The Photoelectric Effect of H^2

LLOYD MOTZ AND WILLIAM RARITA
Columbia University, New York, N. Y.

(Received June 19, 1937)

The total photoelectric cross section of H^2 for an exponential potential is computed with the best available nuclear constants. The value obtained for Th C′ γ-rays is 13.1×10^{-28} cm^2.

SINCE recent work[1] has indicated that the exponential potential is well adapted for exact calculations in the study of the two-, three-, and four-body nuclear problems, the photoelectric cross section of H^2 is calculated in this paper with the above potential in anticipation of more accurate experimental data. It is to be hoped that these more precise data will cause the photoelectric effect to play an important role in the development of nuclear theory.

THEORETICAL FORMULAE

The method used in this paper follows the treatment given by Bethe and Bacher.[2]

Assuming the potential for the triplet state of H^2 to be $-Be^{-2r/b}$ (B is the depth and b is the range of the interaction), we first compute the cross section for an electric dipole transition, $\sigma_{\text{E.D.}}$:

$$\sigma_{\text{E.D.}}=\frac{\pi}{3}\times\frac{e^2}{\hbar c}\times\frac{2\pi\hbar\nu}{k}\times|M_{\text{E.D.}}|^2, \qquad (1)$$

$$M_{\text{E.D.}}=\int_0^\infty u_0 r u_E dr.$$

In the above equation we have used nuclear units,[3] to be retained throughout the entire paper.

$e^2/\hbar c$ is the fine structure constant; $2\pi\hbar\nu$ is the energy of the γ-ray; and $k=(E)^{\frac{1}{2}}$, where E is the energy of the neutron and proton in the final state relative to their center of mass.

u_0 is the normalized radial wave function of the ground state of H^2 and is given by the Bessel function:

$$u_0=NJ_p(\beta e^{-r/b}).$$

N is a constant of normalization; $\beta=b(B)^{\frac{1}{2}}$; and $p=b(\epsilon)^{\frac{1}{2}}$, where ϵ is the binding energy of H^2.

u_E is the radial wave function of the final state of H^2. In the zeroth order, i.e. for the neutron and proton considered as being free

$$u_E\equiv u_E{}^0=-\cos kr+\sin kr/kr.$$

The detailed evaluation of $M_{\text{E.D.}}$ is reserved for the appendix.

Assuming the potential for the singlet state of H^2 to be $-B'e^{-2r/b}$, we calculate the photoelectric cross section for a magnetic dipole transition, $\sigma_{\text{M.D.}}$:

$$\sigma_{\text{M.D.}}=\frac{m}{M}\times(\mu_P-\mu_N)^2\frac{|M_{\text{M.D.}}|^2}{|M_{\text{E.D.}}|^2}\sigma_{\text{E.D.}}, \qquad (2)$$

$$M_{\text{M.D.}}=\int_0^\infty u_0 v_E dr.$$

m and M are the electron and proton masses, respectively; and μ_p and μ_n are the proton and neutron nuclear magnetic moments, respectively, expressed in nuclear magnetons.

v_E is the radial wave function of the final state, normalized to unit amplitude at infinity:

$$v_E=\frac{|\Gamma(1+iq)|}{|J_{iq}(\beta')|}Im[J_{iq}(\beta')J_{-iq}(\beta' e^{-r/b})],$$
$$v_E\rightarrow\sin(kr+\delta_0)\quad\text{as}\quad r\rightarrow\infty,$$
$$q=bk,\quad \beta'=b(B')^{\frac{1}{2}}.$$

Im indicates the imaginary part.

The explicit expansion for $M_{\text{M.D.}}$ appears in the appendix. For completeness, we give the cross section for an electric quadrupole transition, $\sigma_{\text{E.Q.}}$:

$$\sigma_{\text{E.Q.}}=\frac{\pi^2}{5\lambda^2}\times\frac{|M_{\text{E.Q.}}|^2}{|M_{\text{E.D.}}|^2}\times\sigma_{\text{E.D.}}$$

$$M_{\text{E.Q.}}=\int_0^\infty u_0 r^2 w_E dr.$$

λ is the wave-length of the γ-ray; and w_E is the radial wave function of the final state. In zero order,

$$w_E\equiv w_E{}^0=\left(\frac{3}{k^2r^2}-1\right)\sin kr-\frac{3}{kr}\cos kr.$$

[1] Rarita and Present, Phys. Rev. **51**, 788 (1937).
[2] Bethe and Bacher, Rev. Mod. Phys. **8**, 82 (1936).
[3] mc^2 for energy $(\hbar^2/Mmc^2)^{\frac{1}{2}}$ for length, and $M/\hbar^2=1$.

Numerical Calculation

We have carried out the calculation for Th C′ γ-rays. Using the best available nuclear constants, $2\pi\hbar\nu=5.12$ (or 2.64 Mev), $E=0.77$, $\epsilon=4.35$, $b=0.193$, $B=242$, $B'=137.6$, $\beta=3.002$, $\beta'=2.264$, $p=0.4026$, $\mu_p-\mu_n=4.9$, we obtain[4]

$$\sigma_{\text{E.D.}}=9.5\times10^{-28}\ \text{cm}^2,\quad \sigma_{\text{M.D.}}=0.38\sigma_{\text{E.D.}},$$
$$\sigma_{\text{E.Q.}}=0.00035\sigma_{\text{E.D.}}.$$

The total cross section is then

$$\sigma_{\text{total}}=13.1\times10^{-28}\ \text{cm}^2.$$

Appendix

(1) Evaluation of N

We have $\int_0^\infty u_0^2dr=1$. Two rapidly converging series can be used to compute N.

$$\frac{1}{N^2}=\frac{b}{2}\sum_{m=0}^{\infty}\frac{(-1)^m(\beta/2)^{2p+2m}}{(p+m)m!}\,\frac{\Gamma(2p+2m+1)}{\Gamma(2p+m+1)\Gamma^2(p+m+1)};$$

or

$$\frac{1}{N^2}=\frac{b\beta}{2p}J_{p+1}(\beta)\left[\frac{\sigma(p+1)(\beta/2)^{p+2}}{1!(p+1)!}-\frac{\sigma(p+2)(\beta/2)^{p+4}}{2!(p+2)!}+\cdots\right],$$

where

$$\sigma(p+n)=\sigma(p+n-1)+\frac{1}{p+n}\quad\text{and}\quad\sigma(p+1)=\frac{1}{p+1}.$$

(2) Evaluation of $M_{\text{E.D.}}$

Let $M_{\text{E.D.}}\equiv M^0_{\text{E.D.}}$ for $u_E=u_E^0$. The expansion of $M^0_{\text{E.D.}}$ converges extremely rapidly:

$$M^0_{\text{E.D.}}=N\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\,\frac{(\beta/2)^{p+2n}}{\Gamma(n+p+1)}\,\frac{2k^2}{[((p+2n)/b)^2+k^2]^2}.$$

We now calculate $M_{\text{E.D.}}$ correct to the first order.

The differential equation for u_E is

$$\left(\frac{d^2}{dr^2}-\frac{2}{r^2}+E\right)u_E=Be^{-2r/b}u_E.$$

We set u_E on the right-hand side of this equation equal to u_E^0, and solve the resulting inhomogeneous differential equation for u_E^1.[5]

Define $M^1_{\text{E.D.}}=\int_0^\infty u_0ru_E^1dr$, then

$$M^1_{\text{E.D.}}=\cos\delta_1\,M^0_{\text{E.D.}}+\frac{NB}{4}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{n+m+1}}{n!}\,\frac{(\beta/2)^{p+2n}(k/2)^{2m-1}}{\Gamma(n+p+1)}$$
$$\times\left\{\frac{\pi\binom{2m+3}{m}(2m+4)!k^3}{8\Gamma^2(m+5/2)}\left[\left(\frac{b}{2}\right)^{2m+5}K(n)-\left(\frac{b}{2}\right)^{2m+7}\sum_{s=2}^{2m+6}K(n,s)\sin(n,s)\right]\right.$$
$$\left.+\frac{\pi\binom{2m}{m}(2m+1)!}{\Gamma(m+5/2)\Gamma(m-\frac{1}{2})}\left[\left(\frac{b}{2}\right)^{2m+4}\sum_{s=2}^{2m+3}K(n,s)\cos(n,s)\right]\right\}$$

with[6]

$$\sin\delta_1=\frac{B}{k^2}\cdot\left\{\frac{1}{2bk}\log[1+(bk)^2]-\frac{bk}{2}\,\frac{1+(bk)^2/2}{1+(bk)^2}\right\},$$

$$K(n)=\frac{((p+2n)/b)^3+((p+2n)/b)3k^2}{[((p+2n)/b)^2+k^2]^2},$$

$$K(n,s)=\frac{[((p+2n+2)/b)^2+s^2k^2]^{1/2}}{(b/2)^s[((p+2n+2)/b)^2+k^2]^{s/2}},$$

$$(n,s)=\arctan\frac{p+2n+2}{sbk}+s\arctan\frac{bk}{p+2n+2}.$$

---

[4] $\sigma^1_{\text{E.D.}}=0.96\sigma^0_{\text{E.D.}}$ for this energy. $\sigma^1_{\text{E.D.}}$ and $\sigma^0_{\text{E.D.}}$ are related to $M^1_{\text{E.D.}}$ and $M^0_{\text{E.D.}}$, respectively. See appendix.

[5] $u_E=u_E^1$ for this approximation.

[6] $\binom{x}{y}=\frac{x!}{(x-y)!\,y!}$.

(3) Evaluation of $M_{\text{M.D.}}$.

$$M_{\text{M.D.}}=-\frac{b(\beta/2)^p}{|J_{iq}(\beta')|\,|\Gamma(1+iq)|\,\Gamma(1+p)}\times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}Im\left[\frac{A_lA_mA_n}{p+2n+2m+iq}\right],$$

$$A_l=\frac{(-1)^l(\beta'/2)^{2l}\Gamma(1-iq)}{l!\,\Gamma(l+1-iq)},\; A_m=\frac{(-1)^m(\beta'/2)^{2m}\Gamma(1+iq)}{m!\,\Gamma(m+1+iq)},\; A_n=\frac{(-1)^n(\beta/2)^{2n}\Gamma(1+p)}{n!\,\Gamma(n+1+p)}.$$

AUGUST 15, 1937 PHYSICAL REVIEW VOLUME 52

Note on Nuclear Photoeffect at High Energies

F. Kalckar, J. R. Oppenheimer and R. Serber
University of California, Berkeley, California

(Received May 19, 1937)

In I we consider nuclear transmutations when the energy is so high that the levels of the compound nuclei formed have a spacing smaller than their breadth, and investigate the relations between the cross sections for the various possible reactions and the decay constants characteristic of the compound nuclei. When the collision may be treated as a resonance effect, these relations take a simple form. In II we apply these considerations to the photoelectric disintegration of nuclei of intermediate atomic weight by 17 Mev γ-rays, and suggest that the yields should show a marked increase for γ-rays of somewhat lower energy. In III we study the formal connection between the evaluation of transmutation probabilities here given and the resonance formulae appropriate for lower excitation energies, and show that both descriptions may be derived as limiting cases from the same formalism.

I

Bohr's analysis[1] of nuclear phenomena has shown the great importance, for an understanding of the processes of nuclear disintegration, of the capture of the incident particle with the formation of an intermediate nucleus having a considerable excitation energy, and so long a lifetime that the subsequent transformation of this unstable structure by the emission of particles (or γ-rays) may be treated as an independent process. A formalism consistent with these ideas, and especially appropriate to the study of the behavior of slow neutrons, has been developed by Breit and Wigner,[2] who have considered the case that the energy levels of the compound nucleus lie far apart compared to their breadth, and have applied the familiar quantum mechanical dispersion formulae to this problem. For sufficiently heavy nuclei and sufficiently high excitation energies the intermediate nucleus will however no longer have well-defined energy levels; instead the states of the system, and even the states of a particular kind, e.g., of a given angular momentum, will form a continuum. One might hope that the disappearance of any characteristic level structure would lead to a simplification in the description of the probabilities of nuclear disintegration, and that the only quantities which then determine these probabilities would be the rates at which a wave packet representing the intermediate nucleus comes apart into the various possible residual nuclei and emitted particles.

This expectation is not however in agreement with the result given by a simple application of the principle of detailed balancing to the processes of formation and disintegration of the compound system. Thus we may divide the states of the compound nucleus into sets (i) each of which is characterized by decay constants which in the limit of close lying levels may be regarded as slowly varying functions of the energy, and by a density of levels $1/s^i$ per unit energy. Then we obtain a relation between the cross section σ_c for the capture of an incident particle and the decay constants $\Gamma_{a0}{}^i/\hbar$ for the reemission of this particle with its original energy:

$$\sigma_c=(\lambda^2/2w_A)\sum_i\Gamma_{a0}{}^i/s^i, \tag{1}$$

[1] N. Bohr, Science, to be published.
[2] Breit and Wigner, Phys. Rev. **49**, 519 (1936).