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## A MODIFIED METHOD OF MEASURING $e/m$ AND $v$ FOR CATHODE RAYS.

By L. T. Jones.

THIS determination of $e/m$ and $v$ is a modification of the usual method employing the simultaneous electrostatic and magnetic deflections. The modification is the result of an attempt to eliminate as nearly as possible the errors of measurement of the deflections and the correction due to the field distribution at the ends of the electrostatic plates. This is brought about chiefly by the position in which the photographic plate was placed.

### The Apparatus.

A glass cylinder 10 cm. in diameter and 27 cm. long (Fig. 1) was closed at each end by a glass plate. Two holes were made in one of the plates to admit the glass tubes carrying the anode, $A$, and the cathode, $C$. The cathode, an aluminum disc .6 cm. in diameter, was carried on an aluminum rod. This rod was encased in a small glass tube which in turn was supported by a larger glass tube waxed to the glass plate where it entered the discharge chamber. The anode was mounted in a similar manner. Both aluminum rods were connected with the outside by platinum wires sealed in glass.

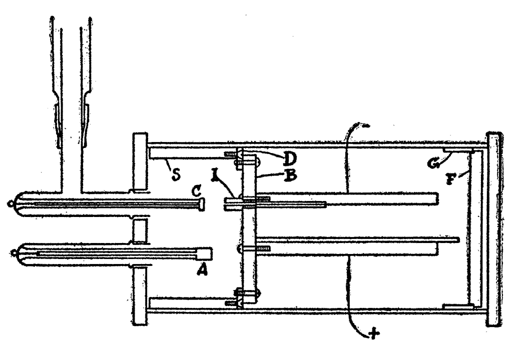

Fig. 1.

A brass ring, $D$, was fastened by sealing wax to the inside of the glass cylinder, and to this were fastened the soft iron shield, $S$, and the ebonite disc, $B$, the latter supporting the electrostatic plates. The electrostatic plates were held to the disc $B$ by brass screws. The potential of the electrostatic plates was supplied through two wires that passed through small holes in the walls of the cylinder. The holes were sealed with wax.

By loosening the screws holding the ebonite disc, $B$, to the brass ring, $D$, the disc and electrostatic plates could be taken as a whole from the cylinder.

At the opposite end a short length of brass cylinder, $G$, was waxed to the inside of the glass cylinder and a hard rubber disc, $F$, turned to fit it, darkened the tube. The glass cylinder was coated on the outside with lamp black and the coating connected to earth. All the metal parts inside the tube, except the electrostatic plates and the discharge terminals, were connected to earth.

## The Electrostatic Plates.

Two electrostatic plates were mounted exactly 1 cm. apart, as shown in Fig. 2. The beam of cathode rays was made to pass along the upper of the two plates at grazing incidence. The photographic plate was placed flat on the lower of the two electrostatic plates. The beam was bent downward, by adjusting the electric field, to strike the photographic plate well within the geometrical limits of the field plates.

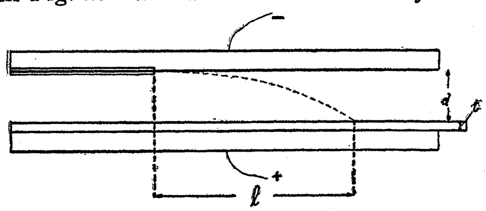

Fig. 2.

The cathode beam emerged from the Thomson plate-tube at a distance of several centimeters from the left end of the electrostatic plates and was then bent downward. Since the plates were plane and parallel the electrostatic deflection was the distance from the upper electrostatic plate to the upper side of the photographic plate.

## The Formula.

Let the two electrostatic plates be separated by a distance $d + t$, $d$ being the air place and $t$ the thickness of the photographic plate, which is of dielectric constant $K$. The two electrostatic plates are kept at constant potentials $V'$ and $V''$.

If the plates are separated by an air space of thickness $d + t$ there is a given electric surface density of charge on the plates and, consequently, a given electric force, $E$, in the space between them. If, now, the dielectric of thickness $t$ is introduced, whose equivalent air thickness is $t/K$, the effective air space will then be reduced from $d + t$ to $d + t/K$. The effective air space has thereby been reduced an air-equivalent amount of $t - t/K$, causing a change in the capacity. Since the potentials of the two plates have remained constant the surface density and hence the electric force have changed. If, now, the air gap between the two electrostatic plates is increased by an amount $t - t/K$, then the capacity, the surface density and the electric force will resume their former values. If, then, while an electric force, $E$, exists between the two plates of potentials $V'$ and $V''$, a dielectric slab of thickness $t$ is introduced and at the same time the plates are further separated by an amount $t - t/K$, making $d + 2t - t/K$ in all, the surface density on the plates and hence the electric force, $E$, will remain constant. The electric force is then given by the equation

$$V' - V'' = E(d + 2t - t/k),$$

or

$$E = \frac{PD \times 10^8}{d + 2t - t/k}, \tag{1}$$

where $PD$ is the potential difference of the two electrostatic plates in volts.

The cathode beam in passing through the uniform electric field, $E$, is accelerated by a constant force and hence follows Newton's second law. The force on the charge $e$ will be

$$Ee = ma, \tag{2}$$

where $a$ is the acceleration toward the positive plate and $m$ is the mass of the electron. Since the electron falls through a distance $d$ in time $t$ we have the distance of fall expressed by the equation

$$d = {}^1/_2 at^2,$$

or

$$a = \frac{2d}{t^2}. \tag{3}$$

If the velocity in the horizontal direction is $v$ and the length of horizontal travel is $l$ we have

$$l = vt,$$

whence

$$\frac{1}{t^2} = \frac{v^2}{l^2}.$$

Substituting this value in equation (3) gives

$$a = \frac{2dv^2}{l^2}.$$

If this value is placed in equation (2) we have

$$Ee = \frac{2dmv^2}{l^2},$$

whence

$$\frac{e}{mv^2} = \frac{2d}{El^2}. \tag{4}$$

If at the same time the moving electron is subjected to the action of a uniform magnetic field of intensity $H$ and its velocity $v$ is perpendicular to the lines of magnetic force, urging the particle in the path of a circle, in the plane of the photographic plate, then the force is given by

$$Hev = \frac{mv^2}{r} \tag{5}$$

where $r$ is the radius of the circle. If the dotted line, Fig. 3, indicates the path of the particle undeflected by the magnetic field, and the circle of radius $r$ the curvature experienced under the influence of the magnetic field of strength $H$ we may represent the horizontal distance traveled by the length $l$ and the magnetic deflection (measured at right angles to the undeflected path) by $z$, since $z$ is small compared with $l$. Then, from Fig. 3,

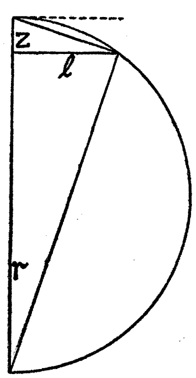

Fig. 3.

$$\frac{z}{\sqrt{z^2 + l^2}} = \frac{\sqrt{z^2 + l^2}}{2r}$$

and

$$\frac{1}{r} = \frac{2z}{l^2},$$

since $z^2$, being small in comparison with $l^2$, may be neglected. Placing this value of $1/r$ in equation (5) we have

$$\frac{e}{mv} = \frac{1}{Hr} = \frac{2z}{Hl^2}. \tag{6}$$

Elimination of $e/m$ between (4) and (6) gives

$$v = \frac{zE}{Hd}.$$

Replacing $E$ by its value given in (1) we get, after simplification,

$$v = \frac{zPD \times 10^8}{Hd(d + 2t - t/K)}. \tag{7}$$

Again, multiplying equations (6) and (7) gives

$$\frac{e}{m} = \frac{z^2PD \times 2 \times 10^8}{H^2l^2d(d + 2t - t/K)}. \tag{8}$$

## The Electrostatic Field.

The two electrostatic plates were rectangular brass plates $7.5 \times 15 \times 1$ cm. Considerable difficulty was experienced in getting the two plates sufficiently plane. The plates were first planed and then finished by "spotting" on a master plate. A slip of soft iron $5 \times 1.5 \times .15$ cm. was inlaid in the upper plate, as shown in Fig. 2, and the plate again surfaced. Several days were required to surface the plates but they were finally finished sufficiently plane that one would raise the other from the table.

A second slip of soft iron was cut out $5 \times 1.5 \times .1$ cm. and one side made plane. A scratch .005 cm. in width and of about the same depth was drawn full length on this surfaced side. This scratch formed the tube through which the cathode rays passed. The iron slip with the scratch was held against the iron slip inlaid in the upper electrostatic plate by ten brass screws. On account of the small diameter of the scratch and its relatively large length it was subsequently found to be easier to make a scratch of about .05 cm. in diameter, close each end with a small bit of solder, cut off the solder flush with the iron surface and then make a small scratch in the bit of solder at each end. A scratch .1 cm. long at each end was found to give perfect satisfaction, and not nearly so much difficulty was experienced in getting the beam to pass through this tube. In adjusting the cathode to send a beam through the tube the electrostatic plates were first mounted in position with the scratch the full .05 cm. diameter. The vessel was exhausted and a potential difference of about 20 volts applied to the electrostatic plates. The wax joint where the glass tube supporting the cathode entered the plate glass end was then softened by heating and the cathode moved about until a phosphorescent spot on the willemite screen, deposited on the opposite glass end plate, showed the presence of the beam. The wax was allowed to cool while the cathode was in the position giving this spot its maximum brightness. The electrostatic plates were then removed by taking out the screws holding the ebonite disc, $B$, to the brass ring, $D$, and the tube made smaller by the bits of solder mentioned above. The plates were then replaced in position and the vessel exhausted. If the spot failed to show on the willemite screen the process was repeated until finally the beam was made to pass through the small tube.

An iron tube, $I$, .5 cm. diameter and 2 cm. long, was screwed into the disc, $B$, to shield the rays from any magnetic effect before entering the confining tube. The cathode was within 1 cm. of the tube $I$.

The electrostatic plates were spaced by four hollow ebonite cylinders, one placed at each corner, and clamped in position by ebonite bolts passing through the cylinders. The length of these cylinders was measured by a micrometer caliper reading to .001 cm. The cylinder was placed between two thin glass plates and the length of the whole measured. The thickness of the plates was then subtracted. Each cylinder was measured on several successive days and the mean of these measurements was taken as the length. When the cylinders were again measured, after having been in the apparatus under pressure for four months, they were found to have shortened by about 1 per cent. All data was taken during the first fifteen days, however, so no correction was made for this change in length. The potential difference of the electrostatic plates was determined as follows: A high potential storage battery, *T*, was used in sending a small current through the two high resistances, *M* and *R*, as shown in Fig. 4. *M* was a resistance of about $2 \times 10^6$ ohms while *R* was an adjustable resistance of about 10,000 ohms. The electrostatic plates were connected directly to the terminals of *M* as shown. By adjusting the value of *R* the potential difference of the terminals of *M* could be kept constant. The potential drop through a small part of *M* was measured by a potentiometer, *P*, against a Weston standard cell of 1.0185 volts at 24° C. The potential difference of the electrostatic plates was thus easily measured to .1 per cent. and by means of *R* the value was kept constant to within .1 volt.

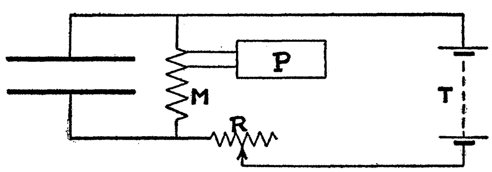

Fig. 4.

## The Magnetic Field.

The magnetic field was furnished by a solenoid of 648 turns and 160.2 cm. length. The solenoid was built in two parts and made to join closely at the middle so as to enclose the whole tube. The length of the solenoid was such that the field could be considered uniform and calculated. From the dimensions of the solenoid the strength of the magnetic field at its center was given by

$$H = 5.083\,I,$$

where *I* is the strength of the current in amperes. The current for the magnetic field was supplied by storage cells of 40 amperes capacity. The current, which varied between .5 and 1.5 amperes, was measured by a Siemens & Halske ammeter reading to .005 amperes.

## Results.

In placing the photographic plate in the apparatus for exposure the plate was placed solidly against the ebonite disc, *B*. The iron confining tube for the cathode beam was 5.08 cm. long and hence a line drawn


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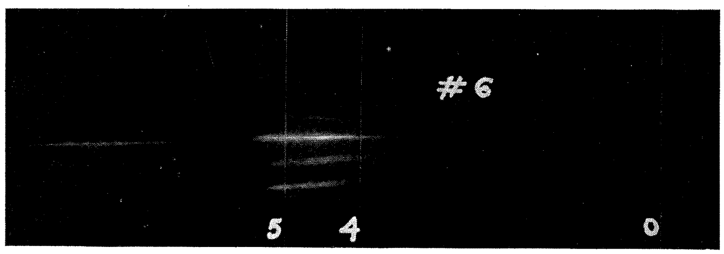


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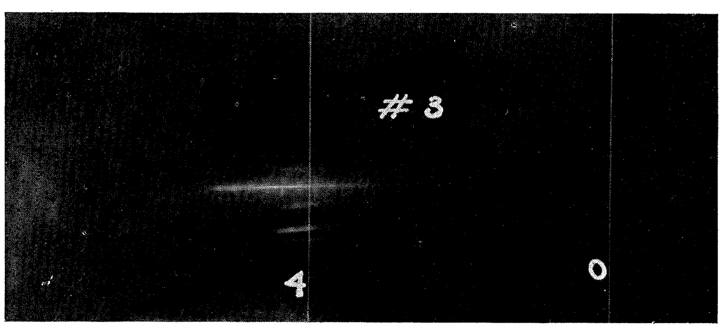


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Fig. 5.

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across the plate 5.08 cm. from the end that touched the ebonite disc established the zero. This line, marked $O$ in the photographs, was then directly under the opening of the tube. The length of horizontal travel, $l$, was measured from this line. In photograph 6 two calculations of $e/m$ were made, where the distance $l$ was 4 and 5 cm. respectively. In each photograph the long streamer, second from the top, is the central one, given by zero magnetic field. The two spots immediately on either side are for the magnetic deflection, direct and reversed. The additional spots seen have no significance relative to the value of $e/m$. The magnetic deflections were accurately measured along the lines drawn parallel to the line marked $O$. The reproductions in Fig. 5 are full size. Twenty photographs were taken in succession. Table I. gives the data relative to all these.

Table I.

| Plate No. | $I$. | $PD$. | $z$. | $l$. | $d$. | $t$. | $d+2t-\frac{t}{K}$. | $v \times 10^{-9}$. | $\frac{e}{m} \times 10^{-7}$. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .460 | 524.0 | .138 | 4.0 | .835 | .165 | 1.292 | 2.866 | 2.114 |
| 2 | .450 | 564.4 | .127 | 4.0 | .820 | .180 | 1.210 | 3.158 | 2.192 |
| 3 | .890 | 498.1 | .245 | 4.0 | .825 | .175 | 1.204 | 2.715 | 1.838 |
| 4 | .8902 | 639.7 | .220 | 4.0 | .830 | .160 | 1.177 | 3.183 | 1.935 |
| 6 | .885 | 425.0 | .251 | 4.0 | .811 | .179 | 1.199 | 2.438 | 1.701 |
| 6*a* | .885 | 425.0 | .3116 | 5.0 | .811 | .179 | 1.199 | 3.027 | 1.678 |
| 7*a* | .850 | 323.5 | .325 | 5.0 | .805 | .185 | 1.206 | 2.506 | 1.508 |
| 7*b* | .850 | 323.5 | .371 | 5.5 | .805 | .185 | 1.206 | 2.861 | 1.624 |
| 7*c* | .850 | 323.5 | .397 | 6.0 | .805 | .185 | 1.206 | 3.061 | 1.563 |
| 8*a* | .8725 | 323.0 | .314 | 4.5 | .805 | .185 | 1.206 | 2.355 | 1.647 |
| 8*b* | .8725 | 323.0 | .372 | 5.5 | .805 | .185 | 1.206 | 2.790 | 1.547 |
| 9 | .879 | 320.3 | .335 | 5.0 | .810 | .180 | 1.200 | 2.470 | 1.482 |
| 10*a* | .864 | 301.0 | .389 | 5.0 | .825 | .165 | 1.182 | 2.734 | 1.937 |
| 10*b* | .864 | 301.0 | .423 | 6.0 | .825 | .165 | 1.182 | 2.973 | 1.591 |
| 11 | .867 | 299.0 | .377 | 5.0 | .826 | .164 | 1.181 | 2.622 | 1.807 |
| 12*a* | .873 | 298.0 | .405 | 5.0 | .826 | .164 | 1.181 | 2.788 | 2.036 |
| 12*b* | .873 | 298.0 | .461 | 6.0 | .826 | .164 | 1.181 | 3.173 | 1.832 |
| 13 | .872 | 298.0 | .382 | 5.0 | .824 | .166 | 1.184 | 2.632 | 1.815 |
| 14*a* | .874 | 284.2 | .4335 | 5.5 | .819 | .171 | 1.189 | 2.947 | 1.837 |
| 14*b* | .874 | 284.2 | .499 | 6.5 | .819 | .171 | 1.189 | 3.278 | 1.735 |
| 15 | .881 | 247.2 | .467 | 6.0 | .817 | .173 | 1.192 | 2.647 | 1.533 |
| 17 | .450 | 248.6 | | | | | | | |
| 18 | .453 | 246.6 | | | | | | | |
| 19 | .4515 | 245.3 | | | | | | | |
| 20 | 1.289 | 296.7 | .558 | 5.3 | .850 | .140 | 1.153 | 2.578 | 1.563 |
| Average | | | | | | | | | 1.748 |

The value of the dielectric constant of the glass plate was that given by Landolt and Börnstein for "spiegel glas." If the value of $K$ was taken as either 5 or 7 instead of 6 the resulting value of $e/m$ is changed by only about .5 per cent. The probable error of the final result, calculated in the usual way from the data in Table I., is 1.5 per cent.


## Summary

The method devised for the determination of $e/m$ and $v$ for cathode rays from a cold cathode is a modification of the usual electrostatic and magnetic deflection photographic method. It has two distinct advantages.

1. Both the electrostatic and magnetic fields are uniform over the entire path of the deflected cathode beam.

2. The electrostatic deflection is kept constant for all strengths of fields employed and thus the inaccuracy in its measurement is eliminated.

The mean of twenty successive photographs gave

$$e/m = 1.75 \pm .03 \times 10^7.$$


I wish to express my appreciation to Dr. C. T. Knipp for his kindly suggestions and to Professor A. P. Carman, Director of the Laboratory, for the facilities offered.

Laboratory of Physics,
University of Illinois,
January 20, 1914.


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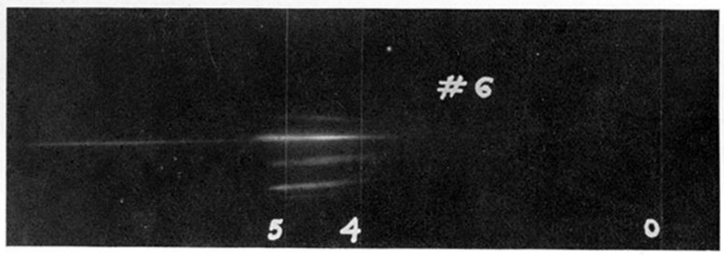


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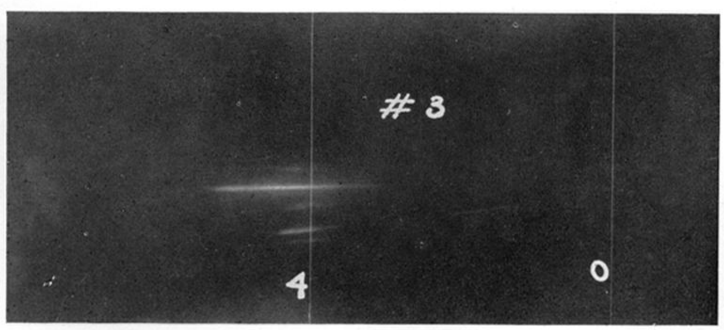


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Fig. 5.