AN ADDITION TO A THEORY OF IONIZATION BY IMPACT.

By H. W. Farwell.

SOME time ago Bergen Davis[1] developed a theory of ionization by impact which has been very suggestive and of particular interest in the consideration of non-elastic impacts. The experiments of Franck and Hertz[2] have shown that in certain gases, such as oxygen, nitrogen, and hydrogen, the impacts are practically all of this character, so that this assumption in Davis's theory is not without support in experiment. The lack of accord between this theory and some of the experimental evidence is due to the fact that the impacts in all cases were originally considered to be non-elastic. In confining attention now to those gases in which the impacts have been shown to be of this kind, there is still cause for differences between fact and theory.

The original paper assumed that there were present in the space ionized all possible ionic paths between zero and infinity, whereas of necessity many of the experiments have been carried out with relatively small distances between the plates of the condenser. It is hardly necessary to point out that under such conditions the average number of ionizing impacts per centimeter can not be determined with accuracy, and differences between theory and results of such experiments may be held against the experiment rather than against the theory.

It seemed worth while to extend the theory to cover all such cases, though the method of calculating the number of ionizing impacts may still cause small differences. Following Davis's reasoning exactly the number of ionizing impacts which an ion makes in traversing a distance dx just beyond a path $\lambda_0 + x$, where λ_0 is the shortest path along which the necessary energy for ionization can be acquired, is given by

$$\frac{1}{l}\epsilon^{-\frac{\lambda_0 + x}{l}}\frac{x}{\lambda_0 + x}\frac{dx}{l},$$

where l is the mean free path of the ion. If now the ion is in a field due to a condenser whose plates are a distance D apart, the quantity x may have any values between 0 and $D - \lambda_0$. The complete expression

[1] Phys. Rev., XXIV., p. 93, Jan., 1907. Also Ann. d. Phys., 42, p. 807, 1913.

[2] Ber. d. Deuts. Phy. Ges., 15, p. 613, 1913. Also p. 929, 1913.

for the number of ionizing impacts per centimeter is then

$$\alpha = \frac{1}{l}\int_0^{D-\lambda_0} \epsilon^{-\frac{\lambda_0 + x}{l}} \frac{x}{\lambda_0 + x}\frac{dx}{l}.$$

This expression is, like the original, not integrable in finite terms, but can be integrated in a form involving the exponential integral.[1]

The solution is

$$\alpha = \frac{1}{l}\left\{\left[\epsilon^{-\frac{\lambda_0}{l}} + \frac{\lambda_0}{l}Ei\left(-\frac{\lambda_0}{l}\right)\right] - \left[\epsilon^{-\frac{D}{l}} + \frac{\lambda_0}{l}Ei\left(-\frac{D}{l}\right)\right]\right\}$$

where $Ei\ (\quad)$ denotes the exponential integral. This is Davis's solution with a correction term added, and reduces to his solution when the distance between the plates is large.

The magnitude of the correction term can be computed for any given conditions, but to avoid tedious calculation the curves below have been

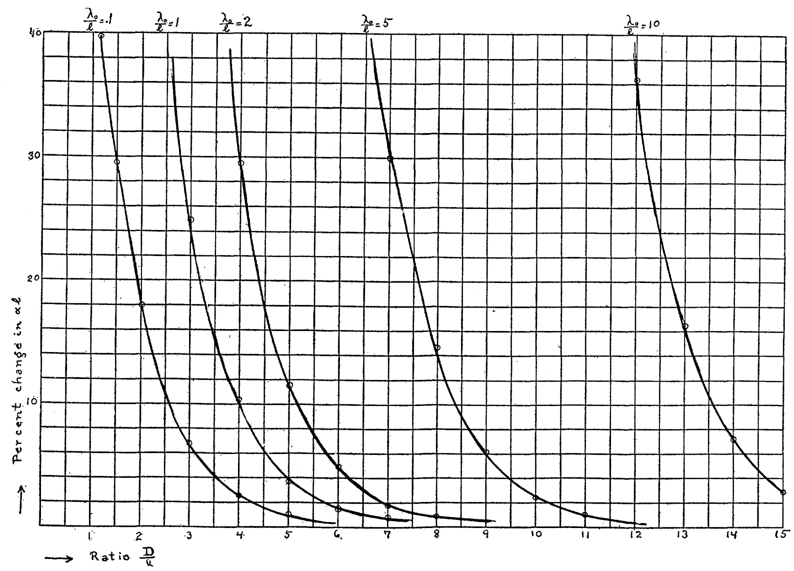

Fig. 1.

plotted to represent the amount of this correction under various conditions. The abscissæ give the ratio of plate-distance to ionic mean free path, and the ordinates give the magnitude of the quantity $\epsilon^{-\frac{D}{l}} + (\lambda_0/l)$ $Ei(-D/l)$ expressed in per cent. of the quantity αl, or in other words the ordinates give the per cent. error in determining α for a given pressure. The curves have been plotted only for the range of values likely to occur

[1] Bierens de Haan, Verh. Kon. Ak. Wet., Vol. II., 1854, p. 214.

in observations of the current between parallel plates, and give at once the error if the observer has a measure of D and l. Curves of this sort may be used to determine the lower limit for plate distance in experimental work. Without mentioning the specific instances it may be remarked that there are numerous cases where experimenters have taken observations with plates separated by such a small distance as to lead to errors in α of fairly large magnitude.

Partzsch[1] has made a similar addition to Townsend's expression for the current

$$n = n_0 \epsilon^{\alpha D}$$

since this expression would indicate an increase in the current even though the applied voltage was less than the ionizing voltage. Expressed in symbols already used, Partzsch's modification may be written

$$n = n_0 \epsilon^{\alpha D\left(1 - \frac{\lambda_0}{D}\right)} = n_0 \epsilon^{\alpha(D - \lambda_0)}.$$

This, like Townsend's, gives from the experimental results the values of α, but the reasoning from it to deduce the ionizing voltage would be open to the same objections as those made by Franck and Hertz[2] to the original form.

This addition to Davis's formula is in itself of minor importance, but it may serve to aid further inquiry into the process of ionization by impact and the character of electron paths.

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[1] Verh. Deutsch. Phys. Gesell., 14, p. 60, 1912.
[2] Ber. Deutsch. Phys. Gesell., Jan. 15, 1914.