Effects of Exchange Forces on the Excitation Function of Li^7 Under Proton Bombardment

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In a recent paper Van Vleck gave an approximate method of estimating the effective potential due to Majorana exchange forces, which acts on a proton or neutron when they collide with a nucleus. The effective potential obtained in this manner varies with velocity. Calculations reported on here show that the effect of this variation with velocity on the shape of the excitation curve of lithium under proton bombardment is small and is of no practical importance. The position of stationary proton levels which are consistent with the excitation curve is affected seriously. Agreement of calculations with the mass defect of Be^8 is materially improved through the use of the exchange potential. With a radius 0.35×10^{-12} cm the empirical mass defect of Be^8 is accounted for perfectly by the calculation which simultaneously fits the excitation curve for the 8-cm particles.

IN a recent paper Van Vleck[1] using the Majorana exchange operator and assuming together with Feenberg[2] a neutron proton interaction of the form $Ae^{-\alpha r^2}$ made calculations of the cross section of heavy nuclei for collisions with slow neutrons. In this work he developed a simple approximate method capable of representing the exchange forces by an equivalent central field which varies with the velocity of the neutrons. Van Vleck's considerations can be applied to protons incident on nuclei simply by interchanging protons and neutrons in his formulas. Calculation by means of them shows that the mean attraction exerted by a nucleus on an incident proton or neutron increases as the velocity of these particles decreases. Similarly a particle in a bound state can be expected to be subjected to a stronger attractive potential than an incident particle. Van Vleck's considerations are only approximate inasmuch as they involve a statistical treatment of particles constituting the nucleus as well as the usual approximations necessary for the reduction of a many body problem to Hartree-Fock fields. They are besides supposed to apply only to large nuclei inasmuch as the surface effects of v. Weizsäcker[3] are neglected. Nevertheless one may suppose that in order of magnitude the difference between the effects of using Wigner and Majorana forces can be estimated also for light nuclei by means of Van Vleck's formulas.

Calculations on the excitation curve of $Li^7+H^1\rightarrow He^4+He^4$ were published recently.[4] In these the field acting on the proton was supposed to be independent of velocity. It was thought of interest to estimate the effect of using Van Vleck's potential on the same problem. One may expect two kinds of effects to arise from the velocity dependence of Van Vleck's potential. Firstly, the excitation functions change and secondly the stationary levels of the protons in the nucleus are affected.

The effect on the excitation curves is usually small and may be expected to be of importance only in those exceptional cases in which the excitation curve depends very critically on the position of a stationary or virtual level. The primary reason for this is the fact that the changes in kinetic energy of incident protons with which one is concerned (1 MEV) are small in comparison with the depth of the stationary levels which are of interest ($\sim$20 MEV). Also the change in the attractive potential is itself smaller than the change in kinetic energy with which it is associated. Thus the main modification due to using the exchange potentials consists in depressing the stationary proton levels which one expects for a given shape of the excitation curve.

As in the previous calculation[4] it is supposed that the chance of disintegration is determined by the probability of finding the incident proton in the nucleus so that the number of disintegrations per second is given by P times the average

[1] J. H. Van Vleck, Phys. Rev. **48**, 367 (1935).
[2] E. Feenberg, Phys. Rev. **47**, 850 (1935).
[3] C. F. v. Weizsäcker Zeits. f. Physik **96**, 431 (1935).
[4] M. Ostrofsky, G. Breit and D. P. Johnson, Phys. Rev. **49**, 22 (1936). This paper will be referred to as (I).

number of incident protons located in the nucleus. The quantity P is taken to be independent of the velocity of the protons.

In agreement with the notation used previously[4] we let U stand for the negative of the potential inside the nucleus so as to have $U>0$. According to Van Vleck

$$U=-A[E(w_+)\mp E(w_-) +2(\alpha/\pi K_p^2)^{\frac{1}{2}}(e^{-w_+^2}-e^{-w_-^2})], \quad K_p\gtrless K \tag{1}$$

$$w_\pm=\frac{|K_p\pm K|}{2\alpha^{\frac{1}{2}}}, \quad E(w)=2\pi^{-\frac{1}{2}}\int_0^w e^{-t^2}dt,$$

$$K=\left[\frac{9\pi N}{4r_0^3}\right]^{\frac{1}{3}}.$$

Here N is the number of neutrons in the nucleus, K_p is 2π times the wave number of the proton when it is in the nucleus and the remaining symbols are defined by the equation itself. The sign before $E(w_-)$ should be taken as $-$ or $+$ depending on whether $K_p>K$ or whether $K_p<K$. From the conservation of energy it follows that

$$\mu T/m=0.0\ 2\ 4\ 8\ 3\ mZ^2Z'^2(K_pa)^2-U, \tag{2}$$

where m is the mass of the proton, μ is the reduced mass, T is the kinetic energy of the proton in the coordinate system in which the nucleus is at rest, while Z and Z' are, respectively, the atomic numbers of the proton and the nucleus. The unit of energy in this equation is the MEV. The quantity a stands for $\hbar^2/\mu ZZ'e^2$. This equation is convenient because K_pa is analogous to the parameter $1/\eta=$"1 3 7" $(v/c)/ZZ'$ which enters into the computation of wave functions in the Coulombian region outside the nucleus. Eqs. (1) and (2) determine U as a function T. Substitution of numbers shows that this relation is nearly linear and that U decreases as T increases. Thus the slower the proton the more effectively it is attracted by the nucleus. With U known as a function of T the collision cross section σ can be calculated in the same way as was done in I. The calculations are summarized in Table I. The values of σ and U obtained by using Eq. (1) are called σ_{ex}, U_{ex} in Table I so as to indicate that they correspond to the use of exchange forces. The second and third columns in Table I correspond to incident protons of zero angular momentum ($L=0$) while the fourth and fifth columns correspond to protons with angular momentum $\hbar$ ($L=1$). The values of A and α were so chosen as to give a good agreement between the theoretical and the experimental yield curves. It will be noted that the values of U_{ex} which correspond to this condition are practically the same as those found previously. All of the calculations were made for a nuclear radius of 0.347×10^{-12} cm. The values of U in Table I correspond to $A=-194.87$ MEV, $\alpha^{-\frac{1}{2}}=1.6\times10^{-13}$ cm for $L=0$ and to $A=-102.43$ MEV, $\alpha^{-\frac{1}{2}}=1.5\times10^{-13}$ cm for $L=1$. This choice of A and α was made rather arbitrarily by assigning a reasonable α and adjusting A to give the right U for $T=0$.

TABLE I. *Calculations of collision cross sections.*

	$L=0$		$L=1$	
T in MEV	U_{ex} in MEV	σ_{ex}/σ	U_{ex} in MEV	σ_{ex}/σ
0	35.00		22.00	
35.4	34.990	1.023	21.990	0.931
56.1	34.980	1.022		
89.0	34.960	1.023	21.975	0.949
141.0	34.940	1.015		
223.5	34.910	1.010	21.940	0.978
354.2	34.865	1.000	21.905	1.000
561.3	34.785	0.984	21.850	1.027
890.0	34.665	0.958	21.764	1.057
1410	34.465	0.916	21.625	1.090
2235			21.406	1.122

The way in which σ_{ex}/σ varies with T in Table I can be understood as the result of progressive shifts in the position of resonance and stationary levels. The importance of such levels for the excitation function was already discussed in I. By a resonance or stationary level in the explanation given below we mean for any T the level which we would have if the value of U_{ex} corresponding to that T were kept fixed for all T. This way of talking about the resonance levels is rational in this connection because the position of the level matters through its effect on the resonance denominator in the formula for σ; the value of the resonance denominator, however, depends only on properties of the wave function at a given energy, i.e., only on U_{ex} at that energy. Proximity to a resonance or stationary level increases the yield. For protons with $L=0$ and $U\sim35$ MEV the closest level is a resonance level located roughly at $T=+10$ MEV. As T increases, the value of U_{ex} decreases, the reson-

TABLE II. *Calculations on position of stationary levels.*

	$L=0$				$L=1$			
	$l=0$		$l=1$		$l=0$		$l=1$	
	U	W	U	W	U	W	U	W
Van Vleck Potential	56.42	−42.85	42.98	−17.13	27.09	−15.45	22.38	−1.25
Constant Potential	35.00	−22.55	35.00	−10.51	22.00	−10.90	22.00	−1.00
	$\Delta W=-20.30$		$\Delta W=-6.62$		$\Delta W=-4.55$		$\Delta W=0.25$	
	$\Delta W'=-19.84$		$\Delta W'=-6.50$		$\Delta W'=-4.40$		$\Delta W'=0.24$	

ance level is moved up and therefore away from the experimental region and consequently σ_{ex}/σ decreases. For protons with $L=1$ the closest level is a stationary level located approximately at -1.5 MEV. As T increases, this level also moves up and therefore this time towards the experimental region. As a result σ_{ex}/σ increases with T.

As has been discussed in (I) the approximate mass of Be^8 can be used as an argument in favor of fitting the excitation function of $L=0$ of the incident protons and using the corresponding value of U. This argument still holds as will be seen from calculations on the positions of stationary levels using Van Vleck's potential. While in (I) the depth of the potential well U was kept constant, we now keep A and α constant and equal to the values which give good fits to the experimental excitation function.

In Table II are given results of calculations on the position of stationary levels using the exchange as well as the constant potentials. Exact radial functions were used inside the "well" and WKB approximations outside. The calculations were checked by using first-order perturbation formulas, and computing the energy difference between corresponding levels with the same L for the exchange potential and the constant potential. The difference between U_{ex} and U was used as the perturbation potential and the wave functions outside the "well" were approximated by ae^{-br}.

In Table II L represents the angular momentum of the incident proton, l the angular momentum associated with the stationary level, while W stands for the energy of that level. The difference in depths of corresponding stationary levels between the Van Vleck and constant "wells" is denoted by ΔW while $\Delta W'$ is the same obtained from the perturbation calculation. All the numerical values are in MEV.

It will be noted that for $L=1$, $l=1$, the change due to using exchange forces rather than a constant potential is insignificant. It is thus still impossible to use this well for obtaining the value $W=-17$ MEV required by the mass defect of Be^8. On the other hand, the value -17.13 for $L=0$, $l=1$, using Van Vleck's potential, agrees very well with what is needed and is seen to be in better agreement with experiment than -10.5 obtained by using the constant potential. The apparently excellent agreement thus obtained is probably somewhat fortuitous. It is nevertheless encouraging that the agreement is made better rather than worse through the use of exchange forces. The value of A which is needed to account for the large U required for $L=0$ is larger than the values needed to account for the mass defects of light nuclei. It agrees better with the values which follow from mass defects of heavy nuclei using statistical considerations as may be expected from the fact that Eq. (1) was also obtained by a statistical method. Thus presumably the value of A used above is too high and a more accurate calculation will give the same potential for a smaller A. This may also be expected from the fact that from the mass defects of light nuclei the statistical method gives a too large estimate of A as compared with more exact methods of calculation.[5]

[5] W. Heisenberg, Zeits. f. Physik **96**, 473 (1935).