THE THEORY OF GALVANOMAGNETIC PHENOMENA IN FLAMES

By C. W. Heaps

Abstract

When positive and negative ions are assumed to move through flame gases under the influence of mutually perpendicular electric and magnetic fields and a retarding force of the nature of viscous resistance, the expression derived for magneto-resistance is found not to agree with experiment. It is then assumed that the Hall e.m.f. produced by the deflection of negative ions exerts a dragging effect on the positive ions, which in turn exert a drag on the upward moving flame molecules and thus alter the upward velocity of the flame gases. Allowing for this change of flame velocity, *equations for the Hall effect and for the magneto-resistance effect* are obtained which are in accord with experiment. The Hall effect is shown to be proportional to the difference of mobilities of positive and negative ions (k_1-k_2) divided by $(1+dR/R)$, where dR/R is the magneto-resistance effect.

THE theory of galvanomagnetic phenomena in flames has been worked out by H. A. Wilson[1] and by E. Marx[2]. The expression given by Wilson for the Hall effect is $Y=-k_2H(X-HV)$, where Y is the vertical potential gradient produced in a flame by a magnetic field H normal to Y and to the horizontal potential gradient X. Here k_2 is the mobility of the negative ion and V is the upward velocity of the flame gases. Values of k_2 determined by using this equation range around 20 m per sec. for one volt per cm. Other methods of obtaining k_2 give considerably higher values, ranging from 60 to 200 m per sec. The expression deduced by Wilson[3] for the magneto-resistance of a flame is $dR/R=H^2k_2^2/12-HV/X$. Here dR is the change produced in R, the resistance of the flame, by the magnetic field H acting normally to X and V. This equation, to agree with experiment, requires a value of V about 100 times as large as the value actually observed. It appears that Wilson's theory is inadequate. Marx has criticized Wilson's methods and derived a different equation, but his theory is no more satisfactory in its results than Wilson's. Marx also, in his deductions, neglects the effect of the Hall electromotive force on the magneto-resistance, a neglect which vitally affects his conclusions.

The theory outlined below is somewhat more general than the theories of Wilson and Marx. Also, an expression for the magneto-resistance of a flame is derived which does not conflict with experiment.

[1] H. A. Wilson, Electrical Properties of Flames, p. 113
[2] E. Marx, Handbuch der Radiologie, p. 749
[3] H. A. Wilson, opus cit.[1], p. 117

Let axes be chosen in a long flat flame so that H is along the z axis perpendicular to the flame, X is along the horizontal x axis in the plane of the flame, and Y and V are along the vertical y axis. Let u_1 and v_1 be the drift velocities of the positive ions along the x and y axes, respectively, under the influence of the electric fields. The corresponding drift velocities of the negative ions are represented by u_2 and v_2. Also let k_1 be the mobility of the positive ions. In the steady state we may set the sum of all forces on an ion in any one direction equal to zero. Hence

$$Xe+Hev_1-eu_1/k_1=0\ ; \tag{1}$$

$$Ye-Heu_1-e(v_1-V)/k_1=0\ ; \tag{2}$$

$$-Ye+Heu_2-e(v_2-V)/k_2=0\ ; \tag{3}$$

$$-Xe-Hev_2-eu_2/k_2=0\ . \tag{4}$$

These equations imply a viscous resistance to the motion of ions through the flame gases. Solving we obtain

$$u_1=\frac{k_1(X+HV+k_1HY)}{1+H^2k_1^2}\ ; \tag{5}$$

$$v_1=\frac{V+k_1(Y-k_1HX)}{1+H^2k_1^2}\ ; \tag{6}$$

$$u_2=\frac{-k_2(X+HV-k_2HY)}{1+H^2k_2^2}\ ; \tag{7}$$

$$v_2=\frac{V-k_2(Y+k_2HX)}{1+H^2k_2^2}\ . \tag{8}$$

These equations are the same as those given by Townsend's theory[4] if we let $k_1=T_1e/m_1$ and $k_2=T_2e/m_2$, the subscripts 1 and 2 referring to the positive and negative ions, respectively, and T representing the free period of an ion.

If the top and bottom of the flame are insulated so that no vertical current can flow we have $n_1v_1=n_2v_2$, where n_1 and n_2 are the respective densities of positive and negative ions. If there is no free charge in the flame $n_1=n_2$, so that $v_1=v_2$. Equating the two expressions for v_1 and v_2 above and solving we get

$$\frac{Y}{HX}=(k_1-k_2)\frac{1+VH/X}{1+H^2k_1k_2}\ . \tag{9}$$

[4] Townsend, Electricity in Gases, p. 100

This is the expression for the Hall effect. Usually the terms VH/X and $H^2k_1k_2$ will be very small.

The expression for the resistance change due to the magnetic field is now easily obtained. Substituting in Eq. (7) the value of Y given by (9), we obtain

$$u_2 = \frac{-k_2(X+HV)}{1+H^2k_1k_2}. \tag{10}$$

The mobility of the positive ion in a flame is much less than that of the negative ion so it is usual to assume that practically all the current is carried by negative ions. Hence if I is the current density and R the specific resistance of the flame, we have $I=X/R=n_2eu_2$. Thus $1/R=n_2ek_2(1+HV/X)/(1+H^2k_1k_2)$.

If R', n_2', k_2' indicate values of the respective quantities when $H=0$, we have $1/R'=n'_2ek_2'$, and

$$\frac{dR}{R'} = \frac{R-R'}{R'} = \frac{n_2'\,k_2'}{n_2k_2}\left(\frac{1+H^2k_1k_2}{1+HV/X}\right)-1\,. \tag{11}$$

If a magnetic field does not change the rate of recombination or ionization we may write $n_2=n_2'$. We may also assume $k_2=k_2'$ for the following reason. k_2 depends on the mass and the mean free period of the ion. The mean free period is the mean free path divided by the velocity of agitation. A magnetic field would convert large numbers of straight free paths into curved paths, but if the mean free path is obtained in the usual way the result is independent of the curvature of the path. Also the velocity of agitation is unchanged by H and hence the free time is unchanged. We would not expect a magnetic field to change the average mass of an ion hence we may assume $k_2=k_2'$. Eq. (11) may then be written in the form

$$\frac{dR}{R'} = \frac{H^2k_1k_2-HV/X}{1+HV/X} \tag{12}$$

The equation fitting the experimental results[3] is $dR/R=3.1\times10^{-9}H^2+1.5\times10^{-5}H$. Neglecting HV/X compared with unity in the denominator of Eq. (12), we may write this equation in the form $dR/R=H^2k_1k_2-HV/X$. Comparing the two equations for dR/R we get $k_1k_2=3.1\times10^{-9}$ and $V/X=1.5\times10^{-5}$. Experiments indicate that to a fair approximation $k_1=10^{-3}k_2$. Hence we get $k_2=176$ m per sec. for one volt per cm. This value is much too large if other experiments are to be relied upon. Also, letting $X=10^9$, the approximate value for which the experimental equation was determined, we find $V=150$ m per sec. This value is about 100 times too large.

The difficulty with the theory appears to lie in the assumption that the upward velocity of the flame gases is constant. If the magnetic field acts on the horizontal current of negative ions with, say, a downward deflecting force, then these negative ions move downward until the Hall e.m.f. is developed. This Hall e.m.f. will urge positive ions in a downward direction and since the positive ions are supposed to be large and numerous it is evident that they will exert a downward drag on the upward moving flame gases. The following forces in the y direction may be supposed to act on the neutral flame molecules in a unit cube: (1) $\partial P/\partial y$, where P is the pressure of the flame gases, (2) $n_1e(v_1-V)/k_1$, representing the viscous resistance to motion of the positive ions through the flame gases, (3) the forces of viscosity tending to retard any relative motion of the flame gases.

In the case of (3) if we think of the air outside the flame as retarding its upward motion much as the walls of a solid tube would retard it, the velocity gradient along z in the flame would be proportional to V. The gradient along x may be assumed zero. The force (3) may therefore be represented by $-AV$, where A is some constant. Hence if $B=n_1e/k_1$ we may write $B(v_1-V)-AV-\partial P/\partial y=0$. Let $V=V_1$ when $v_1=V$. Then $\partial P/\partial y=-AV_1$ and we get $V=Bv_1/(A+B)+AV_1/(A+B)$, or

$$V=Cv_1+V_0 \tag{13}$$

where $C=B/(A+B)$, and $V_0=AV_1/(A+B)$. The term containing C is by no means negligible. J. S. Watt has observed[5] that when a current of about 40 milliamperes is sent through a flame like the one considered above (but into which salt is sprayed), then if H is about 5000 gauss the value of V is made almost zero for one direction of H. If either the current or the field is reversed the flame velocity is much increased. In other words, changing the sign of v_1 very appreciably affects V.

If in Eqs. (6) and (8) we substitute the value of V given by (13), equate v_1 and v_2 as before, and solve the resulting equation, we get

$$\frac{Y}{XH}=(k_1-k_2)\left(\frac{1-C+V_0H/X}{1-C+k_1k_2H^2}\right). \tag{14}$$

The terms V_0H/X and $k_1k_2H^2$ are very small, of the order 10^{-3} or 10^{-4} for ordinary magnetic fields. Hence, unless C is nearly unity its presence will produce very little effect on the value of Y/XH. If $1-C$ is small enough the value of Y/XH should vary slightly with the direction of H. But C will not be comparable with unity unless B is large compared with

[5] J. S. Watt's paper will follow this in the issue for January 1925.

A. A large value of n_1 is associated with a large value of B, so that for strong ionization there should be an asymmetry in the Hall effect. Watt has observed in a strongly salted flame that the Hall effect may be greater for one direction of the field than for the other.

The expression (12) for the magneto-resistance of the flame will now require modification. From Eqs. (3), (4), (13), and (14), remembering that $v_1 = v_2$, we get

$$u_2 = -k_2\left(\frac{(1-C)X+HV_0}{1-C+k_1k_2H^2}\right) . \tag{15}$$

Assuming as before that $n_2 = n_2'$ and $k_2 = k_2'$, we get

$$\frac{dR}{R} = \frac{H^2k_1k_2 - HV_0/X}{1-C+HV_0/X} . \tag{16}$$

To compare this equation with experiment we may substitute the following values for the quantities on the right hand side: $k_1 = 2\times 10^{-7}$ cm per sec., $k_2 = 10^{-4}$ cm per sec., $X = 10^9$, $V_0 = 100$ cm per sec. The value assumed for V_0 is of course apt to be very much in error. We can only be sure that it is less than 200 cm per sec., the velocity which has been found for freely moving flame gases. The value of X is roughly that used by Wilson in his experiment. The numbers assigned to k_1 and k_2 are what other experiments have indicated to be about correct. If now we arbitrarily take $1-C = 0.006$ and plot Eq. (16) the resulting curve is approximately coincident with the curve drawn by Wilson to represent his experimental results.

It appears, therefore, that C is very nearly unity in the case of a Bunsen flame such as Wilson used. Nevertheless, using the values of k_1 and k_2 given above and taking $H = 5000$, the quantity $1-C$ in Eq. (14) is about 10 times larger than $k_1k_2H^2$. It is also perhaps 10 times larger than V_0H/X so that to a fairly close approximation the expression Y/XH gives the value of $k_1 - k_2$.

The accurate expression for the Hall effect may be written in a more convenient form for experimental work than the form of Eq. (14). From (16) we have

$$1-C = \frac{Rk_1k_2H^2}{dR} - \frac{HV_0R}{XdR} - \frac{HV_0}{X} .$$

Substituting this value of $1-C$ in Eq. (14) and simplifying gives

$$\frac{Y}{XH}\left(1+\frac{dR}{R}\right) = k_1 - k_2 . \tag{17}$$

This relationship is independent of C and all the quantities on the left hand side are easily determined experimentally. In calculating $k_1 - k_2$ the value of dR/R must of course be obtained for the same direction of H and X as the value of Y.

The conclusion is, then, that the upward velocity of the flame gases is an important factor in the theory of galvanomagnetic phenomena in flames. This upward velocity may be materially changed by the electrodynamic action of the magnetic field on a current in the flame, but if this effect of the field is allowed for, the resulting equation for magneto-resistance agrees with experiment. The equation for the Hall effect is found to contain a term depending on magneto-resistance.

The Rice Institute,
Houston, Texas.
August 18, 1924.