C are, should aid the fixing of the scheme of levels and thereby the determination of $D(X)$.

State A appears to be following a normal course in the latter part of its observed range. Extrapolation gives about 19,000 cm^{-1} as its heat of dissociation, which places its total energy of dissociation at $40{,}260+19{,}000=59{,}260$ cm^{-1}. In both schemes in Fig. 4 it is assumed that this extrapolated value of $D(A)$ is approximately correct in determining which pair of atoms give A. In either case, however, if state A were assumed to dissociate into the next lower atomic pair, the values of $D(A)$ would not be impossibly low.

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

A Note on the Diffraction of Cathode Rays by Single Crystals

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The black and white line parabolas and circles observed in the diffraction patterns of electrons are shown to be a kind of envelope of the family of Kikuchi lines due to a set of co-zonal planes. That the parabolas appear isolated from the generating Kikuchi lines is due to an effect similar to that which in the case of x-rays accounts for the reduction in density observed at the intersection of two lines in the diffraction pattern. The circle is in fact the envelope of a family of parabolas, its appearance isolated from the generating lines being accounted for in the manner mentioned above. It is shown that the circle is the boundary of a region within which occur no Kikuchi lines of a certain set.

1. INTRODUCTION

When a beam of high voltage cathode rays is sent against a surface of a single crystal at a grazing angle and the scattered beam received on a photographic plate, a diffraction pattern, consisting of Kikuchi lines and spots,[1] is obtained. As the envelopes of certain families of these Kikuchi lines there are observed several parabolas and sometimes also circles. In a previous paper,[2] an account of an investigation on the parabolas in the diffraction patterns for rocksalt and calcite was given. The present paper is concerned with a more general investigation of these curves. The energy of the rays used was of the order of 70 kv.

[1] S. Kikuchi, Jap. J. Phys. **5**, 83 (1928–29). S. Nishikawa and S. Kikuchi, Proc. Imp. Acad. Japan **4**, 475 (1928).

[2] K. Shinohara, Sci. Papers, Inst. Phys. and Chem. Res. **20**, 39 (1932).

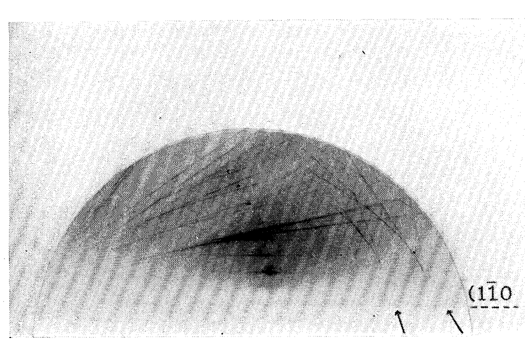

FIG. 1. Zincblende $\lambda=0.043$A (75 kv).

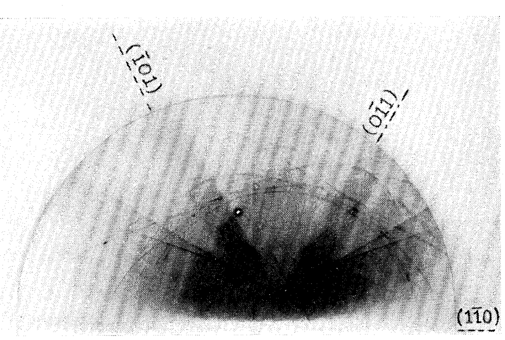

FIG. 2. Zincblende $\lambda=0.044$A (71 kv).

Parabolas

2.

In Figs. 1 and 2, which are photographs taken with the cleavage face of zincblende, there may be seen many black curves looking like parabolas. Their axes are the intersection lines of net-planes of large spacing, *viz.*, the $(1\bar{1}0)$-, $(0\bar{1}1)$- and $(\bar{1}01)$-planes, with the photographic plate. Some of the curves look quite continuous, but most of them show more or less sudden breaks at several points. Some, such as are indicated by the arrows in Fig. 1, are actually formed of short segments of straight lines. It can be seen that these segments are parts of Kikuchi lines and further that the lines belonging to each curve are due to net-planes containing a common zone axis.

White curves also were frequently observed. For example, in Fig. 1 two of them appear in the right corner but may be difficult to see in the reproduction. They appear in case a common zone axis makes a rather large angle with the primary beam, as in this photograph. The geometrical interpretation of their formation is similar to that of the black curves. No white curves had been observed when the previous paper was written, as no photographs having the zone axis suitably directed had been obtained at that time.

Emslie[3] recently observed similar curves in his photographs for stibnite and galena. He considered them to be due to the diffraction of electrons confined to a single net-plane. It was because of the apparent continuity of the curves in his photographs that he preferred this explanation to that of the envelope, and, just on this account, it does not account for the fact that some of the curves have sudden changes of curvature at several points. Moreover, it also seems difficult to explain the existence of white curves. As a matter of fact, the curves seem continuous and without sharp breaks at the intersection points of the generating Kikuchi lines. However this may be expected from the dynamical theory which takes into account the interaction of the electrons with the crystal, as has been shown in the previous paper on the parabolas in the pattern for rocksalt.

[3] A. G. Emslie, Phys. Rev. **45**, 43 (1934).

Reduction in Density of Kikuchi Lines

3.

At the intersection of two Kikuchi lines there is always the disturbing effect of a third plane. Let (l_1, l_2, l_3) and (m_1, m_2, m_3) be the indices of two net-planes and $\mathfrak{l}$ and $\mathfrak{m}$ two radius vectors in the reciprocal lattice, so that

$$\mathfrak{l}=l_1\mathfrak{b}_1+l_2\mathfrak{b}_2+l_3\mathfrak{b}_3,$$

and

$$\mathfrak{m}=m_1\mathfrak{b}_1+m_2\mathfrak{b}_2+m_3\mathfrak{b}_3,$$

where $\mathfrak{b}_1$, $\mathfrak{b}_2$ and $\mathfrak{b}_3$ are the reciprocal axes. Then,

$$\mathfrak{k}_l=\mathfrak{k}_0+2\pi\mathfrak{l}, \tag{1}$$

and

$$\mathfrak{k}_m=\mathfrak{k}_0+2\pi\mathfrak{m}, \tag{2}$$

where $\mathfrak{k}_0$, $\mathfrak{k}_l$ and $\mathfrak{k}_m$ are, respectively, the propagation vectors of the diffracted beam moving in the direction of an intersection of two Kikuchi lines and of the two incident beams which give rise to this common diffracted beam. Therefore,

$$\begin{aligned}\mathfrak{k}_l&=\mathfrak{k}_m+2\pi(\mathfrak{l}-\mathfrak{m})\\&=\mathfrak{k}_m+2\pi\mathfrak{h},\end{aligned} \tag{3}$$

if we write $\mathfrak{h}$ for $\mathfrak{l}-\mathfrak{m}$. From Eqs. (2) and (3) we see that the beam moving initially in the direction of $\mathfrak{k}_m$ is simultaneously reflected by the $(-m_1, -m_2, -m_3)$- and (h_1, h_2, h_3)-planes; in other words, the reflection of the beam from the $(-m_1, -m_2, -m_3)$-plane into the direction of the intersection (i.e. k_0) will be disturbed by the (h_1, h_2, h_3)-plane. The same is true also for the beam moving in the direction of $\mathfrak{k}_l$. As will be shown in §5, this disturbing plane is common for every intersection of two consecutive Kikuchi lines which generate a parabola. Now, the effect of the disturbance increases in general with the reflecting power of the (h_1, h_2, h_3)-plane and is expected to be especially large if the plane is one of cleavage; that is, when the axis of the parabola becomes the intersection line of the cleavage face with the photographic plate. The behavior of black or white lines at such an intersection is shown diagrammatically in Fig. 3. On the other hand, it is also observed that two Kikuchi lines sometimes cross each other without a marked disturbance. In such a case, the angle

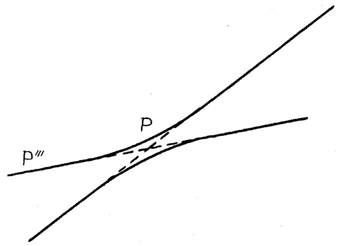

FIG. 3. The broken lines show the behavior when the manner of reflection is not disturbed.

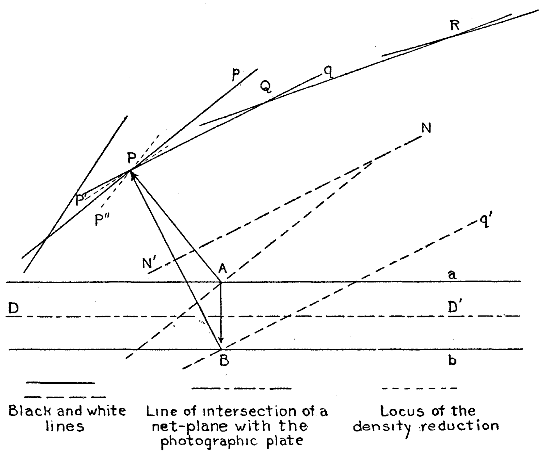

FIG. 4.

between them is usually large and consequently, the reflecting power of the disturbing net-plane becomes small.

4.

A qualitative investigation of the effect of the disturbance can be made in the following way. In Fig. 4, let DD' be the disturbing net-plane and P, one of the points of intersection of the Kikuchi lines. We shall assume, in accordance with the dynamical theory of electron diffraction, that the lines have finite breadth. Then we may expect that the intensity of the beam reflected from the direction given by A or B into P will have two maxima close to one another, separated by a narrow region of reduced density, as has been shown in a previous paper,[2] and studied experimentally by Kikuchi and Nakagawa[4] for the reflection of a beam by a cleavage plane. This reduction in density is due to the reflection of the beam by the plane DD' and the position of the minimum coincides approximately with the direction of the reflected beam which in its initial direction exactly satisfies Bragg's condition for the plane DD'. Now, Bragg's condition for DD' is satisfied by any point on a and b, and therefore there will be a locus of the minimum or density reduction corresponding to each and crossing p and q, respectively, in the neighborhood of P. The locus of the density reduction which intersects p in P may be either PP' or PP''. Making use of the fact that the directions of the incident and the reflected beams are related by $\mathfrak{k}_l = \mathfrak{k}_0 + 2\pi\mathfrak{l}$ and of the condition that the tangential components should be continuous at the surface, we can show by a simple calculation that the first situation occurs when the expression

$$D = [l_t - l_n\,(\alpha + 2\pi l_t)/\gamma_l]/[l_t - l_n\,(\alpha/\gamma)]$$

is positive at P and the second when it is negative, where α and γ are the tangential and normal components of the propagation vector of the incident beam; γ_l, the normal component of that of the reflected beam which is in the direction of P; and where l_t and l_n, respectively, are the components of l in the direction of α and γ. The sign of the normal components is taken so as to make γ_l positive.

In applying this to an intersection of two lines, two cases may be considered. In the first case the disturbing plane is the cleavage plane while in the second it is not. In the first case, D becomes positive for p and negative for q at P and further, as can be seen from a simple calculation, the locus of the density reduction coincides for the two lines. Thus, the appearance of the black lines at this point will be that shown in Fig. 3. The further fact that the branch near P''' (Fig. 3) becomes quite faint will be due to the dynamical interaction of the electrons with the crystal. The same will be true also for white curves, since, according to a calculation similar to that given in the previous paper, the extinction and absorption lines at P resemble one another in general features but are inverted in intensity. In the second case, when the disturbing plane is not a cleavage plane but a plane such as $(0\bar{1}1)$ or $(\bar{1}01)$ in Fig. 2, D is in general positive both for p and q at P and the behavior of the curve near this point is more complex than in the first case.

[4] S. Kikuchi and S. Nakagawa, Sci. Papers, Inst. Phys. and Chem. Res. **21**, 256 (1933).

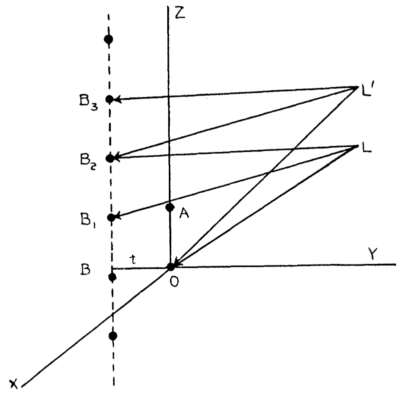

FIG. 5.

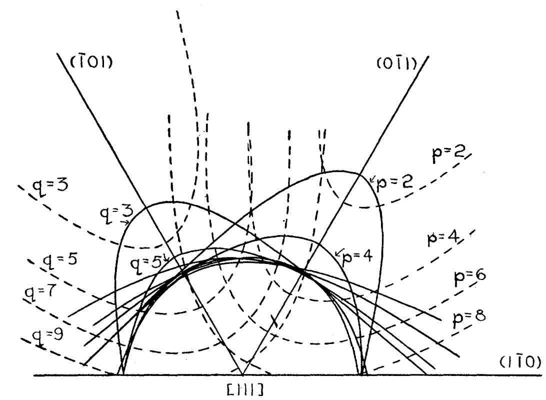

FIG. 6.

FORMATION OF PARABOLAS

5.

Let us now make some geometrical considerations of the formation of parabolas with the help of a reciprocal lattice. Take a series of lattice points B_1, B_2, B_3, $\cdots$ (Fig. 5) along a straight line and let LO, corresponding, for example, to P in Fig. 4, be the common emergent direction of the beams reflected by two net-planes B_1 and B_2. The disturbing plane, given by Eq. (3), is represented by A (Fig. 5). If we now alter the direction of the beam to be reflected by B_2 until it coincides with $L'O$, where it is reflected by two net-planes B_2 and B_3, the process corresponds to tracing out the line PQ from P to Q. By a series of segments such as PQ a kind of envelope $PQR\cdots$ is formed. As the points B_1, B_2, $\cdots O$ and A lie in one plane, the net-planes concerned contain a common zone axis perpendicular to this plane.

Now, because of the disturbance at the intersection P, Q, etc., the angles at these points are rounded with the result that the curve becomes almost continuous and approximates in its appearance an actual envelope defined as the locus of the ultimate intersections of a family of lines formed by varying the value of the parameter continuously; i.e., the envelope when the Kikuchi lines are supposed to lie continuously satisfying a certain geometrical condition necessary to form it. This condition may be satisfied by taking lattice points distributed continuously along B_1B_2. As a consequence, A is made to approach extremely near to O. The locus of the direction LO will be a cone, whose intersection with the photographic plate is the envelope.

Take the axes X, Y, Z as in Fig. 5 with the X axis in the direction of the common zone axis and the Z axis parallel to B_1B_2. Then the equation of the cone whose vertex is at O and whose generating line has the direction LO, is given by

$$k^2z^2+4\pi tky(x^2+y^2+z^2)^{\frac{1}{2}} -4\pi^2t^2(x^2+y^2+z^2)=0, \quad (4)$$

where t is equal to OB, the length of the perpendicular drawn from O to B_1B_2 and k is the wave number (i.e., 2π divided by the wave-length). For high voltage cathode rays, we can take k large compared with πt. Further we shall assume the X axis to be approximately normal to the photographic plate as in most of the curves observed. Then, taking x large compared with y and z, and neglecting small quantities in Eq. (4), we obtain

$$k^2z^2+4\pi tkxy=4\pi^2t^2x^2. \quad (5)$$

When the x axis is perpendicular or nearly perpendicular to the plate we may replace x by a, the distance of the crystal from the plate, and the curve formed by the intersection of this cone with the plate becomes a parabola given by

$$k^2z^2+4\pi tkay=4\pi^2t^2a^2,$$

and coincides with the curve represented by Eq. (2) in the previous paper.

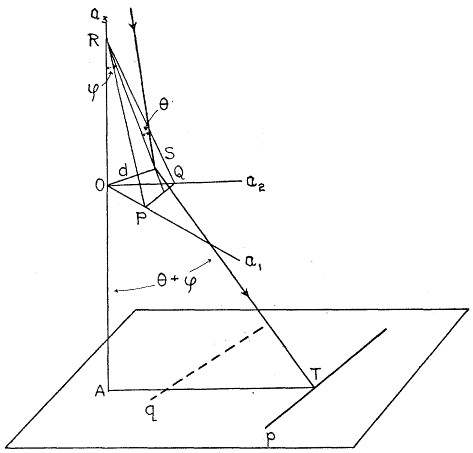

FIG. 7.

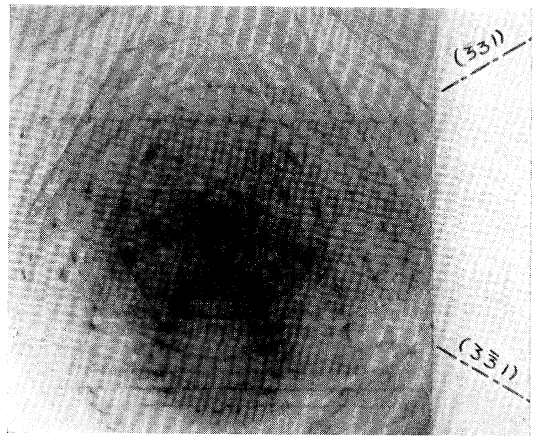

FIG. 8. Mica $\lambda=0.056$A (45 kv).

As an example we have in Fig. 2 a series of zone axes parallel to the $(0\bar{1}1)$ plane, having the indices $[p\ p+1\ p+1]$, such as [233], [455], [677], etc., and others, having indices $[q\ q+1\ q]$, parallel to the $(\bar{1}01)$ plane, as [343], [565], [787], etc., all making small angles with the [111] axis. In this case we obtain the series of parabolas shown in Fig. 6, in which the solid lines represent the black lines and the broken lines the white lines. The white curves, though observable in the original negative, will be difficult to see in the reproduction (Fig. 2).

CIRCLES

6.

In Fig. 2, besides the parabolas already described, there is observed a circle with its center at the intersecting point of the [111] axis with the photographic plate. This circle appears to be a sort of envelope of a family of parabolas. It will be seen that there are no black lines in the circle except those running radially. The circle will be taken as a kind of envelope of straight lines and gives the boundary within which black lines of a certain set of planes do not enter.

In Fig. 7, let $\mathfrak{a}_1$, $\mathfrak{a}_2$ and $\mathfrak{a}_3$ be three translations of the lattice, of which $\mathfrak{a}_3$ represents a special direction such as [111]. Further, let PQR be the net-plane which makes an angle φ with $\mathfrak{a}_3$ and has the indices (lmn). By reflection from this plane, a pair of black and white lines p and q will be formed on a photographic plate which we shall take, for simplicity, normal to $\mathfrak{a}_3$. Let A be the point of intersection of $\mathfrak{a}_3$ with the plate and T, the foot of the perpendicular drawn from A to the line p which lies farther from A, then the beam ST is determined by $\theta+\varphi$ where θ is the angle given by Bragg's condition $\lambda=2d\sin\theta$. From the figure we see that $\sin\varphi=nd/|\mathfrak{a}_3|$, therefore

$$\cos(\theta+\varphi)=\left(1-\frac{n^2d^2}{|\mathfrak{a}_3|^2}\right)^{\frac{1}{2}}\left(1-\frac{\lambda^2}{4d^2}\right)^{\frac{1}{2}}-\frac{n\lambda}{2|\mathfrak{a}_3|}.$$

If we take various net-planes having a common value for n, there is a minimum for $\theta+\varphi$. Considering l and m to vary continuously, it occurs at

$$d^2=\lambda|\mathfrak{a}_3|/2n \tag{6}$$

and
$$\cos(\theta+\varphi)=1-n\lambda/|\mathfrak{a}_3| \tag{7}$$

at this limit. Thus we see from Eq. (6) that the net-planes whose black lines approach this limit depend on the wavelength of the cathode rays and from Eq. (7), that the minimum of $\theta+\varphi$ is given by a right cone having OA as its axis, whose intersection with the photographic plate will be a circle. When $n=1$ the circle just coincides with that observed in Fig. 2. The radius of the circle is determined by $|\mathfrak{a}_3|$, n and the wavelength and for various values of n a number of concentric circles can be expected to appear. The arrangement of the lattice points on the plane parallel to $\mathfrak{a}_1$ and $\mathfrak{a}_2$ does not affect their radii but may slightly affect the appearance of the circles.

As we can see in Fig. 2, the circle is quite continuous and is also isolated from the other lines. It is due to a large number of black lines concerned in its formation and the effect of the density reduction, which, in this case, arises from several net-planes having large reflecting powers, while in the case of a parabola, it arose from a single net-plane.

Another example of such circles can be seen in the pattern (Fig. 8) obtained by transmitting a beam of electrons through a thin sheet of mica. The center of the concentric circle is at the intersecting point of the [$\bar{1}03$]-axis, the axis parallel to the intersection of the (331)- and ($3\bar{3}1$)-planes with the photographic plate. Among the circles, those with large radii are quite continuous, but those with small radii look rather like polygons, as the black lines concerned in their formation are few in number.

Emslie has observed, in the paper already cited, a similar circle for stibnite and considered it to be due to the localizing of an electron's wave packet to the neighborhood of a single row of atoms. The radius of the circle due to the diffraction of such an electron coincides with that in our case. But this explanation, as in the case of parabolas, cannot account for the fact that some of the circles have many sudden breaks. As a matter of fact, we see by a close examination that even for the circle in Fig. 2, the radius of curvature is not exactly constant. Further, the value of the refractive index which Emslie measured from the contraction of the radius of the circle turned out too large but this we may expect from our theory as the contraction of the radius also will be caused by an effect related to the density reduction.

In conclusion the author wishes to express his sincere thanks to Professor S. Nishikawa for his kind guidance throughout this work.

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Equation of a Wave in a Medium with Velocity a Function of Time

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The equation of a wave in a medium for which the velocity is a function of time is obtained. On purely physical considerations for the case of $v=v_0e^{-\alpha t}$ and a simple periodic oscillation at the source, the equation is $y=a\sin 2\pi\nu_0[t-(1/\alpha)\ln(1+(\alpha x/c)e^{-\alpha t})]$. From other considerations the differential equation obtained is $\ddot{y}+\alpha\dot{y}=c^2e^{-2\alpha t}y''$ which is satisfied by the equation above.

In a recent article[1] the writer discussed the characteristics of a wave in a medium with the velocity decreasing slowly with time. It was shown that a wave once emitted would continue with a constant wavelength. The velocity function there considered was

$$v=c(1-\alpha t), \tag{1}$$

and it was shown that for an observer at a distance x from the source, there is a decrease in received frequency proportional to the distance to the source. In the case of light this means a redward shift and it was pointed out that one could thus account for the redward shift of light from nebulae without resorting to an expanding universe, the calculated value of α being 1.81×10^{-17} reciprocal seconds.

It has become desirable to find the form of the function $y=f(x, t)$ for a wave in such a medium and we shall take the velocity function as

$$v=ce^{-\alpha t}, \tag{2}$$

this v being more flexible mathematically than the linear function formerly used. Here again, as in any case where the velocity is a function of time and not of position, a wave once emitted continues with unchanged wavelength, but the wavelength as emitted at the source gradually changes as the velocity changes. The instan-

[1] Wold, Phys. Rev. **47**, 217 (1935).

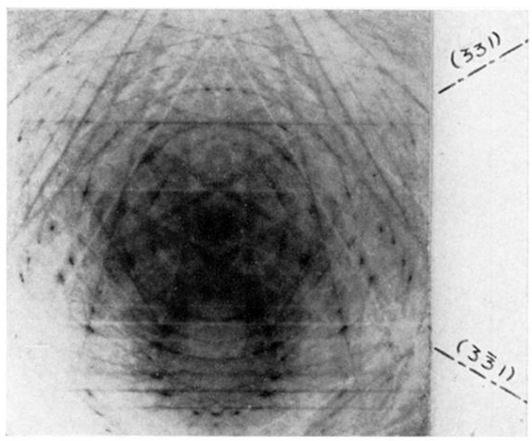

FIG. 8. Mica $\lambda = 0.056$A (45 kv).

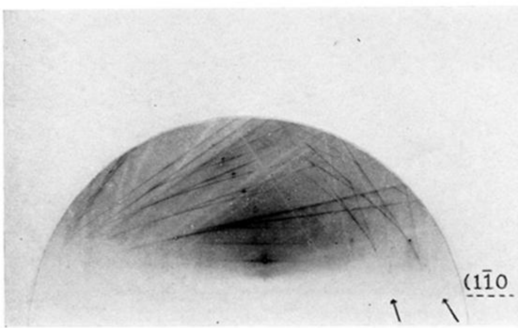

FIG. 1. Zincblende $\lambda=0.043$A (75 kv).

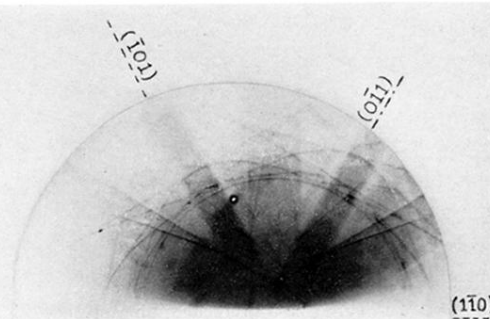

FIG. 2. Zincblende $\lambda=0.044$A (71 kv).