THE DISTRIBUTION OF THE HEAT EMISSION IN THE MAGNETIC HYSTERESIS CYCLE*

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Abstract

Warburg has shown that for a complete magnetic hysteresis cycle an amount of energy equal to the area of the *I-vs-H* loop is dissipated and appears as heat. But for a part of a cycle theory only states that energy $=\int_H^{H+\Delta H} HdI$ is supplied to the iron. Part of this energy appears as heat, while the rest is stored as magnetic potential energy and is released in another part of the cycle. The distribution of the heat emissions for successive steps along the hysteresis cycle was obtained experimentally from the rises in temperature produced in the iron or steel. The corresponding values of $\int HdI$ were obtained from the hysteresis loop. The difference between these quantities gave the potential energy throughout the cycle. The potential energy was greatest for $H=H_{max}$, zero for $H=0$. The heat emission was zero from $H=H_{max}$ to $H=0$, increased up to a little beyond $H=-H_c$, ($H_c=$ the coercive force), and then decreased to $H=-H_{max}$. A cooling was never observed. The results are discussed in connection with a recent theory of atomic behavior in ferromagnetic metals.

I. Introduction

The subject of magnetic hysteresis owes its complete development to Ewing,[1] who obtained the well-known "hysteresis loops," curves showing the cyclic relation between the magnetic force, H, and the magnetization, I. Ewing gave the name "hysteresis," meaning "to lag behind," since he found that I always fell behind H in the cycle.

It had earlier been shown by Warburg,[2] and later independently by Ewing,[3] that when a ferromagnetic material is taken through a continuous cycle of values of the magnetizing force, H, energy is dissipated through hysteresis. Warburg found this energy, in ergs per unit volume, to be equal to the area of the *I-vs-H* hysteresis loop, that is, $\oint HdI$, and this law is known by his name. This is for a whole cycle, at the end of which the material is in the same state as at the beginning, hence this energy must appear as heat, and cause a rise in the temperature of the material. Attempts have been made to check Warburg's Law from this rise in temperature, and Adelsberger[4] has recently done so to within 2 per cent.

If the magnetic force is increased from H to $H+\Delta H$, the energy supplied per unit volume is given by

$$w=\frac{1}{4\pi}\int_H^{H+\Delta H} HdH+\int_H^{H+\Delta H} HdI \qquad (1)$$

* Part of a dissertation presented for the degree of Doctor of Philosophy at Yale University

[1] J. A. Ewing, "Magnetic Induction in Iron and Other Metals," (1900).

[2] E. Warburg, Ann. d. Physik (4) **13**, 141–164 (1881).

[3] J. A. Ewing, Proc. Roy. Soc. **34**, 39–45 (1883).

[4] U. Adelsberger, Ann. d. Physik (4) **83**, 184–213 (1927).

where the first term represents the energy increment in vacuo, and is reversible, while the second represents the energy increment due to the presence of the iron or steel, part of which may be dissipated. Hence $\int HdI$ measures the energy supplied to the metal in any part of a hysteresis cycle. Now for this case theory does not give the heat emission, because it is not known how much of this energy is stored as magnetic potential energy. To obtain this potential energy theoretically involves a knowledge of the mechanism of magnetism, still far from completely explained. The purpose of the present work was to measure the heat emission, Δq, due to hysteresis in successive small steps of a cycle. Having obtained this and the corresponding energy supplied, the magnetic energy could be determined from

$$\Delta U = \int_{H}^{H+\Delta H} HdI - \Delta q \qquad (2)$$

where U is this potential energy. A knowledge of this function should throw important light on the fundamental mechanism of magnetism.

II. The Experimental Procedure

Experimentally the problem had two parts, (1) obtaining the hysteresis loops characteristic of the steel used, and (2) determining the temperature rise of the steel due to hysteresis as each step was taken along a loop.

The specimen was in the form of a ring, and the magnetic measurements were made by the ballistic method, with an over-damped d'Arsonval galvanometer.

For a whole cycle the heat emission $Q = \oint HdI$. In the case of soft iron, where $\oint HdI$ is about 10^4 ergs, the rise in temperature produced is only about 3×10^{-4} degrees C. But $\oint HdI$ increases with the hardness, and reaches 10^6 ergs in the case of the "K. S. Magnet Steel" reported by Honda and Saito.[5] Through the courtesy of the Bell Telephone Laboratories, cast rings of this steel were obtained. The analysis supplied was as follows: 33.44% Co, 6.51% W, 1.81% Cr, 0.74% C, 0.12% Mn, 0.01% Si, 0.007% P, traces of Ni and S, and 0% Mo, V and Cu. The rings as cast were 1.5″ (3.81 cm) in outer diameter with a cross-section $\frac{1}{4}$″ (0.635 cm) square. Tested by the ballistic method the steel gave hysteresis loops whose areas are given in Table I.

Table I. *Areas of the loops for the magnet steel.*

H_{max}	$\oint H\,dI$
560 gauss	510,000 ergs
1000 "	750,000 "
1190 "	840,000 "

Density of steel = 7.8 gm/cm³, Specific heat = 0.12 cal/gm.

[5] L. Honda and S. Saito, Phys. Rev. (2) **16**, 495–500 (1920).

The maximum magnetic force which could be applied was limited by the number of turns of wire which could be wound through the center of a ring before it was completely filled, and by the current which could be sent through this winding without overheating. Black enameled copper wire (B. and S. No. 18) was used, wound uniformly in seven separate layers. Each layer was connected through a reversing switch in series with the next. A special type of reversing switch was designed to avoid breaking the current. This was effected by making the switch short-circuit its section of the coil before reversing it. A constant current was supplied from high-capacity storage batteries, H being varied from $+H_{max}$ to $-H_{max}$ in seven steps by reversing successively each of the switches, while an eighth switch permitted a complete reversal of H in one step. As the number of turns varied for the different layers practically any desired initial value of H or increment ΔH could be produced by proper handling of the switches.

The space available for winding was slightly increased by grinding the inner circumference of the rings until the difference between the inner and outer radii was only 0.118″ (0.300 cm). This also insured a sufficiently small variation in H (which is $\propto 1/r$), from inner to outer circumference. This variation was less than that occurring for any single step of the hysteresis loop.

A careful search was made for other possible sources of heat than magnetic hysteresis. Precautions were first taken against eddy or Foucault currents. These were reduced both by laminating the ring, and by introducing a choke-coil of large self-inductance into the primary circuit to delay the rate of change of the induction through the steel. The lamination of the rings was difficult on account of the extreme hardness of the steel. Each ring was finally sliced in two by a rapidly rotating Alundum saw, a rubber disc containing carborundum imbedded in it. After slitting several rings they were ground down to a thickness of 0.07 cm, and in order that the steel should have sufficient heat capacity four such rings were insulated with an enamel baked on at 100°C, and placed together to form a single laminated ring. This lamination was found sufficient to eliminate eddy current heating, as the introduction of a choke coil, sufficient further to reduce this heating to 0.1% of its value without the choke coil, produced no alteration in the heating actually observed for any step of a cycle. The magneto-caloric effect of Weiss and Piccard[6] is a reversible thermal effect accompanying magnetization for such large values of H that the *I-vs-H* curve has become closed; in the region of smaller values of H where the branches of the hysteresis loop are separate this effect is included in the hysteresis heat.

The changes of temperature of the steel were recorded by copper-constantan thermocouples. Four thermocouples were used in series, four alternate junctions being soldered to the four laminae of the ring, and the other set of four junctions being sandwiched between two mica rings of the

[6] P. Weiss and A. Piccard, Comptes Rendus **166**, 352–354 (1918); Archives des Sciences (4) **45**, 329-335 (1918).

same size, placed next to the steel but separated from it slightly by little cubes of cork. Both steel and mica rings were included in the primary winding. The thermocouples were in series with a very sensitive low resistance Leeds and Northrup galvanometer, suitable for measuring small changes of e.m.f. The sensitivity of the galvanometer was 33 mm per microvolt, and the thermoelectric power of one thermocouple was 4.2×10^{-5} volts/°C. This gave a total temperature sensitivity of 2×10^{-4}°C for the series, whereas the heat rise per cycle calculated from values of $\oint H\,dI$ in Table I was of the order of 2×10^{-2}°C.

For such a sensitive battery of thermocouples it was essential to maintain the junctions at very steady temperatures. For this reason the primary winding and reversing switches described above were employed in order that the magnetic force could be changed without altering the current through the winding. By keeping this current constant it gradually heated the winding and rings up to a steady state when the heat was conducted away at the same rate at which it was generated. That this was necessary is seen from the fact that the energy dissipated in hysteresis, even for a complete loop, is 0.01 percent of that supplied per second by the magnetizing current.

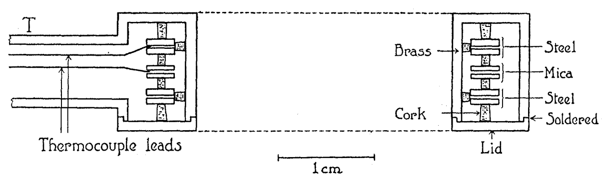

Fig. 1. The vacuum box.

This arrangement did not maintain the temperature of the thermocouples steady enough to prevent drift of the galvanometer. Another difficulty was that the thermocouples recorded only a few per cent of the predicted heating because the heat emitted in the steel was conducted away before the thermocouples and galvanometer (period 8 sec.), could record it. It was therefore necessary to insulate the rings. Experiment and theory showed that the best material insulators, such as silk, wool, or asbestos, were not sufficiently non-conducting, and that layers sufficiently thick to insulate had thermal capacities of the same order as that of the rings themselves. A vacuum box was therefore designed to hold the rings, Fig. 1. The box was of brass, with a lid which was soldered on after the rings and thermocouples had been introduced. The rings were kept 1 or 2 mm from the walls by a few cubes of cork. Two copper wires led in to the thermocouples from the galvanometer through a wax stopper and tube T. The whole box was evacuated through the tube T by means of an oil pump with liquid air trap, all joints being made vacuum tight with hard wax. The vacuum attainable was limited by the vapor pressure of this wax, but was sufficient to eliminate the convection and the heat capacity of the air, and greatly reduced the conduction. It should be noted that the mica rings with the junctions between them were placed between two pairs of steel rings to give as symmetrical an arrangement as possible. With both sets of junctions vacuum insulated and inside the same brass enclosure very steady conditions were now obtained. The only loss of heat was along the thermocouple wires between alternate junctions, and this was reduced by using fine wire (B. & S. No. 40) for thermocouples.

The primary winding was wound outside the box. In order to obtain the large values of $H_{\max}$ desired the whole coil was placed in a water bath. A current of 30 amperes was then possible, and kept the water boiling quietly inside the winding. This insured a steady temperature of the box. As about 300 turns could be wound on, values of H up to 1200 gauss were obtained.

III. Results and Discussion

The ballistic galvanometer was calibrated by producing a known change of flux through the circuit in series with it, and gave: 1 mm deflection $=422.7$ maxwells.

Hysteresis curves for the magnet steel were obtained for $H_{\max}=560$, 1000, and 1190 gauss, these same cycles being used for the measurements of the heat emission. The steel was also demagnetized by reversals and the initial magnetization curve found.

In the measurements of the heat emission a half cycle was covered in seven steps by reversing the switches in turn. Each reading was taken from six to ten times. A typical set of readings for the 1000-gauss cycle is given in Table II.

Table II. *Typical set of readings for the 1000 gauss cycle.*

H in gauss	1000 to 600	600 to 360	360 to 180	180 to −150	−150 to −420	−420 to −610	−610 to −1000
	0 mm	0 mm	0 mm	1.0 mm	5.0 mm	1.0 mm	1.0 mm
	0	0	0	1.0	4.5	3.0	1.0
	0	0	0	0	4.5	3.0	0.5
	0	0	0	1.0	4.0	4.0	1.0
	0	0	0	0.5	5.0	3.0	1.0
	0	0	0	0.5	3.0	4.5	2.0
	0	0	0	0	5.0	3.0	0.5
	0	0	0	1.5	5.0	2.5	1.0
	0	0	0	1.0	5.0	3.0	1.0
	0	0	0	0.5	4.0	3.0	1.0

The average values of each set of readings were taken, those obtained for the initial magnetization curve and for the 1000-gauss cycle being shown in Fig. 2.

The energy input, $\int H\,dI$, was measured from the graphs for each step. Assuming Warburg's Law, the area of the loop was equated to the total heat emitted in going around it, and each value of $\int H\,dI$ changed to corresponding galvanometer deflections in mm. Thus for the 1000-gauss cycle,

Total area of loop $=747{,}000$ ergs
Total heat emitted $=18.4$ mm deflection
40,500 ergs $=1$ mm deflection

Fig. 3 gives $\int H\, dI$ starting from demagnetization and going to $H=1000$, and then around half a cycle to $H=-1000$.This measures the total energy supplied to the steel at each point. The other curve shows the total heat emitted in the same process. The difference between these two curves is the energy stored potentially in the steel at each point, and this is represented in Fig. 4. Similar curves were obtained for the 560- and 1190-gauss cycles.

It is first evident that the energy supplied, $\int H\, dI$, and the energy appearing as heat are not in general equal for a step along a hysteresis cycle. As the heat emission lags behind, energy must be stored in the steel in some

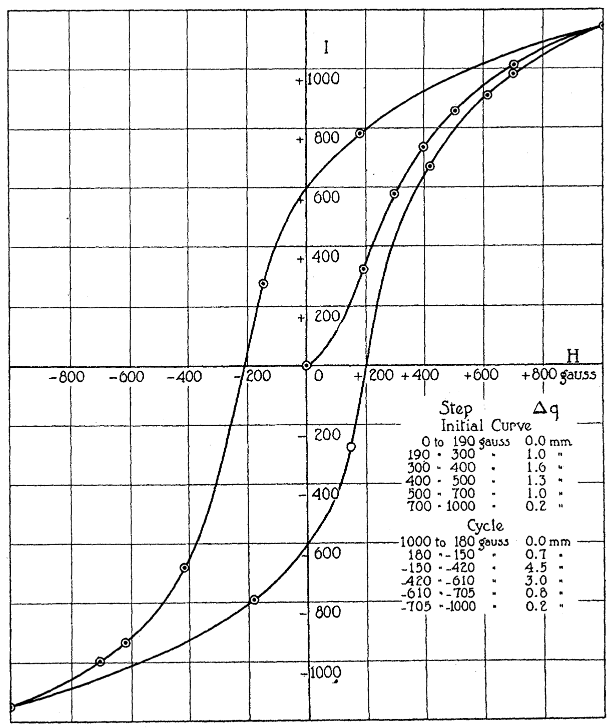

Fig. 2. The heat emission for the 1000-gauss cycle.

way. This agrees with the molecular theories of magnetism, an increase in magnetization setting up stresses and strains between the little magnets. A removal of the magnetic force relieves these strains and the energy is released. This agrees with the fact that the energy was always found to be greatest for $H=H_{max}$ and zero for $H=0$.

If magnetism is regarded as an atomic phenomenon, as the work of Stern and Gerlach[7] and others indicates, its explanation should be intimately connected with atomic theories. McKeehan[8] has presented a theory based

[7] O. Stern and W. Gerlach, Zeits. f. Physik, **9**, 349–352 (1922).

[8] L. W. McKeehan, Phys. Rev. (2) **26**, 274–279 (1925).

on the quantum hypothesis. Whether one uses the Bohr theory or the more recent theories of Heisenberg and Schroedinger the essential idea is that of discrete quantum states. In the case of a paramagnetic atom the resultant magnetic moment can take certain limited orientations with respect to the field at the atom considered, changes in the magnetization of a body as a whole resulting from various atoms suddenly undergoing a change from one orientation to another. Such discontinuous changes should be accompanied by a change in energy. Where the magnetization is changing most rapidly the heat emission might therefore be expected to be greatest. Such seems

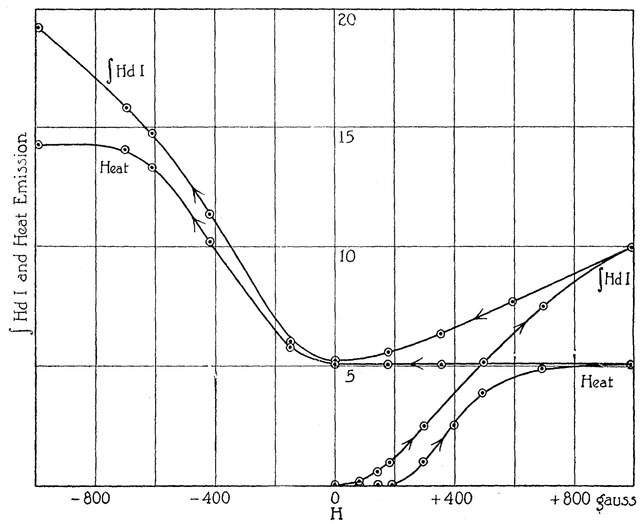

Fig. 3. The heat emission and $\int HdI$ for the initial curve and a half cycle.

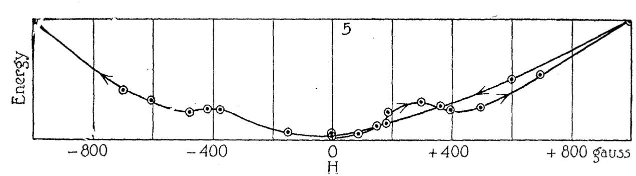

Fig. 4. The magnetic potential energy.

to be a possible explanation of the maximum slopes of heat-emission curves and minima in the energy curves around $H=400$, corresponding to the steepest part of the hysteresis loop and just beyond.

In no cases was a cooling of the steel observed, although from $H=H_{max}$ to $H=0$ energy is taken from the steel, i.e., $\int H\,dI$ is negative. Adelsberger[4] reports such a cooling, but in the present work the sensitivity and the number of steps per loop were probably not great enough to detect the small amounts of cooling which he found. In any event it appears that the energy taken from the steel is entirely supplied, or at least nearly so, from the amount stored potentially in going from $H=0$ to $H=H_{max}$, and it is significant that this amount is just sufficient for the purpose. This probably means that the atomic magnetic axes have, at $H=0$ on the hysteresis loop, returned in most cases to parallelism with their initial directions. The residual magnetization shows, however, that the directions agreeing more closely with the magnetic force just previously applied are, in general, preferred.

From $H=0$ to $H=-H_c$ (H_c=the coercive force), a small amount of heat appears, while from the steepest part to a little beyond the rate of emission of energy is greater than the rate of supply, $\int H\,dI$, and the potential energy is temporarily reduced, producing the hump and dip in the energy curve, Fig. 4. There is a similar minimum for the energy curve of the initial branch, and it occurs at a similar point, beginning where the slope of the *I-vs-H* curve is greatest. Only half of a complete cycle is shown, the other half being symmetrical and giving a minimum which would come almost on top of that of the initial branch. Finally, for large negative values of H the heat emission decreases and the potential energy increases to a maximum for $H=-H_{max}$.

In conclusion I should like to express my sincere gratitude to Professor McKeehan for suggesting the problem, and for his generous help and criticism. Acknowledgment is also made of the financial assistance given by the Loomis Fellowship, established by Francis E. Loomis and Henry B. Loomis.

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