michigan.

[8] See *Handbuch der Physik*, Vol. 22 (1926), p. 76.
[9] R. A. Tricker, Proc. Roy. Soc. **A109**, 384 (1925).

APRIL 1, 1938 PHYSICAL REVIEW VOLUME 53

A New Mass Spectrometer with Improved Focusing Properties

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(Received February 7, 1938)

The use of crossed electric and magnetic fields for a mass spectrometer is discussed. It is shown that this arrangement has perfect focusing properties; the focusing depends only on the m/e of the ion selected, and not on the velocity or direction of the charged particles entering the analyzer. The projection of the path in the plane perpendicular to the magnetic field is a trochoid. The theory necessary for the design of the apparatus is developed in some detail. A method of drawing the trochoids is described as well as a chart which is a great help in rapidly correlating the many variables. It is shown that there are two types of path to be considered, the curtate and the prolate. The former was employed in the first model constructed and gave encouraging results in spite of some structural difficulties encountered. The second apparatus was the prolate type and worked exceptionally well. Some typical mass spectra are shown. It was found that a distribution in energy amounting to 50 percent of the potential accelerating the ions had no effect on the resolution.

WHATEVER be the source of ions employed in mass-ray analysis one is always faced with a velocity distribution and an angular divergence of the charged particles. So-called "double-focus" mass spectrographs have recently been constructed by Bainbridge and Jordan,[1] Mattauch,[2] Dempster[3] and Aston.[4] References to other theoretical work bearing on these methods are to be found in these papers as well as in two recent publications by Cartan.[5] Considerable success has attended these efforts to construct a true e/m selector which is independent to some extent of the velocity and direction of the incident ions. However, in all the field combinations except the one which is the subject of this paper, the focusing that is obtained is not perfect and is limited to a small range of initial velocity and direction. Since there is an arrangement of fields which in theory possesses all the desirable qualities mentioned

[1] Bainbridge and Jordan, Phys. Rev. **50**, 282 (1936).
[2] Mattauch, Phys. Rev. **50**, 617 (1936).
[3] Dempster, Proc. Am. Phil. Soc. **75**, 755 (1935).
[4] Aston, Nature **137**, 357 (1936); Proc. Roy. Soc. **163A**, 205 (1937).
[5] Cartan, *Spectrographie de Masse*, (Hermann, 1937) (Actualités Scientifique 550); J. de phys. **8**, 453 (1937).