YOUNG'S MODULUS DETERMINED WITH SMALL STRESSES

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Abstract

An interferometer method similar to that used by E. Grüneisen was used to determine the extensions of metallic rods of brass, steel, copper, aluminum and nickel under small stresses. The largest stresses applied were considerably less than those usually used in commercial testing. For the substances examined Young's Modulus was found to increase very rapidly as the stress increased from zero, reach a maximum at a comparatively small stress, and then to decrease almost exponentially to the ordinary value. The annealing of the rod may be connected with this divergence from Hooke's law with small stresses, as slight differences were found on reannealing some of the rods used.

IT HAS been known[1] for some time that the value of Young's Modulus computed from the velocity of weak longitudinal vibrations in rods was appreciably higher than that obtained by ordinary static methods. The differences expected in the adiabatic and isothermal moduli[2] are small compared to those found experimentally by dynamic and static methods, although values found by these methods are often quoted as the adiabatic and isothermal moduli respectively.[3] Therefore it was decided to measure the elasticity statically at small stresses, comparable with those in a sound wave. Following E. Grüneisen[4] an interferometer was used to determine the strain.

In Fig. 1 the rods A are under test and the plates B are fixed to their ends. There is a universal joint at the point C. G and H are plates with holes and lock screws to clamp them to the rods. The holes are bevelled so that contact with the rods A can be made at one point only. By means of rods P the plate G supports another plate E, having a circular hole in its centre under which is supported a three-quarter silvered mirror T. The plate H has three levelling screws supporting a plate Q, on which is placed the other interferometer mirror R, similar to T. N is a right-angled prism held in position by a fixed external support and L is a mirror also supported from the outside. S is the source of light, a mercury arc, used with a filter F.

Monochromatic light is incident on L and reflected through the partly silvered mirrors to N, which totally reflects it to the telescope M. The telescope is adjusted for parallel light and circular interference fringes can be produced by making the mirrors R and T parallel with each other. The telescope is equipped with an eye-piece having both movable and fixed cross

[1] R. J. Lang, Proc. Roy. Soc. Can. **16**, 163 (1907). G. W. Pierce, Am. Acad. Proc. **63**, **1** (1928). E. Grüneisen, Ann. der Physik **22**, 801 (1907).

[2] G. F. C. Searle, "Experimental Elasticity" p. 22.

[3] E. Grüneisen, reference 1, p. 843.

[4] E. Grüneisen, reference 1, p. 826.

hairs. W is a scale pan attached to the center of the lowest plate by means of a universal joint not shown in the diagram. As weights are applied at W the rods A stretch, the mirrors separating by the extension in the length from G to H. This causes the fringes to move out from their center. Three vanes not shown in the diagram are clamped to the bottom plate. These move in an oil bath to damp the motion of the apparatus after the application of a weight at W.

Measurements were made by taking the diameters of two successive fringes in the arbitrary units of the screw attached to the movable cross hair. Then a weight was applied to W and the number of fringes passing the cross

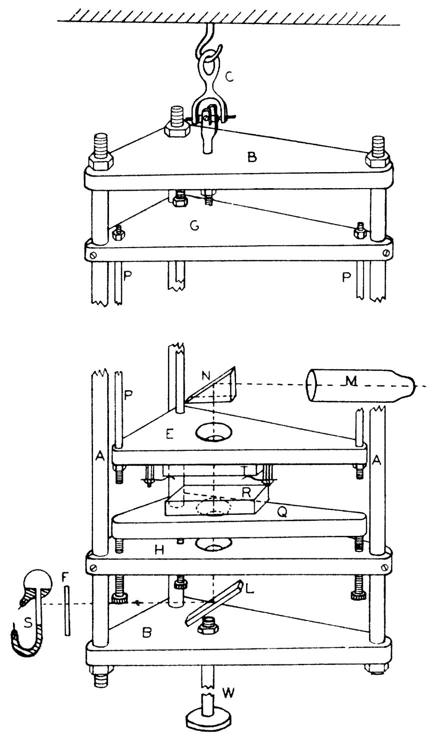

Fig. 1. Diagram of interferometer.

hairs counted. The diameter of the fringe which remained between the positions previously occupied by the two fringes measured was then taken. If readings were taken on fringes further from the center than the fourth or fifth, no appreciable error was introduced by taking the fractional fringe shift as proportional to the displacement over the width of one fringe. Thus if d_1 and d_2 be the diameters of two successive fringes, and, on the application of a weight, n fringes and a fraction pass the cross hairs, if d_3 be the diameter of a fringe occupying a position between d_1 and d_2, then the fringe shift is $n+(d_3-d_1)/(d_2-d_1)$.

Since a shift of one fringe is caused by a motion of the mirrors *R* and *T* of one-half wave-length, the extension in the part of the rods from *G* to *H* can be computed, and Young's Modulus found in the usual way by dividing the stress by the strain.

Care was taken to keep the apparatus at a nearly constant temperature as the difference in the lengths of the rods *P* and *A* produces a different thermal expansion. The two sets of rods must be of the same material, also, so that their thermal expansion coefficients will be equal. It was found that if one set were of brass and the other of steel, temperature changes of one degree

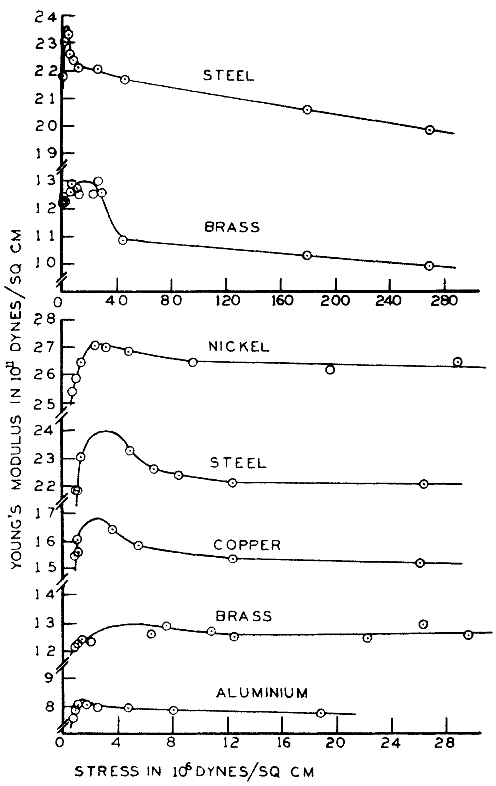

Fig. 2. Curves of Young's Modulus.

per hour were quite sufficient to invalidate the results since it required two or three minutes to take a reading.

After a reading was taken the weight was removed and the diameters of the fringes checked to see if the rods had returned to their original lengths. if they had not, the reading was thrown out. It was found quite easy to measure the displacement of the mirrors to one two-hundredth of a wave-length.

The stress on the rods was adjusted to any desired value by applying weights at *W* and the elasticity measured by applying a small additional weight and measuring the shift in the fringes. The stress was computed from the sum of the original weights at W, one-half of the small additional weight and the weight of the parts of the apparatus supported by the rods including one-half of their own weight.

In this way Young's Modulus was determined for five materials at various stresses. The rods used were about 40 cm long and 0.95 cm in diameter. In Fig. 2 curves of Young's Modulus against stress are plotted for the various materials.

Professor Huntley of Armour Institute, Chicago, kindly determined Young's Modulus for two of the specimens at three very much higher stresses than it was possible to use with the apparatus just described. The rods used were cut from the same piece as the ones tested with the interferometer. These values are the three with largest stresses plotted in Fig. 2 for steel and brass, respectively. By means of Kundt's tube the velocity of sound was measured in each material. The air in the tube was carefully dried and Young's Modulus, E, calculated from the relation

$$v=\sqrt{E/d},$$

where v is the velocity of sound and d is the density. The values obtained by this method are given in Table I.

TABLE I. *Young's Modulus as determined from velocity of sound.*

Material	v(cm/sec)	d(gm/cm^3)	E(dynes/cm^2)
Brass	3.63×10^5	8.45	11.1×10^{11}
Steel	5.15×10^5	7.81	20.75
Copper	3.89×10^5	8.61	13.01
Aluminum	5.155×10^5	2.70	7.19
Nickel	4.905×10^5	8.82	21.25

For two of the materials rods were excited into longitudinal vibrations at various frequencies up to 17000 per second, by a method described by Boyle and Lehmann.[5] The rods were cut in half and each glued, end on, to a thin piece of quartz cut with its faces perpendicular to an electric axis. This constituted a parallel-plate condenser which was connected in parallel with the variable condenser in the tank circuit of an ordinary five watt radio oscillator. Then the faces of the quartz disc were vibrated longitudinally by the piezoelectric pressures and the frequency of the electric oscillations varied until the rods came into resonance. If this frequency were in the audible range of frequencies the rods gave off a distinct sound. A precision wave-meter was used to determine the frequency of the Hertzian oscillations which was, of course, the frequency of the mechanical vibrations of the rod. In agreement with Professor Pierce,[6] this was found to be constant at all of the frequencies tried and the same as that given by Kundt's tube, although the stresses were probably much smaller.

[5] Boyle and Lehmann, Trans. Roy. Soc. Can., Sect. III, Vol. XIX (1925).
[6] Pierce, Am. Acad. Proc. **63**, 40 (1928).

A set of steel rods were annealed in an electric furnace and the tests again carried out. Young's Modulus was found to be slightly higher for each value of the stress that it was previously. Otherwise the behavior was exactly similar.

From these results it appears that Hooke's Law does not hold exactly for very small stresses but that Young's Modulus has a maximum at some small but finite stress. The elasticity decreases rapidly to the ordinary value as the stress is increased from this value and decreases even more rapidly as the stress is decreased from the point of maximum elasticity. It will be noted that Young's Modulus determined from the velocity of sound is somewhat lower than the values from the interferometer method but higher in the cases of brass and steel, than the values obtained with large stresses.

In conclusion the author wishes to acknowledge with thanks the assistance given by Professor Huntley of Armour Institute, mentioned above, and to thank Professor Dempster of this Laboratory for his untiring interest in the problem and his many very valuable suggestions.