lying on the light cone. It is possible to take into account the effect of the field on the $p-E$-space; since an amount $e\varphi_{a/c}$ is added to the momentum-energy of a particle of charge e in a field of potential φ_a, it is evident that the field produces a translation, proportional to the charge on the particle, of all the hyperboloids, and at the same time leaves the light-cone unchanged. This translation can thus cause a material particle to have a representative vector lying *on* or even *outside* the light-cone so that Synge's method is inapplicable in this case.

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The Widths of X-Ray Bands in Solids

In a recent number of this journal,[1] O'Bryan and Skinner have presented some interesting experimental and theoretical data on x-ray emission bands in solids, and in a companion paper Jones, Mott and Skinner[2] have considered their interpretation in the light of Bloch's theory.

I have also been making some rough calculations recently, particularly on the widths rather than the forms of these bands. From perturbation theory and also from a simple application of the Bloch theory one can derive the approximate formula $\Delta E=3p\beta/4$ for the "half-width" of a band (in ergs) where p is the number of closest neighbors to an atom in the lattice, and $\beta=|\int\psi_a{}^*V\psi_b d\tau|$ is the fundamental integral of the Bloch theory. By using the simple wave functions and shielding constants of Slater[3] one can derive the following rather approximate expression for β:

$$\beta=(Z^*e^2/4\pi a_0)\cdot 2^{2n^*+1}\cdot(R_0/a_0)^{n^*-2} \times e^{-R_0/a_0}[\Gamma(n^*+2)/\Gamma(2n^*+1)],$$

where Z^* and n^* are the effective nuclear charge and principal quantum number for a valence electron, R_0 is the average internuclear distance, and Γ represents the ordinary gamma-function.

O'Bryan and Skinner find that the simple formula $\Delta W=(h^2/2m)(3\rho N/8\pi)^{2/3}$ for the "total" width, derived from the Sommerfeld theory, gives an unexpectedly good check. The half-widths computed from the approximate values of β, while of the correct order, are not in as good numerical agreement with experiment as the above formula for ΔW. The relation between "half" and "total" width is ambiguous, depending on the intensity distribution in the band. We can account for it roughly by writing $\Delta W=\sigma\Delta E$ where σ is expected to be of the order of 2 to 3. If we assume both formulas to be valid, they imply a relation between N, the number of valence electrons per atom, and $R_0(=\rho^{-1/3})$, the average internuclear distance in the lattice. This can be written as

$$R_0{}^2\beta pN^{-2/3}=(2h^2/3m\sigma)(3/8\pi)^{2/3}=(\text{a constant})/\sigma$$

or approximately

$$Z^*N^{-2/3}p\cdot 2^{2n^*+1}\cdot(R_0/a_0)^{n^*}\cdot e^{-R_0/a_0}[\Gamma(n^*+2)/\Gamma(2n^*+1)]$$
$$=(4\pi/e^2)(2h^2/3m\sigma a_0)(3/8\pi)^{2/3}$$
$$=80.6/\sigma=A.$$

Table I contains the values of A computed by means of the above equation, as well as the values of the ratio σ deduced therefrom:

TABLE I.

Element	Li	Be	Na	Mg	Al	Si
R_0 (Angstroms)	3.03	2.28	3.72	3.22	2.85	2.35
A	9.1	14.8	23.6	47.9	62.1	29.2
$\sigma=80.6/A$	8.9	5.5	3.4	1.7	1.3	2.8

The values for σ are of the correct order of magnitude. The variation shown in the table is not surprising when one considers that we have combined results from two limiting theories, one valid for tightly bound electrons, the other for "free" electrons, and that the formula is very sensitive to the numerical values of the quantities involved.

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[1] O'Bryan and Skinner, Phys. Rev. **45**, 370 (1934).
[2] Jones, Mott and Skinner, Phys. Rev. **45**, 379 (1934).
[3] Slater, Phys. Rev. **36**, 57 (1930).