

The Effect of Tension on the Electrical Resistance of Single Antimony Crystals

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The adiabatic tension coefficient of resistance of single antimony crystals has been determined for various orientations. Since antimony crystals have the same type of symmetry as bismuth, the curves connecting the coefficient with the primary and secondary orientations have the same general shape as in the case of bismuth and again uphold the theory put forth by Bridgman. The observed values of the coefficients for antimony when the tension and current are parallel to the trigonal axis and per-

pendicular to it are different both in magnitude and in sign, whereas in the case of bismuth the two were both negative and very nearly equal in magnitude. The six coefficients necessary completely to determine the behavior of the resistance when deforming forces are applied to the antimony crystals have been found (1) without correcting for strain and (2) correcting for the changes arising from the strain produced by the tension.

IN a previous paper¹ the author has studied experimentally the effect of tension on the electrical resistance of single *bismuth* crystals. The experimental dependence of the tension coefficient on the principal and secondary orientations of the crystals with respect to the cylindrical axis was later found by Professor Bridgman² to be in agreement with a geometrical theory based on the symmetry of the crystal. The object of the present paper is to examine the effect of tension on the resistance of single crystals of *antimony* which has the same type of symmetry as bismuth and to see whether the results obtained are consistent with the theory. The values of the six tension coefficients are found numerically, both as observed directly and as corrected for the change in resistance due to the strains in the crystal produced by the tension.

PROCEDURE

Experimentally the procedure is somewhat more difficult than in the case of bismuth. The specific resistance of antimony is roughly one third that of bismuth so that the resistance of a similar piece of antimony is approximately one third as great. Furthermore, since the tension coefficient of resistance of polycrystalline antimony is of the order of one tenth³ that of

polycrystalline bismuth, it is to be assumed that the tension coefficients of antimony will be of the order of one tenth of those of bismuth. Consequently, the absolute change in resistance to be measured will be about one thirtieth that studied in the case of bismuth. This is counteracted in some degree by the fact that twice and even four times as great a tension (force per unit area) as had been necessary to use with bismuth was here safely applied. On the other hand, antimony is more brittle and so is more difficult to handle.

Therefore the apparatus had to be studied afresh to see in what way the sensitivity could be increased—and preferably to about ten times its previous value. Considerable increase in sensitivity was obtained by using twice as large, and sometimes four times as large, a current as in the work with bismuth. The most obvious way further to increase the precision of the method was to use a more sensitive galvanometer, but even the new Leeds and Northrup galvanometer was rated to have only twice the voltage sensitivity of the older galvanometer already in use. It was pointed out in the previous paper¹ that when the *deflection* method was used the voltage sensitivity of the galvanometer depended on the resistance r of the compensating circuit Q across which the galvanometer was connected and was in fact proportional to the sum of this resistance r and the resistance of the galvanometer itself. It is thus of advantage to use a small r . Moreover, in comparing small changes of re-

¹ Mildred Allen, Phys. Rev. **42**, 848 (1932).

² P. W. Bridgman, Phys. Rev. **42**, 858 (1932).

³ P. W. Bridgman, Proc. Amer. Acad. **57**, 41 (1922).

sistance it seemed desirable to attempt to use the galvanometer so that its voltage sensitivity remained constant. This was effected by choosing r as 3 ohms plus half the length of the slide wire in the auxiliary circuit, and by roughly balancing the potential fall across the crystal against the auxiliary potential difference by varying the large resistance governing the current in the auxiliary circuit. The final precise balancing could then be carried out by moving the contact on the slide wire a small distance. For this reason the large resistance in the auxiliary circuit Q was chosen to be a 10,000 ohm variable radio resistance. The voltage sensitivity of the galvanometer, with the scale at 5 meters as before, was 2.15×10^{-7} volts per cm. Readings then were satisfactorily large as they varied from 0.15 cm to 4.00 cm. With this *deflection* method it proved to be possible to measure the tension coefficients to within about 1×10^{-6} in absolute value where the coefficient itself varied from $+9.0 \times 10^{-6}$ to -35.0×10^{-6} . The *null* method used before was discarded, since the changes of resistance were too small to give more than a millimeter's displacement on the balancing slide wire of the main circuit; its only use in this set of experiments was to check the sign of the change of resistance.

A further precaution was to decrease the size of the air chamber containing the crystal and the surrounding copper cylinder, so that temperature equilibrium was obtained more rapidly. The substitution of a mercury thermostat for the metal one and of a Burgess vacuum switch for the tilting mercury switch made the temperature less variable and entirely independent of the room temperature. This greater stability of temperature was of importance in measuring the markedly smaller effect and was reflected in the character of the deflection runs made with alternate readings with and without tension, Fig. 1, which in this work with antimony usually showed a definite periodic variation. This periodicity must arise primarily from the periodic changes in temperature produced by the regular increases and decreases in heating current governed by the thermostat. In addition there is usually to be noted a small gradual displacement of the whole curve up or down, and this drift in a given direction can reasonably be ascribed to the

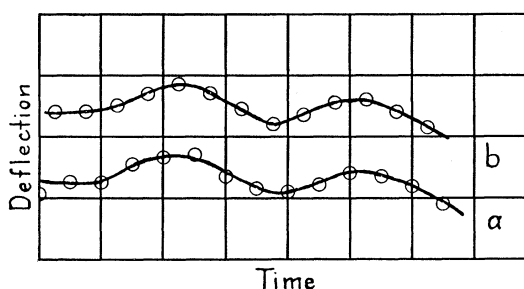


FIG. 1. *a* without tension; *b* with tension.

zero drift of the galvanometer and to the unequal changes of voltage in the two balancing circuits arising very probably from the polarization of the dry cell of the auxiliary circuit Q . As is seen in Fig. 1 this drift is very small when temperature equilibrium has been thoroughly established before beginning to take readings.

The crystals used were of Kahlbaum antimony. Some of them had been made and used by Professor Bridgman and were generously put by him at the writer's disposal; they had diameters of approximately $1/8''$. Others were crystallized by the writer, following the procedure indicated by P. W. Bridgman⁴; most of these latter had a diameter of approximately $1/16''$. The advantage of the smaller diameter was the greater resistance giving a correspondingly greater change of resistance; the disadvantage, the greater difficulty in handling this brittle metal.

RESULTS

I. Without correcting for strain

Twenty-one separate individual single crystals of antimony have been studied and their tension coefficients measured. Because of the smallness of the effect it proved not to be sufficient to measure the coefficients once with the current flowing in but one direction. With most of the crystals the coefficient was determined with one tension and then under identical conditions with the current reversed; to check this result this entire process was then repeated with either the magnitude of the current or of the tension changed. The two determinations were likely not to agree if the crystal were slightly cracked. Furthermore the accuracy with which the coeffi-

⁴ P. W. Bridgman, Proc. Amer. Acad. 63, 351 (1929).

cient could be determined under given conditions without reversal of the current was found by taking two determinations with only the galvanometer leads reversed in the two cases. In only a few cases did the difference between these two readings differ by more than 0.3×10^{-6} in absolute value. On the other hand, the readings with the current reversed were likely to differ by a considerably larger amount, usually of the order of 1.0×10^{-6} . These differences were thus larger than the error to be expected as intimated with the galvanometer leads reversed. It seemed at one time that these discrepancies were proportional to the magnitude of the current used, and so were to be ascribed to a tension coefficient of the Peltier effect, but further work showed this explanation to be untenable, so that probably they arise from some irregular heating effect and are eliminated by taking the average of the readings with the direct and reversed currents.

The values of the tension coefficient were then to be studied on the basis of Professor Bridgman's theory² as functions of the primary orientation of the crystal θ and of the secondary orientation φ . The angle θ was defined as the angle between the trigonal axis of the crystal and the cylindrical axis. The angle φ was defined as the angle between the projection of the cylindrical axis on the principal cleavage plane (which is perpendicular to the trigonal axis) and the projection on the same plane of the normal to that secondary cleavage plane which makes an angle of about 71° with the principal cleavage plane. The relation derived by Professor Bridgman was the following:

$$\beta = (1/\rho_\theta) \{ \rho_{11} \sin^4 \theta + \rho_{33} \cos^4 \theta + (2\rho_{13} + \rho_{44}) \sin^2 \theta \cos^2 \theta - 2\rho_{14} \sin^3 \theta \cos \theta \cos 3\varphi \}, \quad (1)$$

where β is the tension coefficient, which is the relative change in resistance $\Delta R/R$ produced by a tension T of 1 kg/cm², the ρ_{rs} 's are five of the six

⁵ Since ρ_{11} and ρ_{33} determine the tension coefficients for $\theta=90^\circ$ and $\theta=0^\circ$, respectively, it appears from Eq. (2) that the tension coefficient for $\theta=90^\circ$ is known with less error than for $\theta=0^\circ$. The coefficient ρ_{14} gives the distance between the curves for $\varphi=0$ and for $\varphi=60^\circ$ for a given primary orientation; this "spread" of the curves is very accurately known. The remaining constant $(2\rho_{13} + \rho_{44})$ determines the exact shape of the curve for $\varphi=30^\circ$ and the error here is seen to be comparatively large.

tension coefficients necessary completely to define the behavior of the resistance of a trigonal crystal under the action of deforming forces, and ρ_θ is the specific resistance at the orientation θ . The expression is the same as that used for bismuth, since bismuth and antimony crystals have the same type of symmetry. The term in φ shows the trigonal symmetry. Since none of the twenty-one readings of the coefficient probably were entirely consistent with any of the others and the tension coefficients were quite small, it seemed worth while to use least squares in evaluating them. The values of the four constants and their probable errors⁵ as found in this way directly from the experimental values are

$$\begin{aligned} \rho_{11} &= (+0.24 \pm 0.09) \times 10^{-10}, \\ \rho_{33} &= (-12.20 \pm 0.33) \times 10^{-10}, \\ 2\rho_{13} + \rho_{44} &= (+5.72 \pm 0.53) \times 10^{-10}, \\ \rho_{14} &= (+5.10 \pm 0.16) \times 10^{-10}. \end{aligned} \quad (2)$$

In Fig. 2 the full lines give the curves computed from these values for $\varphi=60^\circ$, $\varphi=30^\circ$, and $\varphi=0^\circ$.

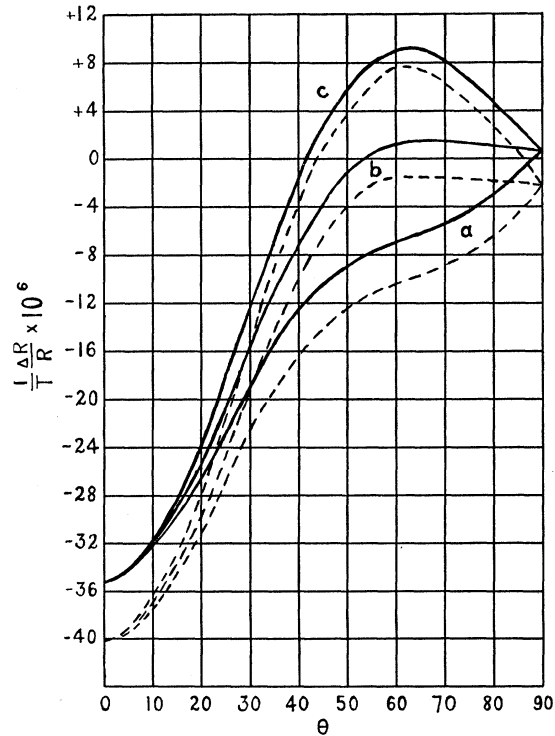


FIG. 2. Dependence of tension coefficients on orientation. *a*, $\varphi=0^\circ$; *b*, $\varphi=30^\circ$; *c*, $\varphi=60^\circ$. Full curves, experimental curves. Dotted curves, corrected for strain.

This single set of curves is sufficient entirely to describe the behavior of the electrical resistance of antimony under tension, since from Eq. (1) it is known that the coefficient always varies between these indicated outside limits proportionately to $\cos 3\varphi$. These curves are like the similar ones for bismuth in that the coefficient has a single constant value for $\theta=90^\circ$ and for $\theta=0^\circ$ (as it must according to the theory) and varies in sign according to the orientation; but they differ in that for bismuth the values of these constants have the same order of magnitude and the same sign, whereas for antimony the absolute magnitude at $\theta=0^\circ$ is more than fifty times that for $\theta=90^\circ$ and the signs of the two are different. A numerical comparison of the observed experimental values and the computed values of the tension coefficient with the constants of (2) is given in Table I and it is seen that the difference between the two only once exceeds 1.1×10^{-6} ,

TABLE I.

θ	φ	Mak- er and size	$R \times 10^3$	$\beta \times 10^6$ expt. uncorrected	$\beta \times 10^6$ theor. uncorrected	$\beta \times 10^6$ expt. corrected for strain	$\beta \times 10^6$ theor. corrected for strain
90°	16°	B L	2.09	+ 1.03	+ 0.54	- 1.92	- 2.40
83°	46°	B L	2.02	+ 3.04	+ 2.74	+ 0.39	+ 0.11
83°	24°	A s	1.98	+ 0.96	- 0.14	- 2.08	- 3.16
79°	17°	A s	2.48	- 2.03	- 1.70	- 5.25	- 4.92
77°	55°	B L	1.96	+ 5.05	+ 5.68	+ 2.83	+ 3.49
75°	40°	B L	2.18	+ 4.02	+ 3.87	+ 1.58	+ 1.44
75°	36°	A s	5.74	+ 1.79	+ 2.83	- 0.78	+ 0.26
73°	03°	B L	1.99	- 4.52	- 4.61	- 8.13	- 8.20
72°	06°	A L	5.51	- 4.43	- 4.57	- 8.00	- 8.16
67°	51°	B L	3.10	+ 6.76	+ 7.88	+ 4.97	+ 6.09
65°	38°	B L	5.96	+ 3.99	+ 3.24	+ 1.74	+ 0.83
64°	34°	A s	4.32	+ 2.64	+ 3.06	+ 0.21	+ 0.65
61°	17°	B L	1.50	- 3.46	- 3.75	- 6.75	- 7.07
62°	07°	B L	1.16	- 6.39	- 5.99	- 10.02	- 9.64
62°	04°	B L	2.01	- 5.64	- 6.34	- 9.32	- 10.03
68°	01°	A s	2.18	- 6.33	- 5.59	- 9.99	- 9.27
54°	53°	A s	1.93	+ 7.25	+ 7.06	+ 5.62	+ 5.45
50°	51°	A L	4.37	+ 5.04	+ 4.98	+ 3.29	+ 3.22
53°	36°	B L	1.41	+ 3.47	+ 1.92	+ 1.16	- 0.38
44°	06°	A s	1.64	- 11.5	- 10.46	- 15.2	- 14.1
22°	56°	A s	0.86	- 21.7	- 21.77	- 25.6	- 25.6

B indicates crystal was made by P. W. Bridgman.

A indicates crystal was made by the author.

L designates a large crystal of approximately 1/8" diameter.

s designates a small crystal of approximately 1/16" diameter.

which considering the smallness of the effect indicates satisfactory agreement. The one reading which differs by more than this amount lies on a very steep part of the curve where a small error in measuring φ necessarily produced a large error in β .

Only two readings were made for θ less than 50° . This came about for two reasons: it is comparatively difficult to grow crystals with the principal cleavage plane nearly perpendicular to the cylindrical length, and, more important, crystals with such orientations are so exceedingly brittle that it is very nearly impossible to mount them in the apparatus without sufficient jarring to break them along a principal cleavage. The point for $\theta=22^\circ$ fitted within experimental error on the curve computed from the points for θ between 44° and 90° , the value of β computed on this basis being -22.9×10^{-6} as compared with the experimental value of -21.7×10^{-6} . This furnishes a check on the correctness of the general trend of the curve which seems entirely satisfactory, although in the final calculation this lowest point was included and the values of the constants modified somewhat thereby.

Completely to determine the six coefficients which are necessary to describe the behavior of a trigonal crystal under the action of forces, the data derived from the effect of tension on antimony crystals must be supplemented by the values of the hydrostatic pressure coefficients when the current is flowing parallel to the trigonal axis and when it is perpendicular to it. Professor Bridgman's best values for these pressure coefficients at 30°C are:

$$\begin{aligned}\theta=90^\circ \quad [(1/p)(\Delta R/R)]_{90^\circ} &= +3.7 \times 10^{-6}, \\ \theta=0^\circ \quad [(1/p)(\Delta R/R)]_{0^\circ} &= +19.4 \times 10^{-6}.\end{aligned}\quad (3)$$

These are taken from a recent paper⁴ in which the pressure coefficients are carefully studied as a function of pressure; these values are extrapolated a short distance to zero pressure to be comparable with the very small forces used in the tension experiments. We have therefore the additional equations as given by Bridgman²

$$+3.7 \times 10^{-6} = -(\rho_{11} + \rho_{12} + \rho_{13})/(\rho_{\theta=90^\circ}), \quad (4)$$

$$+19.4 \times 10^{-6} = -(2\rho_{13} + \rho_{33})/(\rho_{\theta=0^\circ}). \quad (5)$$

This gives finally for the six tension coefficients of resistance *uncorrected* for the changes of resistance arising from the change in dimensions caused by the application of the forces:

$$\begin{aligned}\rho_{11} &= +0.2 \times 10^{-10}, & \rho_{12} &= -4.6 \times 10^{-10}, \\ \rho_{33} &= -12.2 \times 10^{-10}, & \rho_{13} &= +2.7 \times 10^{-10}, \\ \rho_{14} &= +5.1 \times 10^{-10}, & \rho_{44} &= +0.2 \times 10^{-10}.\end{aligned}\quad (6)$$

It is to be noted that these are of the order of magnitude of one tenth those found in the case of bismuth as was anticipated from the polycrystalline tension coefficients. This polycrystalline coefficient was found by Professor Bridgman to be approximately 5×10^{-6} and this lies within the limiting values found in this set of experiments. One would be inclined to explain his one reading of 20.0×10^{-6} as arising from a crack in the antimony which was not otherwise detectable, particularly as this metal is so very brittle.

II. Correcting for strain

In the case of bismuth the changes of resistance arising from the deformation of the crystals were so small compared with those actually produced by the tension that, being within experimental error, they could very properly be neglected.⁶ In

the case of antimony, however, with the increased precision of the method, the deformation of the crystals gives changes greater than the experimental error, sometimes of the order of magnitude of the tension coefficients themselves, and should be considered, although in practice the observed change of resistance will always include that arising from the change of dimension. As a matter of fact, since these corrections do not follow the same law of variation with the orientations as do the tension coefficients, Professor Bridgman's equation is rigorously only applicable when these corrections have been made.

Correcting for strain is not an entirely simple matter. As Professor Bridgman has pointed out in the case of hydrostatic pressure⁷ the change of resistance due to the strain produced by the tension will consist of three parts: (1) that arising from change in length; (2) that from change in cross section; and (3) that from change in the orientation θ . These will be considered separately.

(1) *To find the relative change of resistance arising from the change in length produced by the application of a tension of 1 kg/cm² as a function of the primary orientation θ and of the secondary orientation φ .* The elastic behavior of antimony⁷ is defined by the equations

$$\begin{aligned}e_{xx} &= s_{11}X_x + s_{12}Y_y + s_{13}Z_z + s_{14}Y_z + 0Z_x + 0X_y, \\ e_{yy} &= s_{12}X_x + s_{11}Y_y + s_{13}Z_z - s_{14}Y_z + 0 + 0, \\ e_{zz} &= s_{13}X_x + s_{13}Y_y + s_{33}Z_z + 0 + 0 + 0, \\ e_{yz} &= s_{14}X_x - s_{14}Y_y + 0 + s_{44}Y_z + 0 + 0, \\ e_{zx} &= 0 + 0 + 0 + 0 + s_{44}Z_x + 2s_{14}X_y, \\ e_{xy} &= 0 + 0 + 0 + 0 + 2s_{14}Z_x + 2(s_{11} - s_{12})X_y,\end{aligned}\quad (7)$$

⁶ This has been shown quantitatively by applying the method of least squares to the *bismuth* data. The numerical values of the coefficients are, giving Professor Bridgman's values for comparison,

	Bridgman	Least Squares Uncorrected	Corrected for strain
ρ_{11}	- 7.7 $\times 10^{-9}$	- 7.2 $\times 10^{-9}$	- 7.7 $\times 10^{-9}$
ρ_{33}	- 6.6	- 6.2	- 6.7
ρ_{14}	+15.7	+13.5	+13.2
ρ_{12}	+ 5.6	+ 4.3	+ 4.2
ρ_{13}	+ 1.8	+ 1.6	+ 1.9
ρ_{44}	-12.3	-10.4	-13.1

The uncorrected values differ somewhat from those given by Professor Bridgman in his theoretical paper² since his values were computed from six graphically interpolated points and these are based on all forty-five directly observed points. The average difference between the experimental values of the tension coefficients of resistance and those computed from these uncorrected constants is 0.6×10^{-5} ; in only four cases is this difference greater than 1.2×10^{-5} . Since the correction of β for strain for bismuth lies between 0.36×10^{-5} and 0.83×10^{-5} , depending on the crystal orientation, it is evident that the strain correction is of the same order of magnitude as the experimental error.

⁷ P. W. Bridgman, Proc. Amer. Acad. 60, 305 (1925).

in which Z represents the direction of the trigonal axis. The constants for this case have the values, T being expressed in dynes/cm²,

$$\begin{aligned} s_{11} &= +17.7 \times 10^{-13}, & s_{33} &= +33.8 \times 10^{-13}, \\ s_{12} &= -3.8 \times 10^{-13}, & s_{44} &= +41.0 \times 10^{-13}, \\ s_{13} &= -8.5 \times 10^{-13}, & s_{14} &= -8.0 \times 10^{-13}. \end{aligned} \quad (8)$$

This immediately gives us, for $\theta = 0^\circ$, when T is given in the practical units of kg/cm²,

$$(1/T)(\Delta l/l) = s_{33} \times 980 \times 1000 = +3.31 \times 10^{-6}, \quad (9)$$

which gives the order of magnitude of the correction to be expected from this source.

To find the most general dependence of $(1/T)(\Delta l/l)$ on θ and φ , it will be convenient to transform from the coordinates XYZ defined by the trigonal axis and the crystalline properties of the antimony to $X'Y'Z'$ axes defined by the characteristics of the cylinder in which the metal is cast. The relative elongation $e_{z'z'}$ will then give the change in length desired. We must first define these two sets of coordinates more precisely. The Z -axis, Fig. 3, is to be parallel to the

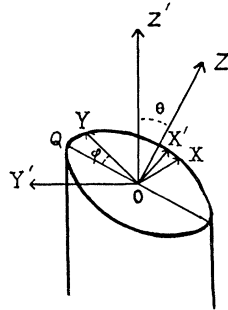


FIG. 3.

trigonal axis of the crystal, the Z' -axis to the cylindrical axis of the casting. The angle between them is then by definition θ . The Y -axis lies in the elliptical cross section of the principal cleavage plane and is the projection on this plane of the normal to the secondary cleavage plane making an angle of about 71° with the principal cleavage plane; thus the Y -axis makes an angle φ with the major axis OQ , by definition. The Y' -axis cannot of course lie in this plane, but it is to be in the plane OQZ' , so that OQ is its projection on the principal cleavage plane. The nine direction cosines necessary completely to define

the transformation are then known and are recorded in Table II. In the new set of coordi-

TABLE II.

	X'	Y'	Z'
X	$\cos \varphi$	$\sin \varphi \cos \theta$	$\sin \varphi \sin \theta$
Y	$-\sin \varphi$	$\cos \varphi \cos \theta$	$\cos \varphi \sin \theta$
Z	0	$-\sin \theta$	$\cos \theta$

nates the equations of elasticity have the form, since the tension is always along the Z' -axis,

$$\begin{aligned} e_{x'x'} &= s'_{13} Z'_{z'}, & e_{y'y'} &= s'_{43} Z'_{z'}, \\ e_{y'y'} &= s'_{23} Z'_{z'}, & e_{z'x'} &= s'_{53} Z'_{z'}, \\ e_{z'z'} &= s'_{33} Z'_{z'}, & e_{x'y'} &= s'_{63} Z'_{z'}, \end{aligned} \quad (10)$$

where the six s' 's define the behavior of the crystal in the primed set of axes and are known functions of the s 's, the transformation of the s 's being given by Voigt.⁸ Carrying out the calculation for s'_{33} , we get for the relative change in length along the axis of the cylinder the equation

$$\begin{aligned} (1/T)(\Delta l/l) &= s_{11} \sin^4 \theta + s_{33} \cos^4 \theta \\ &\quad + (2s_{13} + s_{44}) \sin^2 \theta \cos^2 \theta \\ &\quad - 2s_{14} \sin^3 \theta \cos \theta \cos 3\varphi. \end{aligned} \quad (11)$$

The term in $\cos 3\varphi$ is in agreement with the known trigonal symmetry.

(2) To find the relative change of resistance arising from the change in cross section produced by the application of a tension of 1 kg/cm². It is to be noted that OY' is the major axis of the horizontal elliptical cross section into which the tension deforms the circular cross section. This cross section originally had the area πr^2 and when strained has the area $\pi r^2(1 + s'_{13})(1 + s'_{23})$ so that

$$(1/T)(\Delta A/A) = s'_{13} + s'_{23}, \quad (12)$$

where it is to be expected that one or both of these s' 's will be negative. Evaluation of $\Delta A/A$ in terms of the s 's gives for the correction due to change in cross section

$$\begin{aligned} (1/T)(\Delta A/A) &= s_{13}(\cos^4 \theta + \sin^4 \theta + \cos^2 \theta) \\ &\quad + s_{12} \sin^2 \theta + (s_{11} + s_{33} - s_{44}) \sin^2 \theta \cos^2 \theta \\ &\quad + 2s_{14} \sin^3 \theta \cos \theta \cos 3\varphi. \end{aligned} \quad (13)$$

⁸ W. Voigt, *Lehrbuch der Kristallphysik* (pp. 589-592), 1910.

The values computed from this formula are negative as was to be expected.

(3) *To find the relative change of resistance arising from the change in the angle θ produced by the application of a tension of 1 kg/cm².* Since the specific resistance of a crystal is given by the relation

$$\rho_\theta = \rho_0 \cos^2 \theta + \rho_{90} \sin^2 \theta \quad (14)$$

we have

$$\frac{d\rho}{\rho} = \frac{2(\rho_{90} - \rho_0)}{\rho_\theta} \sin \theta \cos \theta d\theta. \quad (15)$$

It is immediately evident, as inspection shows to be true also in the case of the previous two corrections studied, that the change in resistance produced by the application of tension is independent of the secondary orientation at $\theta = 0^\circ$ and at $\theta = 90^\circ$. In the case of this third correction $d\rho$ vanishes at both these limiting points. It remains to find the value of $d\theta$ produced by the tension; $d\theta$ signifies a rotation about the X' -axis.

In determining $d\theta$, it must be borne in mind that the displacement of a point during the deformation of a body is compounded of a pure strain and of a rotation, the former being defined by the six elastic constants and the latter by three additional constants. Let us consider a small section in the middle of the cylindrical casting of the crystal, and let 0 be the very middle point which, since the tension consists of two equal and opposite forces, must remain unchanged in position. Let Q_2 be a point on the principal cleavage plane through 0 and in the plane $Y'OZ'$, Fig. 3. Q_1 is a point on the cylindrical axis having the same Z' coordinate as Q_2 relative to 0; this may be called ζ . The angle Q_1OQ_2 is by definition $(90 - \theta)$. Similarly we may call η the relative Y' coordinate of Q_2 .

According to Love, the most general displace-

ment of a point Q after deformation when 0 is kept constant is given by the three expressions:

$$\begin{aligned} e_{xx}\xi + \frac{1}{2}e_{xy}\eta + \frac{1}{2}e_{xz}\zeta - \eta\omega_3 + \zeta\omega_2 &= dx', \\ \frac{1}{2}e_{xy}\xi + e_{yy}\eta + \frac{1}{2}e_{yz}\zeta - \zeta\omega_1 + \xi\omega_3 &= dy', \\ \frac{1}{2}e_{xz}\xi + \frac{1}{2}e_{yz}\eta + e_{zz}\zeta - \xi\omega_2 + \eta\omega_1 &= dz', \end{aligned} \quad (16)$$

where ξ, η, ζ are the coordinates of Q relative to 0 and the ω 's are the three additional constants defining the rotation. Applying these equations to the point Q_1 when a tension is acting along OQ_1 , i.e., along OZ' , we have the conditions, since $\xi_1 = \eta_1 = 0$,

$$\begin{aligned} dx_1' &= 0 \quad \text{or} \quad \frac{1}{2}e_{x'z'} + \omega_2 = 0, \\ dy_1' &= 0 \quad \text{or} \quad \frac{1}{2}e_{y'z'} - \omega_1 = 0, \\ dz_1' &= e_{z'z'}\zeta. \end{aligned} \quad (17)$$

Applying Eq. (16) to the point Q_2 gives the relations, ξ_2 being zero,

$$\begin{aligned} dx_2' &= \frac{1}{2}e_{x'y'}\zeta \cot \theta + \frac{1}{2}e_{x'z'}\zeta - \zeta \cot \theta \omega_3 + \zeta \omega_2, \\ dy_2' &= e_{y'y'}\zeta \cot \theta + \frac{1}{2}e_{y'z'}\zeta - \zeta \omega_1, \\ dz_2' &= \frac{1}{2}e_{y'z'}\zeta \cot \theta + e_{z'z'}\zeta + \zeta \cot \theta \omega_1. \end{aligned} \quad (18)$$

The assumption that there is no motion in the X' direction leads to a perfectly consistent result, i.e., $\frac{1}{2}e_{x'y'} = \omega_3$. From the last two conditions in Eq. (18) we can get the new angle between OQ_1 and OQ_2 , since

$$\tan(\theta + d\theta) = \frac{\zeta + dz_2'}{\eta + dy_2'} \quad (19)$$

so that we have the relation

$$d\theta = \sin \theta \cos \theta \{e_{z'z'} - e_{y'y'} + e_{y'z'} \cot \theta\}. \quad (20)$$

This means that the final change in resistance due to change in angle is

$$\frac{1}{T} \frac{\Delta \rho}{\rho} = 2 \sin^2 \theta \cos^2 \theta \frac{\rho_{90} - \rho_0}{\rho_\theta} \{s_{33}' - s_{23}' + s_{43}' \cot \theta\}, \quad (21)$$

and the problem again reduces to the determination of the s 's involved. On this basis, the final expression for the correction of the specific resistance of the crystals arising from the change in the angle θ is

$$\begin{aligned} \frac{1}{T} \frac{\Delta \rho}{\rho} &= 2 \sin^2 \theta \cos^2 \theta \frac{\rho_{90} - \rho_0}{\rho_\theta} \{ (s_{11} - s_{13}) \sin^4 \theta - (s_{33} - s_{13} - s_{44}) \cos^4 \theta + (s_{11} - s_{33} + s_{44}) \sin^2 \theta \cos^2 \theta \\ &\quad - 2s_{14} \sin \theta \cos \theta \cos 3\varphi \}. \end{aligned} \quad (22)$$

Thus the total effect on the resistance produced by the *strain* caused by the application of tension to trigonal crystals is given by the Eqs. (11), (13) and (22). Since $R = \rho l/A$, the relative change of resistance is

$$\Delta R/R = \Delta l/l - \Delta A/A + \Delta \rho/\rho, \quad (23)$$

the correction for the change in cross section being subtracted. The numerical values of these corrections in the case of antimony are plotted in Fig. 4 in the same way as the tension coeffi-

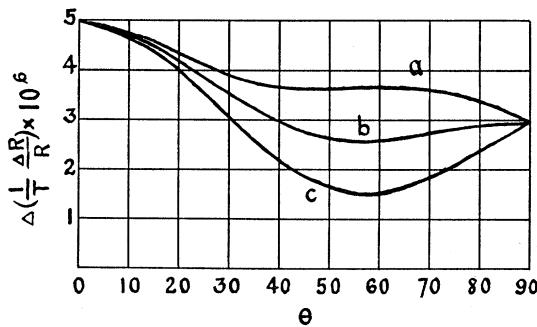


FIG. 4. *a*, $\varphi = 0^\circ$; *b*, $\varphi = 30^\circ$; *c*, $\varphi = 60^\circ$. Strain corrections of the tension coefficients.

cients were plotted in Fig. 2. These data are to be used to correct the four constants determined from the experiments with tension, the strain corrections being subtracted from the experimental values and least squares being applied again to these corrected observations. The resulting constants and their probable errors are

$$\begin{aligned} \rho_{11} &= (-1.07 \pm 0.09) \times 10^{-10}, \\ \rho_{33} &= (-13.90 \pm 0.54) \times 10^{-10}, \\ 2\rho_{13} + \rho_{44} &= (+4.34 \pm 0.34) \times 10^{-10}, \\ \rho_{14} &= (+5.81 \pm 0.16) \times 10^{-10}. \end{aligned} \quad (24)$$

The curves in which these corrections have been made are indicated in the dotted lines of Fig. 2 and the corrected values of the tension coefficients in the last two columns of Table I.

The two constants derived from pressure measurements must also be corrected for strain. The analysis for this correction for pressure has already been carried out by Professor Bridgman.⁷ The correction for angle is again zero for the limiting cases $\theta = 0^\circ$ and $\theta = 90^\circ$ and that arising from the combined changes in length and cross section is

$$\begin{aligned} ((1/\rho)(\Delta R/R)) &= 2(s_{11} + s_{12}) \cos^2 \theta \\ &\quad + 2s_{13} \sin^2 \theta + s_{33}(1 - 2 \cos^2 \theta). \end{aligned} \quad (25)$$

Evaluating this correction in the two limiting cases which are needed, gives the values

$$\begin{aligned} \theta = 90^\circ \quad ((1/\rho)(\Delta R/R))_{90} &= +5.4 \times 10^{-6}, \\ \theta = 0^\circ \quad ((1/\rho)(\Delta R/R))_0 &= +20.0 \times 10^{-6}. \end{aligned} \quad (26)$$

With all these *corrections for strain* the final values of the tension coefficients of resistance are

$$\begin{aligned} \rho_{11} &= -1.1 \times 10^{-10}, & \rho_{12} &= -3.3 \times 10^{-10}, \\ \rho_{33} &= -13.9 \times 10^{-10}, & \rho_{13} &= +2.5 \times 10^{-10}, \\ \rho_{14} &= +5.8 \times 10^{-10}, & \rho_{44} &= -0.6 \times 10^{-10}. \end{aligned} \quad (27)$$

These may be considered the tension coefficients of specific resistance.

CONCLUSION

In this work with antimony, a crystal of trigonal symmetry, Professor Bridgman's theory of the change of resistance has again been found to be in agreement with experimental results, as in the case of bismuth. The values of the six coefficients defining the change of resistance under the action of stresses have been found directly and have also been corrected for the change arising from strain. The latter procedure is the more defensible from a logical point of view, since the corrections for strain do not depend on the crystal orientations in exactly the same way as do the tension coefficients, so that the uncorrected readings can be expected to satisfy the Bridgman theory only approximately. However, in practice, since the corrections arising from change in cross section and from change in angle differ only slightly in their functional dependence on θ and φ from that of the tension coefficients and the correction for length not at all, both the uncorrected and the corrected observations can be represented within experimental error by Professor Bridgman's equation, the constants involved being of course different in the two cases. This view is upheld by the fact that the probable errors are very nearly equal in the two cases.

It is again a pleasure to thank the Director of the Laboratory, the authorities of Harvard University and particularly Professor Bridgman for the privilege of carrying on this work in the Research Laboratory of Physics.