Space Density of Cosmic-Ray Particles

A portion of the primary cosmic radiation entering our earth exists in the form of charged particles. If it exists in this form in interstellar space, its existence necessitates a certain volume density of particle radiation there. The object of the present note is to show that this density is large enough to insure that if the particles were all of one sign, there would exist differences of potential of enormous amounts between points in space separated by distances which are relatively small on an astronomical scale.

Suppose $\alpha ds d\omega$ is the number of cosmic-ray particles crossing the element of area ds within a solid angle $d\omega$, whose axis is perpendicular to ds. Then the number of rays crossing ds within the solid angle $\sin\theta d\theta d\varphi$ is $\alpha\cos\theta\sin\theta d\theta d\varphi ds$, where θ is measured from the normal to ds. The number of these to be found at any instant within the cylindrical element of volume $dsdx$, of length dx perpendicular to ds is $\alpha\cos\theta\sin\theta d\theta d\varphi dxds/c\cos\theta$, where c is the velocity of light. The total space density (number of particles per cc) contributed by the particles from all directions, including both sides of ds is

$$D=\frac{2\alpha}{c}\int_0^{2\pi}d\varphi\int_0^{\pi/2}\sin\theta d\theta=\frac{4\pi\alpha}{c},$$

a result analogous to the well-known corresponding formula in heat radiation.

If the particles all have a charge e, the volume density ρ of electricity which they constitute is

$$\rho=4\pi\alpha e/c.$$

At any point in interstellar space let us draw a sphere with center O and radius R. The field at any point inside the sphere, and so the potential difference between the center and any point P on the boundary is composed of a part resulting from the electricity outside the sphere, and a part resulting from that inside. The difference of potential V between P and O caused by the latter is

$$V=\frac{4}{3}\pi\rho\int_0^R\frac{r^3}{r^2}dr=\frac{8\pi^2e\alpha R^2}{3c}.$$

Now if the potential difference between P and O is reduced below V by the field caused by the charge outside the sphere, the corresponding difference of potential will be increased by a practically equal amount in the case of the difference of potential between O and a point Q diametrically opposite to P. In case one has any doubts on this point, imagine the sphere R surrounded by a concentric sphere S of radius large compared with R. The electricity between S and R produces no field inside R, and the field resulting from the more distant electricity outside S produces a field, which is sensibly uniform inside R.[1] The result of these considerations is to the effect that there will be points on R which differ in potential from the center O by an amount comparable with V.[2]

Now if we take for α a value no larger than that corresponding to the earth's surface for cosmic rays, i.e., $\alpha=0.7\times10^{-2}$; and if we express V in volts and replace R by the value R_0 measured in light years, we have

$$V=\frac{300\times8\pi^2\times4.7\times10^{-10}\times0.7\times10^{-2}\times(3\times10^{10}\times365\times86400)^2R_0^2}{3\times3\times10^{10}},$$

$$V=7.7\times10^{17}R_0^2\text{ volts.}$$

Thus V amounts to about 7.7×10^{17} volts for $R_0=1$ light year. Since α should probably be taken as larger than the above value for a point outside the atmosphere,[3] the above value of V certainly represents a lower limit. A sphere one light year in radius is a relatively small sphere as astronomy counts distances. Hence, we see that the assumption of particles of only one sign in sufficient amount to account for cosmic-ray intensities would lead to conclusions prohibited by the possibilities of nature. Of course the presence of charges of both signs would alleviate the difficulty without the necessity of assuming that both signs of charge had energy sufficient for cosmic rays. Such a situation would presumably prevail if positive and negative particles were created in space or were released from atoms existing there. Any attempt to picture particles of one sign as emanating from the stars or other heavenly bodies would result in a condition in which the particles would become bound to their respective stars as a sort of space charge unless both signs of charged particles were emitted.

In conclusion it may be remarked that the author's recent suggestion[4] as to the possibility of the emission of high energy charged particles by the heavenly bodies is by no means limited to particles of one sign.

W. F. G. Swann

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[1] We wish to avoid any vague statements based upon implications that O is the "center" of an infinite distribution of charge.

[2] It is not contended that there is a unique solution of the problem for a charge which extends to infinity in some unspecified manner. There most certainly is not. The potential difference between O and the sphere R will depend upon the specific problem. All that is maintained is that there are points on the sphere of radius R for which the potential differs from that at O by the amount specified.

[3] It will be recalled that in line with the conclusions of G. Lemaitre and M. S. Vallarta, Phys. Rev. **43**, 87 (1933), the intensity at the earth's surface should be equal to that at infinity except, of course, for absorption.

[4] W. F. G. Swann, Phys. Rev. **43**, 217 (1933).