# CALCULATION OF A DAMPING RECTANGLE TO PRODUCE CRITICAL DAMPING IN A MOVING COIL GALVANOMETER.

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IF a moving coil galvanometer is used ballistically, it is ordinarily desirable to have the coefficient of damping small, principally because much damping reduces the throw produced by a given quantity of electricity; there are, however, certain conditions under which it is expedient to have the instrument critically damped. When the galvanometer is to be used for steady deflection work it is often not only expedient but imperative to have the motion just aperiodic. The advantages of critical damping have been discussed by O. M. Stewart[1] and Jaeger.[2]

The usual procedure in changing from a ballistic to a deflection galvanometer is to increase the damping to aperiodicity by attaching to the coil a closed wire rectangle, usually of the same mean area as the coil. However, since a certain coil may be used with different kinds and lengths of suspension fibers and, furthermore, since there is considerable initial damping when the galvanometer is used on a closed circuit of high resistance, the damping rectangle usually furnished by the manufacturers may or may not make the coil just aperiodic. In most cases the boundary condition between periodic and aperiodic motion is not realized. A method is given in this paper by means of which the proper diameter of a wire for a damping rectangle may be found in order that, for given conditions, the galvanometer may be critically damped.

We shall assume at the outset that the magnetic field in which the coil is suspended is uniform, *i. e.*, that the damping rectangle which is to be attached to the face of the coil will swing in a field of the same strength as does the "mean" turn of the coil. In a previous paper[3] the author

[1] PHYS. REV., 16, O.S., 1903, p. 158.

[2] Ztschr. f. Instrumentenk., 23, 1903, pp. 261 and 353.

[3] Klopsteg, PHYS. REV., 2, N.S., 1913, p. 390. After the publication of the paper here cited, and after the present paper had been submitted for publication, my attention was called by Dr. I. Fröhlich of Budapest to his paper, which I had overlooked, appearing in "A Mathematikai és Természettudományi Értesítö", Vol. VII, 1889, p. 44, under the title, "Zárt elektromos vezetök lengése homogén mágnesi térben", dealing with the vibrations of a closed conductor in a homogeneous magnetic field. Dr. Fröhlich develops an equation

developed an equation for field intensity, based upon critical damping, of the form

$$H = \frac{2\sqrt{10^9 I_0}}{nld}\sqrt{\frac{R_0}{T_0}(\pi - \lambda)}, \tag{1}$$

in which $I_0$, $n$ and $ld$ are, respectively, the moment of inertia, number of turns and mean area of the coil, $T_0$ is its period and $\lambda$ its logarithmic decrement on open circuit, while $R_0$ is the total resistance in ohms of the circuit containing the galvanometer. The resistance $R_0$ is such that damping is critical. If in this equation we place $n = 1$ and solve for $R_0$, the result will be the resistance of a single closed turn of wire such that the induced current in it due to the cutting of the field may be of proper magnitude to bring about the desired effect. The equation becomes

$$R_0 = \frac{(Hld)^2 T_0}{4 \times 10^9 I_0(\pi - \lambda)}. \tag{2}$$

The damping rectangle having the same mean area as the coil, the total length of wire will be $2(l + d)$; the diameter of the wire is then, in centimeters,

$$D = \sqrt{\frac{8\rho(l + d)}{\pi R_0}}, \tag{3}$$

$\rho$ being the specific resistance of the metal from which the wire is drawn. Combining (2) and (3),

$$D = \frac{1}{Hld}\sqrt{\frac{3.2 \times 10^{10}\rho I_0(l + d)(\pi - \lambda)}{\pi T_0}}. \tag{4}$$

In galvanometer fields of the type represented in Fig. 1, with the usual

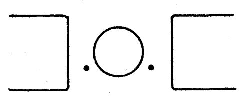

Fig. 1.

which gives the relationship between the intensity of the field and quantities to be obtained by measurement or calculation from the closed conductor. Briefly, his method is based upon the difference in the periods of the closed conductor when vibrating in zero field and in the field under investigation, whereas my formula makes use of the difference in the logarithmic decrements when the coil vibrates on open and closed circuits, respectively. In my paper (p. 392, eq. (10)) it was shown that the difference in periods corresponding to any two logarithmic decrements is very small, which fact alone would make the method of Dr. Fröhlich impracticable; on the other hand, the comparatively large differences in the logarithmic decrement are readily measurable with great accuracy. Another objection to using a closed conductor is the inflexibility of the method, on account of the fixed resistance of that conductor. For the best results by such a method, the damping must be great enough to make an appreciable difference in the period, but not so great as to make the vibrations die out rapidly. This means, in its practical application, that any one closed conductor could be used over a very limited range only; whereas, in the description of the damped coil method, it was shown that it is entirely practicable to use the same coil for measurements of a few gausses up to several thousand. Dr. Fröhlich gives no experimental data. Had he tried his method he would have been at once confronted by the above mentioned limitations.

type of coil used in such fields, the vertical wires of the rectangle are approximately in the position shown by the dots in the figure when the coil is undeflected. The damped coil method[1] of determining $H$ for equation (4) gives as its value the field intensity at the position of the "mean" turn of the coil at rest. Evidently the value so obtained is higher than the field strength at the position of the rectangle, consequently equation (4) will give a value of $D$ which is too small. To correct for this difference, the field was measured in several galvanometers of this type both at the null position of the coil and at the position of the rectangle. The ratio between these two values was found[2] to be 1.30. Usually the damping rectangle is made of copper wire; in this case $\rho = 1.69 \times 10^{-6}$ (15° C.); introducing this value and the ratio 1.30 into equation (4) it becomes

$$D = \frac{170}{Hld}\sqrt{\frac{I_0}{T_0}(l+d)(\pi-\lambda)}. \tag{5}$$

To apply formula (5), $H$ is determined by the method mentioned above, $l$ and $d$ are readily found by means of a stage microscope, while $\lambda$ and $T_0$ are obtained in the usual manner. It should be observed that $\lambda$ is not necessarily the logarithmic decrement on *open* circuit; when, for example, the galvanometer is used on a closed circuit of such resistance that damping is not critical, $\lambda$ is the logarithmic decrement for this special condition. $I_0$ is found by the method of comparing the periods of the coil and of a disk of known moment of inertia, each in turn suspended from the same fiber. The value of $D$ having been calculated, the wire gauge corresponding most nearly with this value is selected for the rectangle.

The addition of the mass of the rectangle to that of the coil has the effect of increasing to some extent the values of both $I_0$ and $T_0$. It is unnecessary to make a second approximation because of the fact that the value of the fraction $I_0/T_0$ is changed only slightly and that this fraction is under the radical sign, the percentage change being thereby halved. Practically, one is also limited to the standard wire gauges, which admits of some latitude in the value of $D$. To determine the effect of the added mass of the rectangle, an experiment was tried, using an

[1] Loc. cit.

[2] For any particular type of field or depth of coil (distance from face to back) the ratio between field strengths may be determined by the damped coil method, the equation for the ratio being

$$\frac{H}{H'} = \sqrt{\frac{T_0'}{T_0}\frac{\Lambda-\lambda}{\Lambda'-\lambda'}}$$

when the same value of $R$ is used in both cases. The value 1.30 is understood to refer to the case only which is discussed above.

"open" rectangle of the wire gauge given by one calculation, and redetermining the quantities appearing in the equation. There was a slight change in the new value of $D$, but not sufficient to necessitate using a different wire gauge. The size found by the first calculation produced damping as nearly critical as could be judged.

When the number of turns in the galvanometer coil is known, a formula more simple in its application—one which involves the determination of neither $H$ nor $I_0$—may be found by making use of the equation[1]

$$H = \frac{2\sqrt{10^9 I_0}}{nld}\sqrt{\frac{R}{T_0}(\Lambda - \lambda_0)}, \tag{6}$$

where $\Lambda$ is the logarithmic decrement corresponding to a total resistance $R$ in the circuit which is not sufficiently small to make damping critical, and $\lambda_0$ is the logarithmic decrement on open circuit. The other quantities are defined as before. From this equation and (2) we find

$$R_0 = \frac{R}{n^2}\frac{\Lambda - \lambda_0}{\pi - \lambda}, \tag{7}$$

which equation, when combined with (3) gives

$$D = \sqrt{\frac{8\rho n^2(l + d)(\pi - \lambda_0)}{\pi R(\Lambda - \lambda_0)}}. \tag{8}$$

Introducing the constant 1.30, explained in connection with (5), and substituting for $\rho$ the value before given, we find that for a copper wire damper

$$D = 0.00269n\sqrt{\frac{(l + d)(\pi - \lambda)}{R(\Lambda - \lambda_0)}}. \tag{9}$$

Evidently, when the galvanometer is used in open circuit work, $\lambda_0$ and $\lambda$ of equations (8) and (9) are equal; when the coil is on closed circuit, $\lambda$ is, as before, the logarithmic decrement for this condition, while $\lambda_0$ remains the logarithmic decrement on open circuit.

The results obtained by equations (5) and (9) were found in good agreement in all cases tried, which included several types of moving coil galvanometers and three kinds of upper suspensions, viz.: 3-mil and 1.5-mil phosphor bronze strip and 1.5-mil phosphor bronze wire.

## SUMMARY.

In brief, this paper gives the development of two equations for finding the proper size of wire to be used in damping rectangles in order that critical damping may be obtained in moving coil galvanometers. The

[1] Loc. cit.

first equation is used when the number of turns in the coil cannot be ascertained, the second when this number is known.

The suggestion is made, in conclusion, that manufacturers be requested to furnish with their galvanometer coils the values of $n$, $l$, $d$ and $I_0$. Such data would minimize the number of quantities to be determined by the experimenter in measuring fields by the damped coil method, and would greatly simplify the obtaining of data for calculations such as are mentioned in this paper.

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