THE MAGNETIC SUSCEPTIBILITY OF GASES.[1]

BY W. P. ROOP.

1. The explanation of the magnetic properties of substances in terms of electronic motions is the objective point of the physical theory of magnetic phenomena. Recent work has been done in developing a theory of ferro-magnetic phenomena by Weiss and in investigating the magnetic properties of organic compounds by Pascal. In each of these cases, however, the results have been useful principally as determining susceptibility as a function of structural relations rather than as a fundamental property of matter. On the other hand, the theory of Langevin and the magneton hypothesis of Weiss,[2] supported by work at low temperatures in the Leiden laboratory, look toward a more general theory of magnetism, so that magnetic phenomena may be expected eventually to occupy a place in the *electro-magnetisches Weltbild* of Abraham.

It is unfortunate, however, that experimental work has been of necessity almost entirely limited to investigations of the magnetic properties of solids, liquids, and solutions. For it is only in gases that the molecule is sufficiently free from the disturbing influences of its neighbors so that a direct study of molecular properties, as distinguished from structural properties, becomes possible.

2. The study of the magnetic properties of gases is difficult on account of the extreme smallness of the quantities to be measured. This difficulty becomes apparent in the large differences between results obtained by different observers. Oxygen is the only gas whose susceptibility has been measured with sufficient accuracy to justify a comparison of results. Data are also available for air and NO, but the behavior of these is due to the oxygen content. With other gases, there is no agreement, even as to sign. A partial summary of values for the susceptibility of oxygen is given on the following page.

3. Before systematic study of the magnetic properties of gases can be expected to yield valid results, a method must be devised which shall be

[1] A preliminary report of work conducted under a grant from the Rumford fund of the American Academy of Arts and Sciences.

[2] For a summary of the work of Langevin and Weiss, with references to the papers of the others, see Phys. Ztschrft., 12, 935, 1911.

Observer.	Mean Value.	Mean Error.	Reference.
Quincke....	$.156 \times 10^{-6}$	4.5 per cent.	Ann. d. Ph., 34, 401, 1888.
DuBois.....	.117	.....	Ann. d. Ph., 35, 137, 1888.
Hennig.....	.120	1.6 per cent.	Ann. d. Ph., 50, 485, 1893.
Curie.....	.167	3.5 per cent.	Ann. Ch. Ph., 5, 289, 1895.
Roop.....	.127	5 per cent.	Phys. Ztschrft., 12, 48, 1911.
Piccard.....	.1407	0.1 per cent.	Arch. des Sciences, 35, 480, 1913.

sensitive enough to be used with other gases than oxygen, and which shall give consistent results with oxygen. A suggestion is given by Drude.[1] A manometer consisting of a closed tube containing two fluids (see Fig. 1) becomes very sensitive if the fluids have a small density difference. For ordinary purposes, such a manometer could not be used, since there would be no way of connecting it with the pressure to be measured. But if one of the bounding surfaces be placed in a magnetic field, the surface is displaced into a new equilibrium position such that the hydrostatic pressure due to the displacement is balanced by the pressure due to the magnetic field. The pressure on the bounding surfaces tending to restore them to their equilibrium positions, for a displacement d, is $dg(\rho - \rho_0)$, where ρ and ρ_0 are the respective densities of the fluids. Since there are two such surfaces, each of which is displaced by the same amount, the total pressure tending to restore equilibrium is $2dg(\rho - \rho_0)$. The magnetic pressure tending to disturb the hydrostatic equilibrium is $(\kappa - \kappa_0)/2 \cdot H^2$, where κ and κ_0 are the respective susceptibilities of the two fluids. Equating these pressures, and solving,

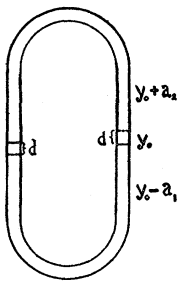

Fig. 1.

$$\kappa - \kappa_0 = \frac{4dg(\rho - \rho_0)}{H^2}.$$

In case the magnetic field intensity H at the other bounding surface is not negligible, this becomes

$$\kappa - \kappa_0 = \frac{4dg(\rho - \rho_0)}{H^2 - H_0^2}.$$

4. Now at the boundary between two gases, there is, of course, no sharp discontinuity. The suggestion of Drude, that a film of liquid be used to separate the gases, seems impracticable. It is possible, however, to carry out the experiment in such a way that the gases only remain in contact with each other for such a short time that diffusion introduces no serious errors.

[1] Physik des Aethers, 1st ed., p. 148.

Consider the effect of diffusion on a boundary initially sharp. In a straight tube, of uniform bore, the diffusion will be symmetrical, so that the section of the gas column originally constituting the boundary remains the section at which the concentrations of the two gases are equal. In what follows we will designate this section as the boundary. On one side the concentration of one gas, and on the other side the concentration of the other gas, will drop off more or less gradually to zero. Let Y be the coördinate parallel to the axis of the tube. Suppose that the diffusion at a given time t has made the mixture of the gases appreciable to a distance a_1 on one side and a_2 on the other side of the boundary at y_0. The density and susceptibility will vary continuously from $y = y_0 - a_1$ to $y = y_0 + a_2$.

On account of the symmetry of the manometer the hydrostatic force caused by the bodily displacement of both gases, together with the region within which diffusion has occurred, up d cm. on one side, and down the same distance on the other, will be the same as if the boundaries were true discontinuities, provided that the diffusion results in the same distribution of gas on one side as on the other, and does not extend beyond the straight parts of the tube.

As regards the magnetic forces, consider the region between $y_0 - a_1$ and $y_0 + a_2$ to be filled with layers of gas mixture of uniform composition enclosed between cross-sections of the tube at intervals of Δy, differing from each other in composition and susceptibility by differential increments. Denote the concentration of one of the gases at point y by c_y. That of the other is then $1 - c_y$. If susceptibility is an additive function in mixtures, the susceptibility of the mixture at y is $\kappa \cdot c_y + \kappa_0(1 - c_y)$. The susceptibility difference between two neighboring layers is therefore $(\kappa - \kappa_0)\Delta c_y$. The magnetic pressure now becomes

$$\frac{\kappa - \kappa_0}{2} \int_{y_0 - a_1}^{y_0 + a_2} H^2 \cdot \frac{dc_y}{dy} dy.$$

Or, if the limits of integration lie within the region over which H has a sensibly uniform value, we have

$$\frac{\kappa - \kappa_0}{2} H^2 \int_{y_0 - a_1}^{y_0 + a_2} \frac{dc_y}{dy} dy.$$

Since the integral gives the concentration difference between the points $y + a_2$ and $y - a_1$, which is unity, the expression for magnetic pressure reduces to that previously given. The conditions, therefore, under which the simple expression for susceptibility is unchanged by diffusion are these:

(1) Diffusion same on two sides of manometer.
(2) Diffusion limited to straight part of tubes.
(3) Diffusion limited to uniform part of field.
(4) Susceptibility of mixtures additive.

5. Of these conditions, (3) is the most questionable. We may determine approximately the extent to which it is met in the following way. Consider the diffusion in a closed tube of bore negligible relative to the dimensions of the manometer. In such a tube it seems justifiable to assume that diffusion proceeds in such a way as not to disturb the uniformity of concentration in any right section of the tube, so that the concentration gradient remains everywhere parallel to the axis of the tube. Let s be the coördinate measured along this axis. The diffusion proceeds subject to the condition

$$\frac{\partial c}{\partial t} = k\frac{\partial^2 c}{\partial s^2},$$

c being concentration of one of the gases, and k the diffusion constant.

Suppose the tube to be half filled with each of two different gases in such a way that the two boundaries separating the two gases are initially sharp. Consider the origin of the coördinate s to be located half-way between the two boundaries. The initial distribution may then be described as follows:

When $t = 0$,

$$c = f_0(s) = 0 \quad \text{where} \quad -l < s < -\frac{l}{2},$$

$$= 1 \quad \text{where} \quad -\frac{l}{2} < s < +\frac{l}{2},$$

$$= 0 \quad \text{where} \quad +\frac{l}{2} < s < +l.$$

Here $2l$ is the entire length of the manometer tube, measured along its axis. Further, for all values of t, at the boundaries, where

$$s = \pm\frac{l}{2}, \quad c = \frac{1}{2}.$$

The Fourier series[1]

$$c = q + \Sigma p_n e^{-k\frac{\pi^2 n^2}{l^2}t} \cos\frac{n\pi}{l}s$$

satisfies all conditions if

$$q = \frac{1}{2}$$

[1] Cf. H. A. Lorentz, Differential- u. Integralrechnung.

and

$$p_n = \frac{1}{l}\int_{-l}^{+l} f_0(s)\cos\frac{n\pi}{l}\,sds$$

$$= (-1)^{\frac{n-1}{2}}\cdot\frac{2}{n\pi} \quad \text{when } n \text{ is odd,}$$

or

$$= 0 \quad \text{when } n \text{ is even.}$$

Since the conditions named, viz., the boundary conditions, the initial conditions, and the condition expressed by the differential equation, determine physically the progress of the diffusion, and since the function represented by the Fourier series satisfies all these conditions, this function must give the true relation existing between the concentration, time, and distribution along coördinate s. Thus we may determine the extent to which diffusion has caused a failure to meet condition (3) at any time t.

6. The method has been tested by the use of a simple apparatus of the form shown in the diagram. The essential part of the apparatus, the gas manometer, is contained between the two three-way cocks, A and B. One arm of the manometer is placed between the poles of a Weiss electro-magnet,[1] with 10 cm. cores. The other arm is at a distance of 3 cm. from the edge of the cylindrical pole-pieces. One side of the manometer is filled with air, the other with CO_2. On turning the cocks A and B so as to connect the two sides of the manometer with each other, the heavier gas settles and fills each side of the manometer in the lower half only, while the upper half of each side is occupied by the lighter gas. If now this process is repeated with the magnet acting on the gases on one side, a different position of equilibrium is reached. The distance between these two equilibrium positions is d. It is determined by forcing the gas out of each side separately into a gas burette by lifting the mercury reservoir, and measuring the volume of air, after having absorbed the CO_2 in the KOH solution with which the burette is filled and operated. The difference between the volume of unabsorbed gas obtained without the field and that obtained with it,

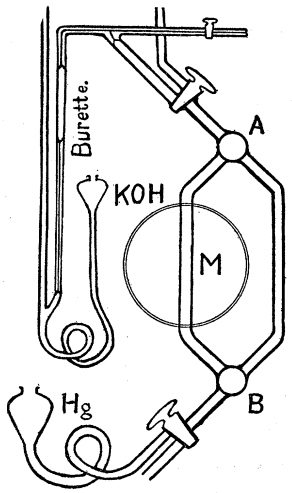

Fig. 2.

[1] A solenoid cannot be used, both on account of temperature disturbances, and because a manometer of excessively large cross-section would be necessary.

divided by the cross section of the manometer tube, gives d. To calculate d, however, it is only necessary to observe the difference in the burette readings in the two cases, and divide by the ratio of the cross-sectional areas of manometer and burette tubes.

7. The total length $2l$ of the manometer was 50 cm. The internal diameter of the manometer tubes was about 4.5 mm., that of the bore of the cocks being 3 mm. The ratio of cross-sectional areas of manometer and burette tubes was found by mercury weighings to be 11.3. It was found to require about 7 seconds after opening the cocks for the boundaries to reach their equilibrium positions. One cock was in each case opened for that length of time, the other, however, was necessarily open a little longer.

The field was determined by means of a bismuth spiral, using the calibration curve furnished by the makers and correcting for the difference (4°) between the temperature of the calibration and that at which the measurements were made. The current necessary was only 2 amperes (capacity of the magnet 20), and the corrected field intensity was 6,220 gausses. The intensity at the other manometer arm, H_0, was 750 gausses. Whence $H^2 - H_0^2 = 38.2 \times 10^6$.

The mean of six observations of burette reading difference is 4.8 cm., whence $d = .425$ cm. This figure leads to the value of 0.030×10^{-6} for the susceptibility difference between air and CO_2, and to this corresponds a value for the susceptibility of oxygen of 0.144×10^{-6}, that of CO_2 being negligibly small.

By substituting the numerical values for the diffusion constant (0.142 C.G.S.) and the dimensions of the apparatus in the equation obtained in paragraph 5, we find that at the end of 7 seconds the air at a distance of 5 cm. from the boundary contains 7 per cent. of CO_2. Doubtless the diffusion somewhat exceeds the calculated amount, on account of the motion of the boundaries from the cocks to their equilibrium positions. Nevertheless the error due to neglecting diffusion altogether must be small, as the result is close to the average of previously obtained values.

SUMMARY.

A new method for determining gas susceptibility is developed which far exceeds previous methods in sensitiveness.

An illustrative set of data taken with air and CO_2 leads to the value for oxygen of

$$\kappa = 0.144 \times 10^{-6}.$$

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